A NUMERICAL SIMULATION AND VISUALIZATION OF BLOOD FLOW THROUGH A STENOSED ARTERY WITH THE DIFFERENT SEVERITY

Siti Zulaiha Binti Abd Rahim & Dr. Norzieha Binti Mustapha

1.1 INTRODUCTION

The cardiovascular system consist of the heart and blood vessels that carrying blood in whole human bodies. The heart is a vital organ in the human body. The heart disease is the most common chronic disease of the coronary arteries is called atherosclerosis. Atherosclerosis occurs when a build-up of plaque or cholesterol deposits on artery walls. Over time, plaque can accumulate, harden and narrow the arteries and impede blood flow to the heart. Coronary artery disease or coronary artery disease (CAD) is the basically can caused heart attacks, strokes, various heart disease including congestive heart failure and most cardiovascular disease in general. A blockage in one or more coronary arteries can cause heart attack suddenly.

In addition, diseased arteries also tend to experience sudden muscle contractions. Thus, a piece of a blood crust can form a contraction, release chemicals which then result in narrowing the artery wall, triggering a heart attack. If the working system of the heart is damage, the normal rhythm of the heart can become chaotic and the heart began to tremble with uncertainty or experiencing fibrillation. This abnormal rhythm known as arrhythmia is a deviation from the normal heart rhythm. This will cause the heart's ability to pump blood effectively to the brain.

1.2 LITERATURE REVIEW

1.2.1 Atherosclerosis

Atherosclerosis is a disease affecting arterial blood vessels (as well as veins that have been surgically moved to function as arteries). A stenosed artery is the result of atherosclerosis that is hardening of the artery due to the growth of a calcified plaque layer on the inner walls of the artery (Layak *et al.*, 2007). Stenosis is defined as a partial occlusion of the vessels caused by abnormal growth of tissues of the deposition of cholesterol as substances on the arterial wall.

Stenosis not only develops in one position of the artery but also it may develop at more than one location of the cardiovascular system. Many researchers such as Misra *et al.*

(2011), Minagar *et al.* (2006) have been carried out a studies to understand the effects of double stenoses on blood circulation in the arteries. However, Ang and Mazumdar (1995) worked on triplet stenoses and their research presented that multiple stenoses have more significant effects on blood flow compared to the sum of the consequences of the individual stenoses.

In fact, the geometry of the stenosis is irregular. Dasgupta *et al.* (2010) observed a few irregular geometries of plaque and the most prevalent one was the Cosine shape geometry. Mustapha *et al.* (2010) carried out a numerical simulation of unsteady blood flow through multi irregular arterial stenoses. In their study, they solved numerically the governing equations with boundary conditions by MAC (Marker and Cell) method and the pressure-Poisson equation has been solved by successive-over-relaxation (SOR). The flow pattern reveals that the separation Reynolds number for the multi-irregular stenoses is lower than those for cosine-shaped stenoses and a long single irregular stenoses, obtained from their study.

1.2.2 Marker and Cell (MAC) Method

The MAC method is a finite difference solution technique for investigating the dynamics of an incompressible viscous fluid.The original Mac method was solved on an Eulerian grid and the free surface was defined by whether a cell contained fluid or did not contained.It was said to contained fluid if the cell contained one or more marker particles and it was defined to be empty if it had no marker particles. Marker and cell accurate resolution of interface (and free surface) by accurately determining the surface normal and curvature, by Sousa *et al.*, 2004.

1.2.3 The Governing Equations

The governing equation for blood flow are the continuity and momentum conservation equations (Sidik (2013)). The governing equations of Newtonian model are stated as follows:

$$r\frac{\partial w}{\partial z} + \frac{\partial (ur)}{\partial r} = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial r} + \frac{\partial (uw)}{\partial z} + \frac{u^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right]$$

$$\frac{\partial w}{\partial t}+\frac{\partial(uw)}{\partial r}+\frac{\partial w^2}{\partial z}+\frac{uw}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right]+G(t)$$

Where $u(r,z,t)$ is the radial velocity component, $w(r,z,t)$ is the axial velocity component. Next, ρ and μ is the density and viscosity of blood respectively. Lastly, $G(t)$ represented as body force. In present study, we consider $G(t)=0$.

The boundary and initial condition are given by:

$u(r,z,t)=0\,, w(r,z,t)=0$

$$u(r,z,t)=0\,,\frac{\partial w(r,z,t)}{\partial r}=0 \qquad \text{for } r=0$$

$u(r,z,t)=2(1-\frac{r}{R^2}),w(r,z,t)=0$ for $z=0$

$$\frac{\partial u(r,z,t)}{\partial z}=0=\frac{\partial u(r,z,t)}{\partial z}\text{for } z=L$$

$u(r,z,t)=0, w(r,z,t)=0, p(r,z,0)=0$ for $z>0$

1.3 SOLUTION PROCEDURE

1.3.1 Geometry of The Stenosis

The graph of stenosis was plotted using Matlab programme. According to Mustapha (2008), the geometry of the stenosis is assumed to be manifested in the arterial segment is described as:

$$R(z)=\begin{Bmatrix} r_0-\frac{\delta}{2}\left(1+\cos\left(\frac{\pi(z-S_1)}{Z_1}\right)\right), & S_1-Z_1<z<S_1+Z_1 \\ r_0 & , \text{otherwise} \end{Bmatrix}$$

where $R(z)$ and r_0 are the radius of the artery with and without stenosis, respectively. Z_1 is the length of the stenosis and S_1 indicates its location, δ is the severity of the stenosis.

1.3.2 Derivation of Governing Equation

The governing equation for continuity equation becomes as:

$$\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\bar{u}}{\bar{r}}+\frac{\partial \bar{w}}{\partial \bar{z}}=0$$

The governing equations for $\bar{r}$ –component and $\bar{z}$-component of momentum together with continuity in non-conservative form written as follows:

$$\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\bar{u}}{\bar{r}}+\frac{\partial \bar{w}}{\partial \bar{z}}=0$$

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}}+\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{r}}+v\left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}-\frac{\bar{u}}{\bar{r}^2}\right)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}}+\bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}}+v\left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}}+\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right)$$

denoted $\bar{u}$ and $\bar{w}$ are the axial and radial velocity components, respectively. While $\bar{p}$ is the pressure and $v=\frac{\mu}{\rho}$, μ as the viscosity of blood.

The boundary condition and sometime initial conditions, dedicate the particular solution to be obtained from the governing equation.

$$u(r,z,t)=0\,, \qquad \frac{\partial w(r,z,t)}{\partial r}=0 \qquad \text{for } r=0$$

$$u(r,z,t)=0\,, \qquad w(r,z,t)=0 \qquad \text{for } t=0$$

$$u(r,z,t)=2(1-\frac{r}{R^2}), \qquad w(r,z,t)=0 \qquad \text{for } z=0$$

1.3.3 Non-dimensionalization of the Equation

The following dimensionless variables are considered as:

$$t=\bar{t}\frac{U}{\tau_0},r=\frac{\bar{r}}{r_0},z=\frac{\bar{z}}{r_0},w=\frac{\bar{w}}{U},u=\frac{\bar{u}}{U},p=\frac{\bar{p}}{\rho U^2},R=\frac{\bar{R}}{r_0}$$

Then, all the non-dimensionalization equations are stated as below:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 (u)}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial Z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)$$

Where Re is the Reynolds number defined by Re=$\frac{U r_0 \rho}{\mu}$.

Non-dimensionalization of the boundary and initial conditions:

$$u(r,z,t) = 0\,, \frac{\partial w(r,z,t)}{\partial r} = 0 \qquad \text{for } r = 0$$

$$u(r,z,t) = 2(1 - \tfrac{r}{R^2}),\, u_x(r,z,t) = 0 \qquad \text{for } z = 0$$

$$u(r,z,t) = 0\,,\ w(r,z,t) = 0$$

1.3.4 Radial Coordinate Transformation

The radial coordinate transformation is given by

$$x = \frac{r}{R(z,t)}$$

Simplifying all the radial coordinate transformation, gives:

$$\frac{1}{R}\cdot\frac{\partial u}{\partial x} + \frac{u}{xR} + \frac{\partial w}{\partial z} - \frac{x}{R}\frac{\partial w}{\partial x}\frac{\partial R}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} = -\frac{1}{R}\frac{\partial p}{\partial x} + \frac{1}{R}\left[-u + wx\frac{\partial R}{\partial z}\right]\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial z} + \frac{1}{ReR^2}\left[\left\{1 + \left(x\frac{\partial R}{\partial z}\right)^2\right\}\frac{\partial^2 u}{\partial x^2} + \left\{\frac{1}{x} + 2x\left(\frac{\partial R}{\partial z}\right)^2 - xR\frac{\partial^2 R}{\partial z^2}\right\}\frac{\partial u}{\partial x} + R^2\frac{\partial^2 u}{\partial z^2} + \frac{u}{x^2}\right]$$

$$\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{R}\left[-u + wx\frac{\partial R}{\partial z}\right]\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} + \frac{1}{\mathrm{Re}R^2}\left[\left\{1 + \left(x\frac{\partial R}{\partial z}\right)^2\right\}\frac{\partial^2 w}{\partial x^2} + \left\{\frac{1}{x} + 2x\left(\frac{\partial R}{\partial z}\right)^2 - xR\frac{\partial^2 R}{\partial z^2}\right\}\frac{\partial w}{\partial x} + R^2\frac{\partial^2 w}{\partial z^2}\right]$$

The non-dimensionalization of the boundary and initial conditions becomes:

$$w(z,x,t) = 0, \qquad u(z,x,t) = 0 \qquad \text{for } x = 1$$

$$\frac{\partial w(z,x,t)}{\partial x} = 0, \qquad u(z,x,t) = 0 \qquad \text{for } x = 0$$

$$w(z,x,0) = u(z,x,0), \qquad p(x,z,0) = 0 \quad \text{for } z > 0$$

The governing equation with the boundary condition can be solved numerically by using finite difference method, finite volume method, finite element method, FIDAP (CFD package), marker and cell (MAC) and successive-over-relaxation (SOR).

1.3.5 Graphical User Interface (GUI)

The equations are solved by using the existing Matlab programming and the result will visualized it by using GUI.A graphical user interface (GUI) is one of tools in Matlab programming used to visualize the results. The process of GUI development are stated as below:

1. Design of GUI and start GUIDE
2. Choose, rearrange and set properties of the components
3. Save and generate file: Code file and FIG-file
4. Add code for components in Callbacks (Code file) and run
5. Compile by using Matlab Compiler (Standalone application)

1.4 NUMERICAL RESULTS AND DISCUSSION

1.4.1 The Geometry of the Stenosis

Figures 1.4.1 shows the geometry of the stenosis on 30%, 50% and 70% area occlusion respectively. It is noticed that the stenosis reaches its critical height at the position $z = 20.0$.

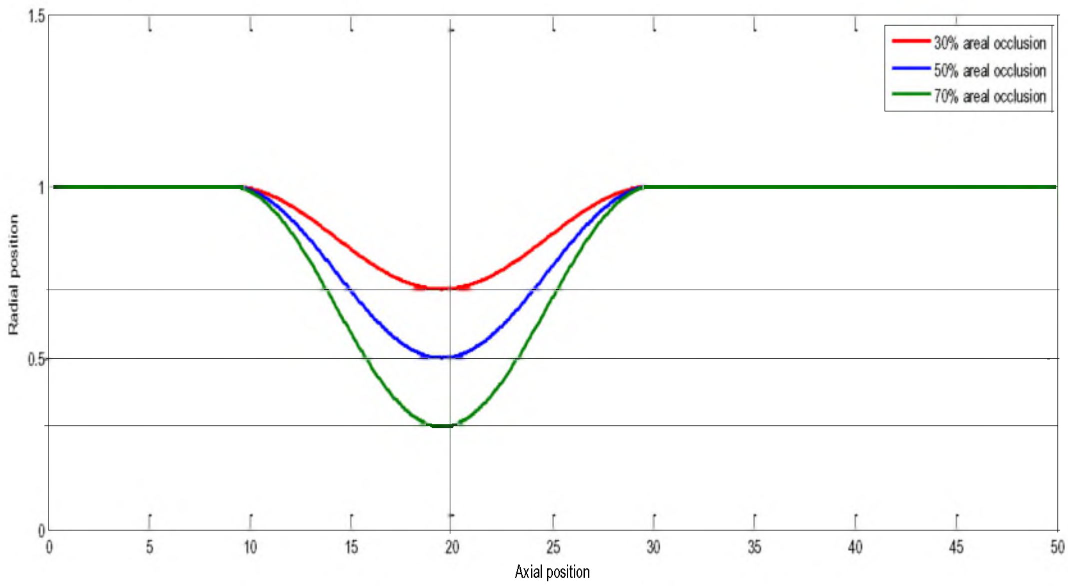


Figure 1.4.1 The geometry of the stenosis with different areal occlusion

1.4.2 The Effect of 30%, 50% and 70% Area Occlusion of Stenosis on Axial Velocity with Different Reynolds Number

Figure 1.4.2(a) – 1.4.2(f) shows how a stenosed artery with 30%, 50% and 70% areal occlusion influences the pattern of the flow field at the lowest and highest Reynolds number, respectively.

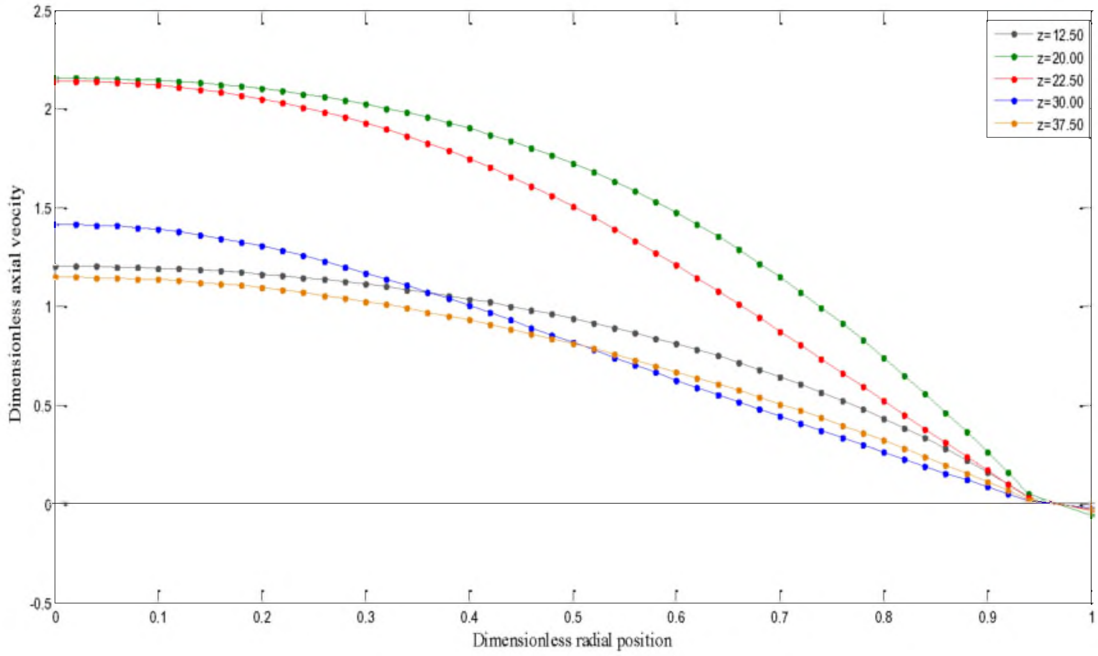


Figure 1.4.2(a) The axial velocity profile at different radial position with 30% area occlusion at Re=100

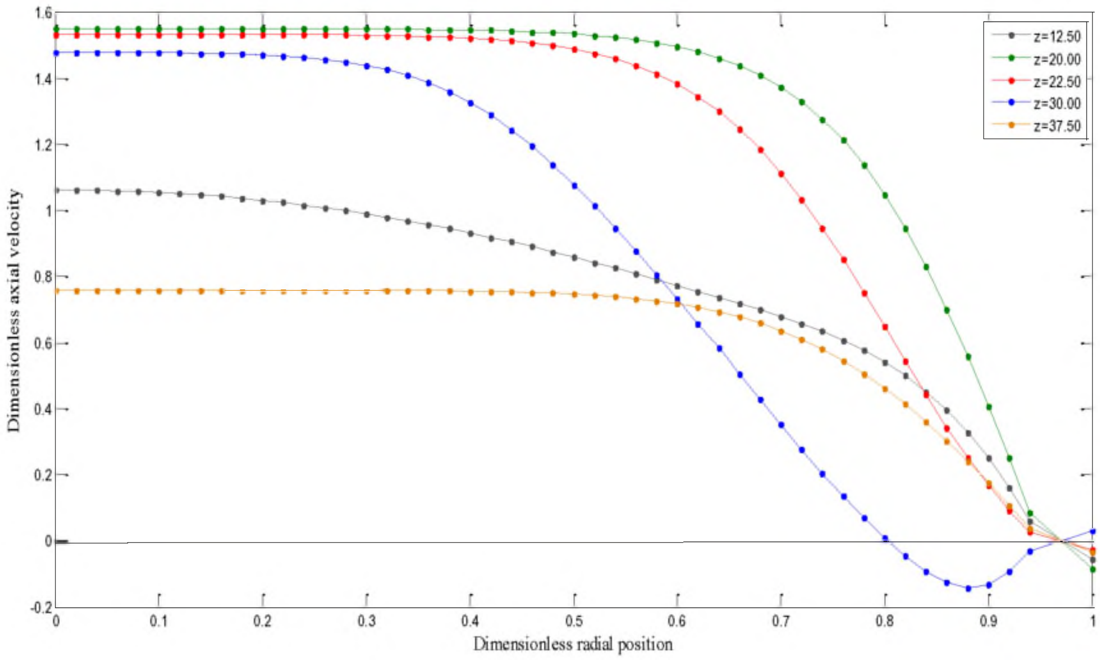


Figure 1.4.2(b) The axial velocity profile at different radial position with 30% area occlusion at Re=1000

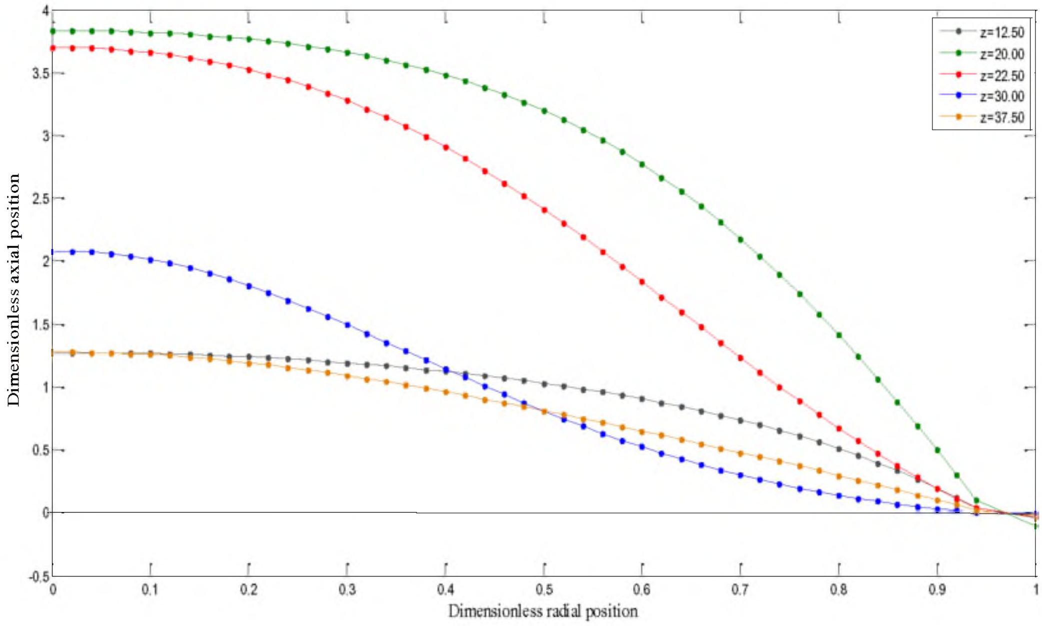


Figure 1.4.2(c) The axial velocity profile at different radial position with 50% area occlusion at Re=100

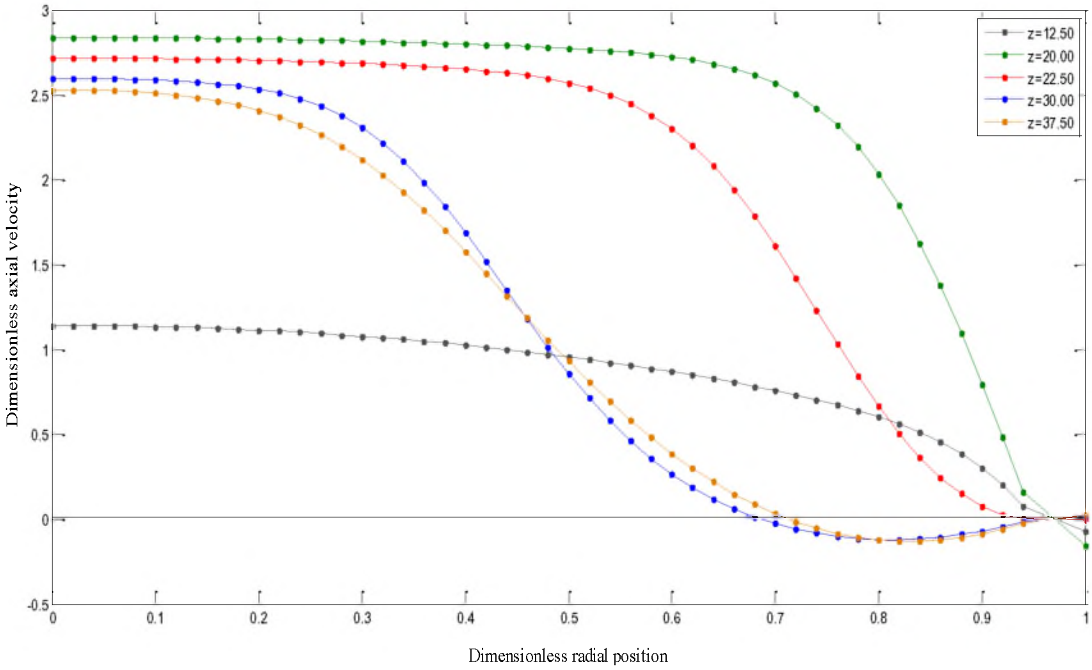


Figure 1.4.2(d) The axial velocity profile at different radial position with 50% area occlusion at Re=1000

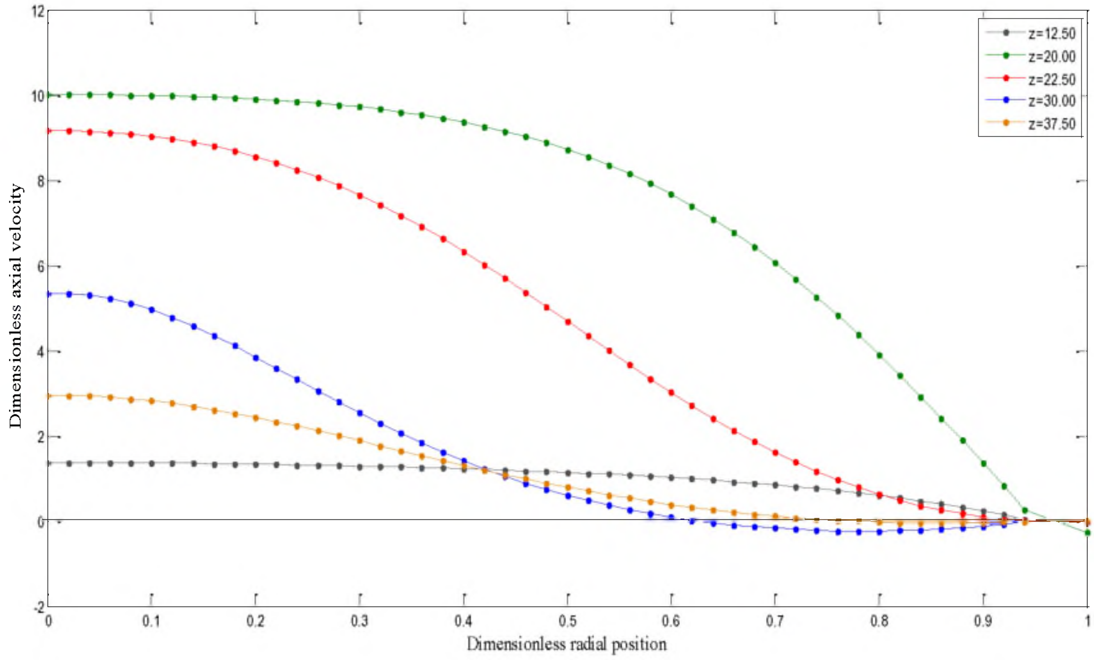


Figure 1.4.2(e) The axial velocity profile at different radial position with 70% area occlusion at Re=100

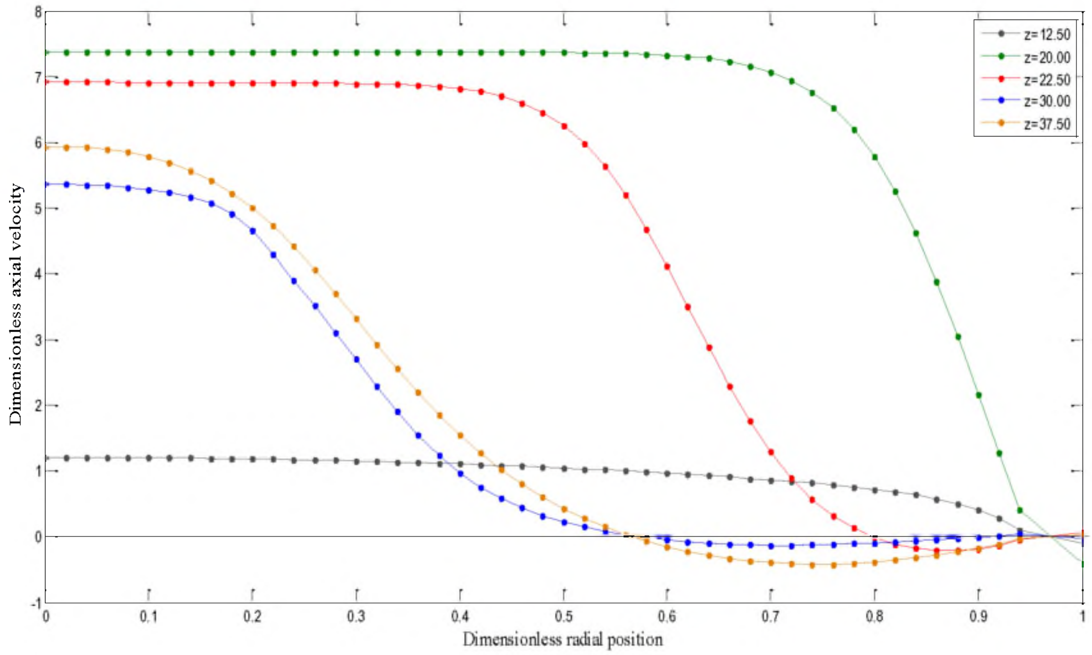


Figure 1.4.2(f) The axial velocity profile at different radial position with 70% area occlusion at Re=1000

From all figures 1.4.2(a) to 1.4.2(f), we conclude that the velocity value of stenosis increase when the area occlusion of stenosis increased. While the value velocity decreased when Reynolds number increased. We also can see that all the curves are found get distorted substantially at higher Reynolds number. Besides that, the negative value velocity show there exist the back flow in artery after blood passed through the stenosed area (z=30.00 and z=37.50).

1.4.3 The Effect of 30%, 50% and 70% Area Occlusion of Stenosis on Wall Shear Stress with Different Reynolds Number

The illustrations on the wall shear stress effected by a stenosed artery with 30%, 50% and 70% area of occlusion at Re=100 and Re=1000 shows in Figure 1.4.3(a) until Figure 1.4.3(f) respectively.

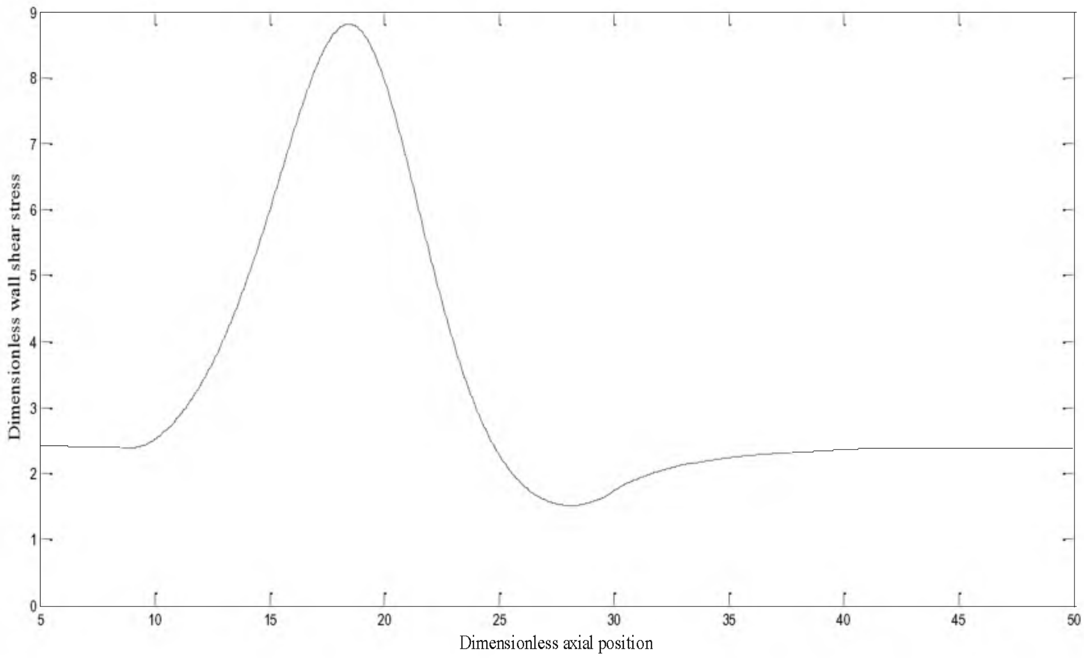


Figure 1.4.3(a) The wall shear stress of a stenosed artery with 30% area occlusion at Re=100

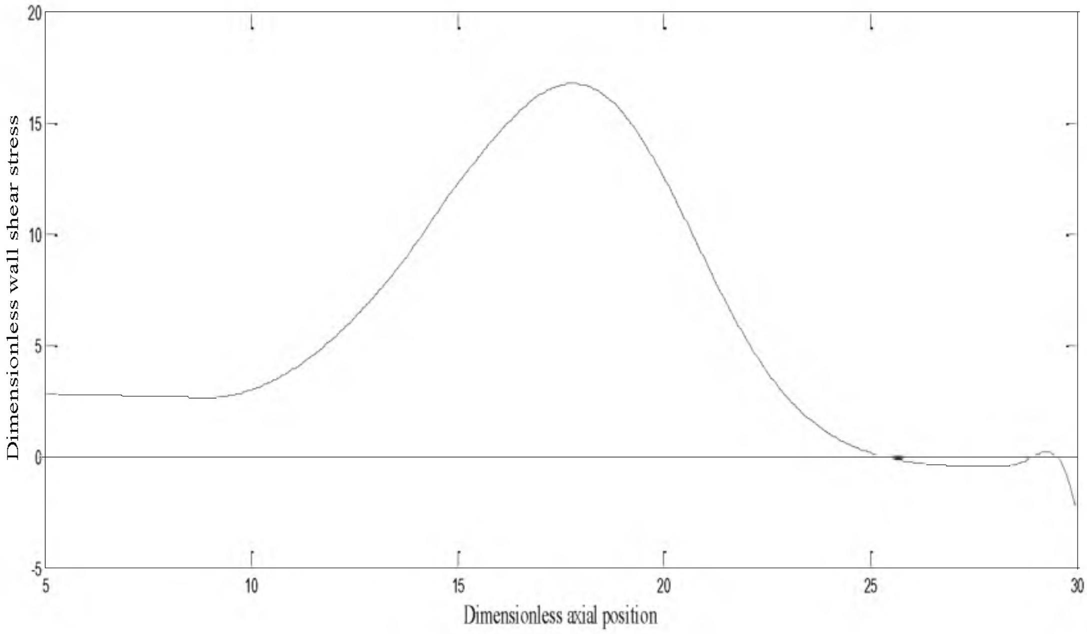


Figure 1.4.3(b) The wall shear stress of a stenosed artery with 30% area occlusion at Re=1000

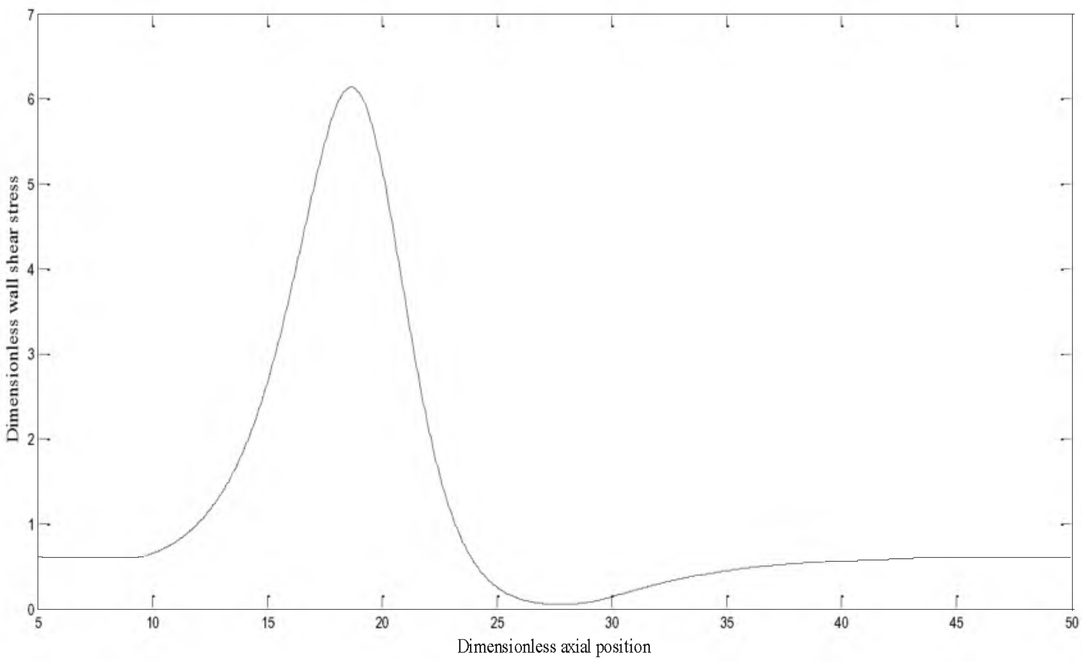


Figure 1.4.3(c) The wall shear stress of a stenosed artery with 50% area occlusion at Re=100

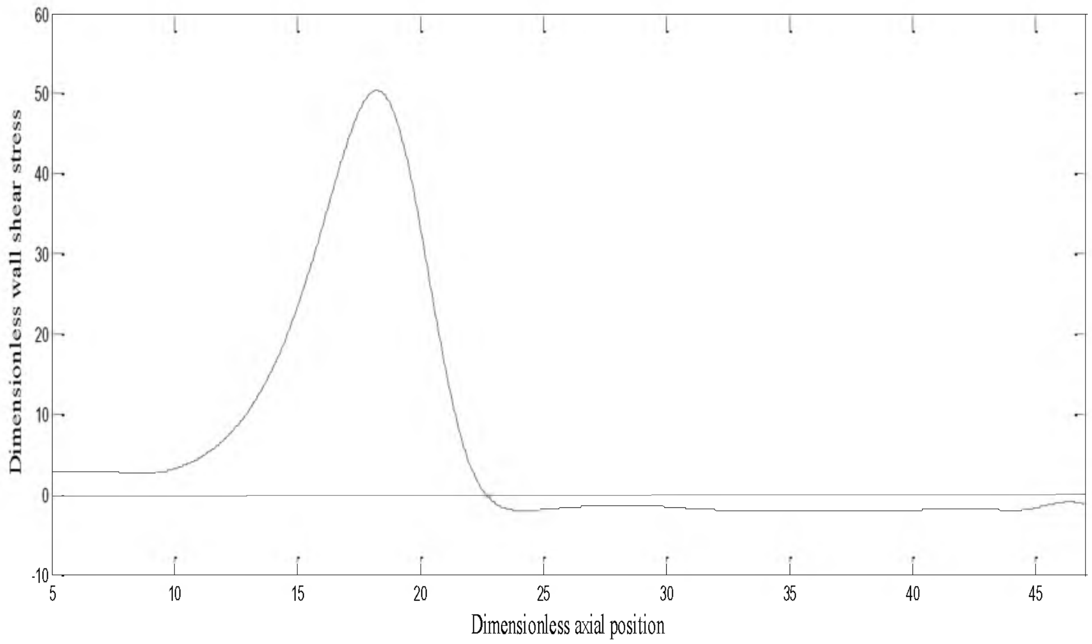


Figure 1.4.3(d) The wall shear stress of a stenosed artery with 50% area occlusion at Re=1000

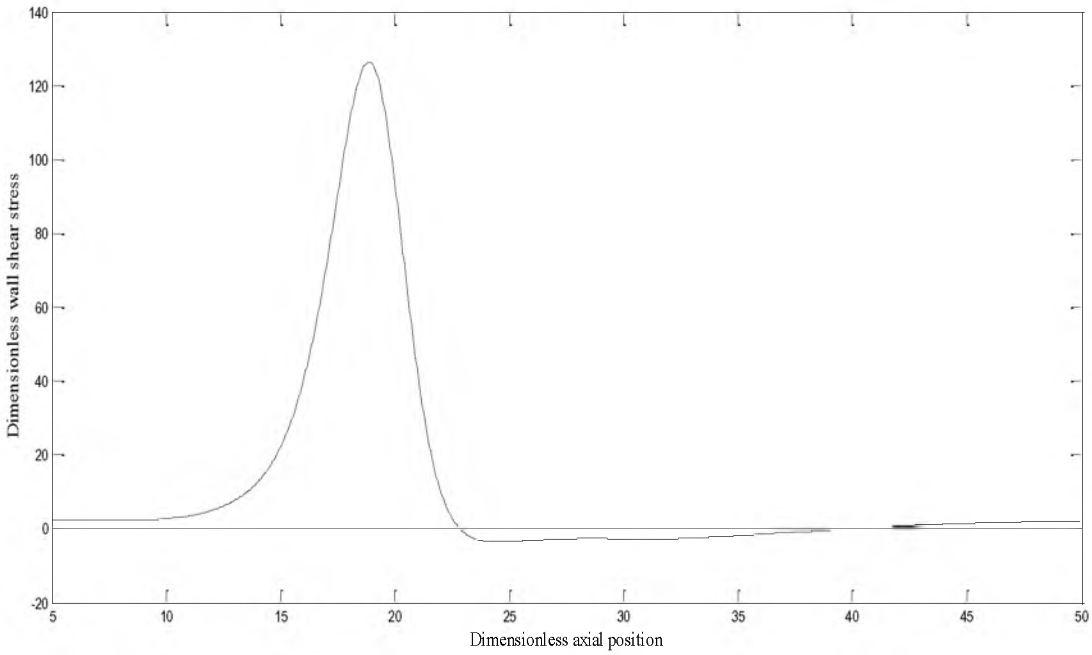


Figure 1.4.3(e) The wall shear stress of a stenosed artery with 70% area occlusion at Re=100

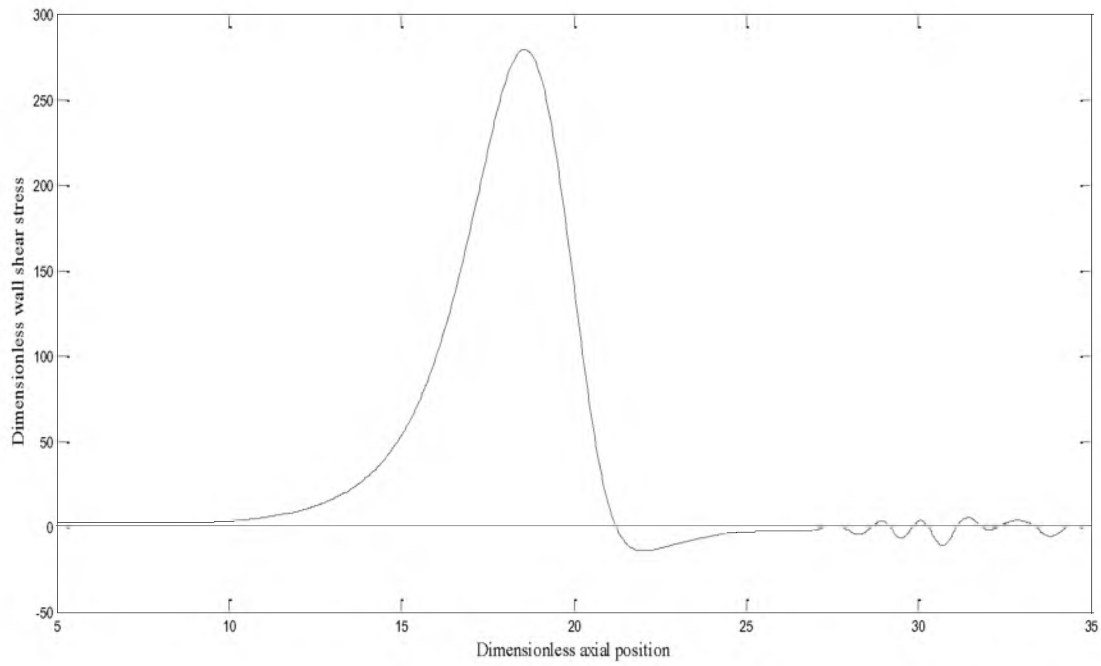


Figure 1.4.3(f) The wall shear stress of a stenosed artery with 70% area occlusion at Re=1000

The graphs shows that the high Reynolds number has high wall shear stress compared to lower Reynolds number. The wall shear stress also increased if the area occlusion become larger.As the area occlusion increase, the recirculation region become large. So that, it can conclude that the larger separated region occur for more severe stenosis and highest Reynolds number.

1.4.4 The Effect of 30%, 50% and 70% Area Occlusion of Stenosis on Pattern Streamlines with Different Reynolds Number

Figure 1.4.4(a) until Figure 1.4.4(f) illustrates how a stenosed artery with 30%, 50% and 70% area occlusion with different Reynolds number influences the pattern of streamlines.

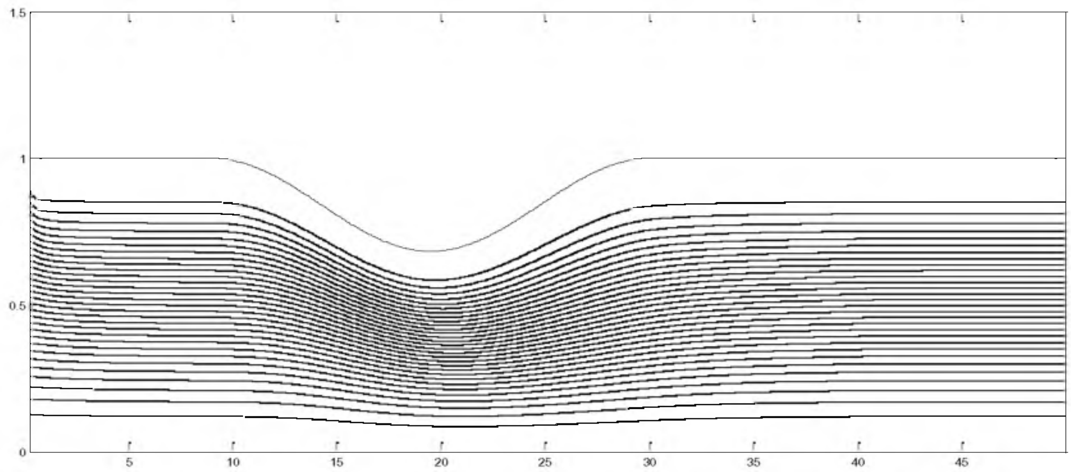


Figure 1.4.4(a) The pattern of streamlines for a stenosed artery with 30% area occlusion at Re=100

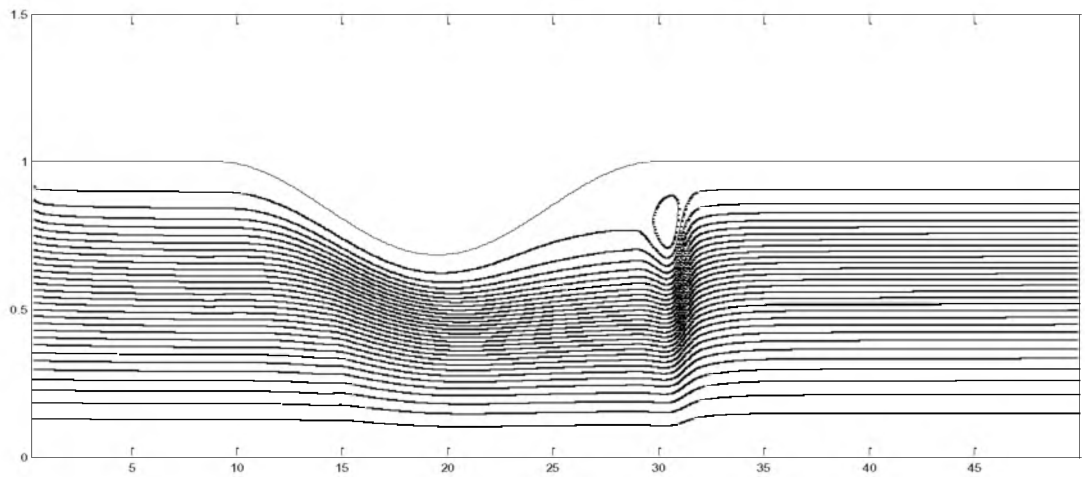


Figure 1.4.4(b) The pattern of streamlines for a stenosed artery with 30% area occlusion at Re=1000

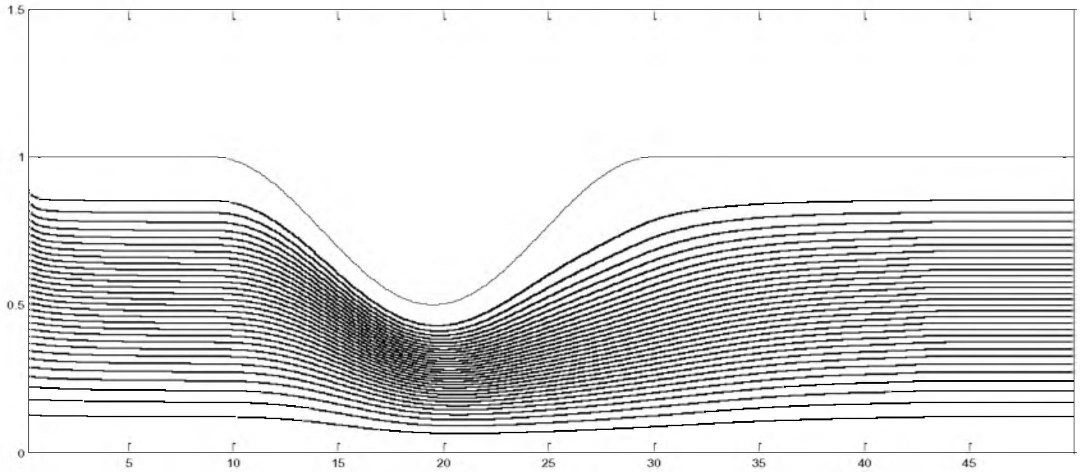


Figure 1.4.4(c) The pattern of streamlines for a stenosed artery with 50% area occlusion at Re=100

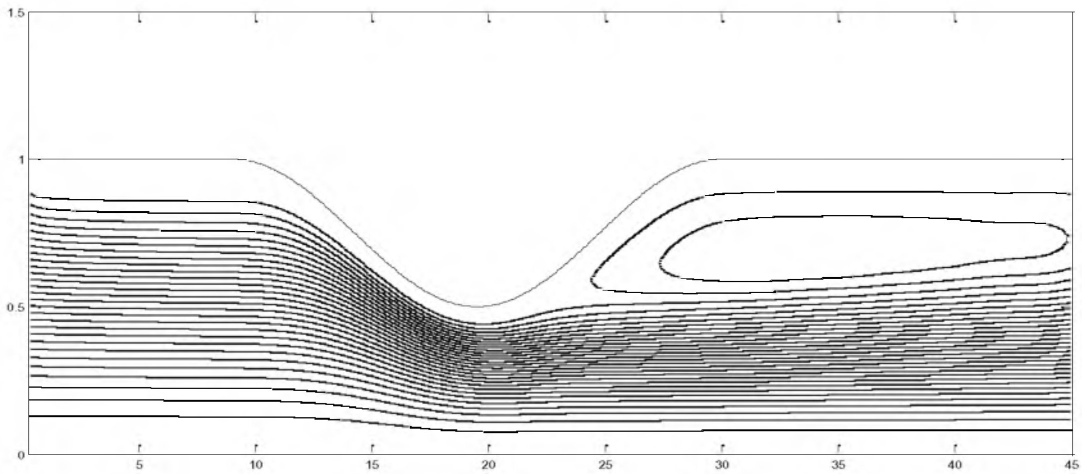


Figure 1.4.4(d) The pattern of streamlines for a stenosed artery with 50% area occlusion at Re=1000

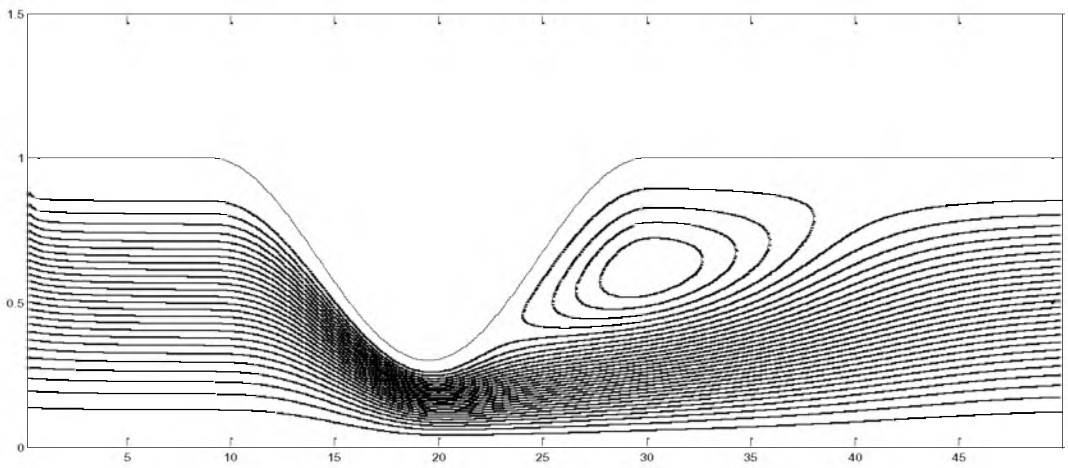


Figure 1.4.4(e) The pattern of streamlines for a stenosed artery with 70% area occlusion at Re=100

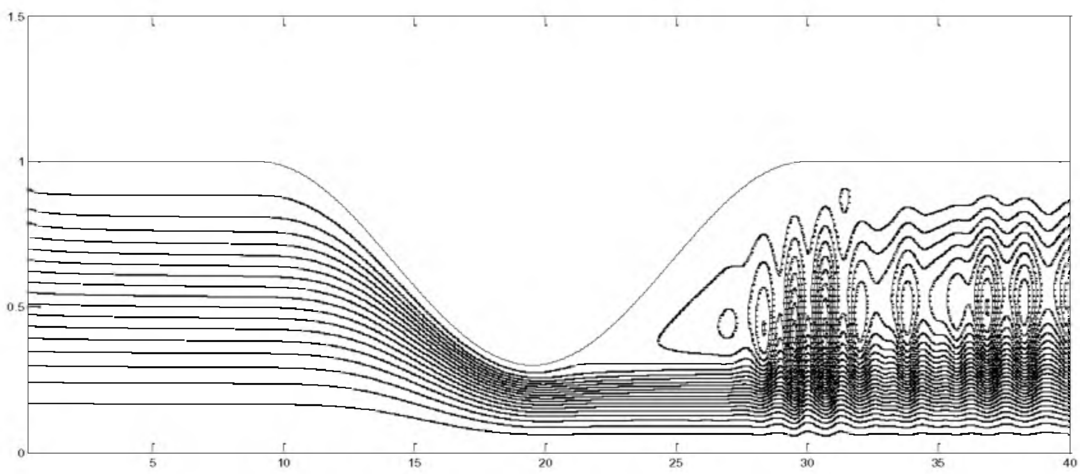
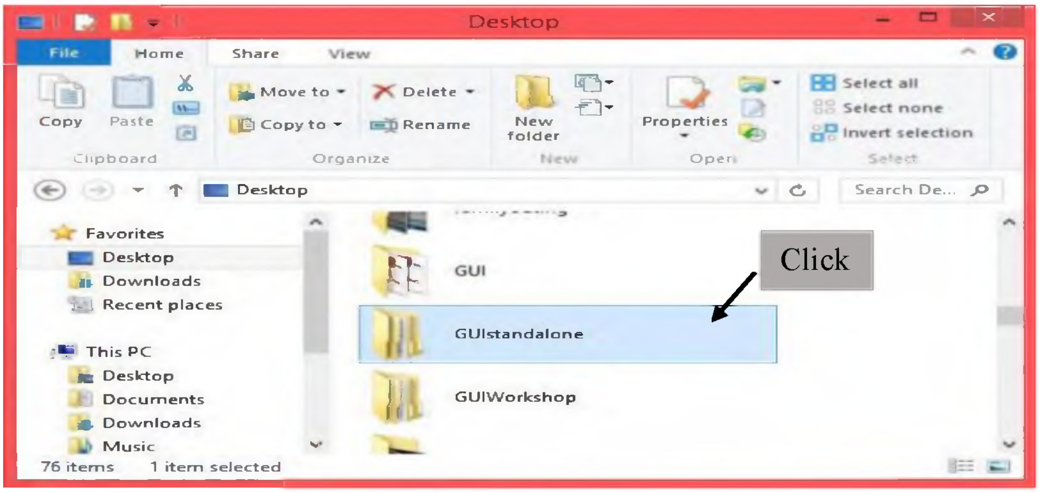


Figure 1.4.4(f) The pattern of streamlines for a stenosed artery with 70% area occlusion at Re=1000

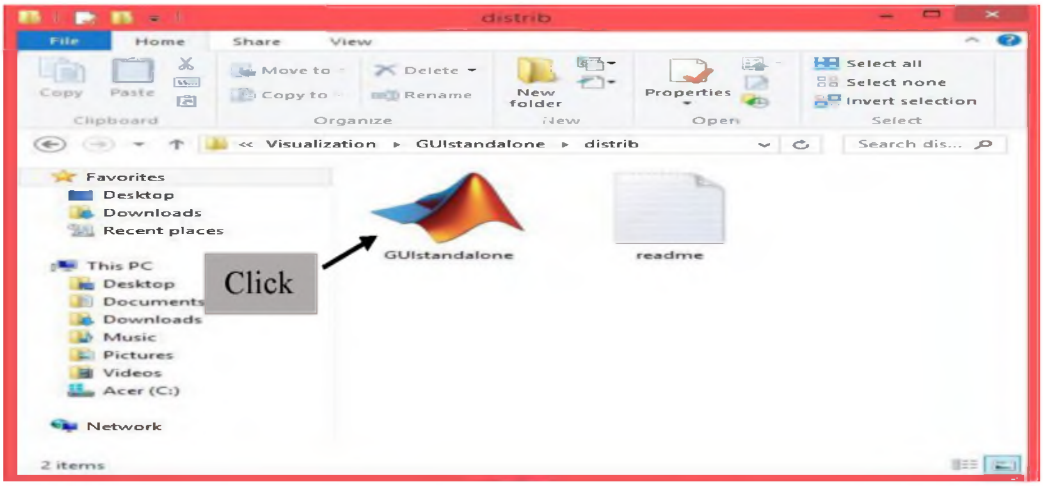
From all the figures 1.4.4(a) to 1.4.4(f), it clearly can be investigate that large recirculation region observed at the highest Reynolds number. It also can be seen that the recirculation region become large at the largest area occlusion. In addition, when Reynolds number and area occlusion increasing, the formation of eddies also increase. So that, it can be says that the more severe the stenoses, the larger the recirculation region.

1.4.5 The Visualization of Blood Flow through a Stenosed Artery

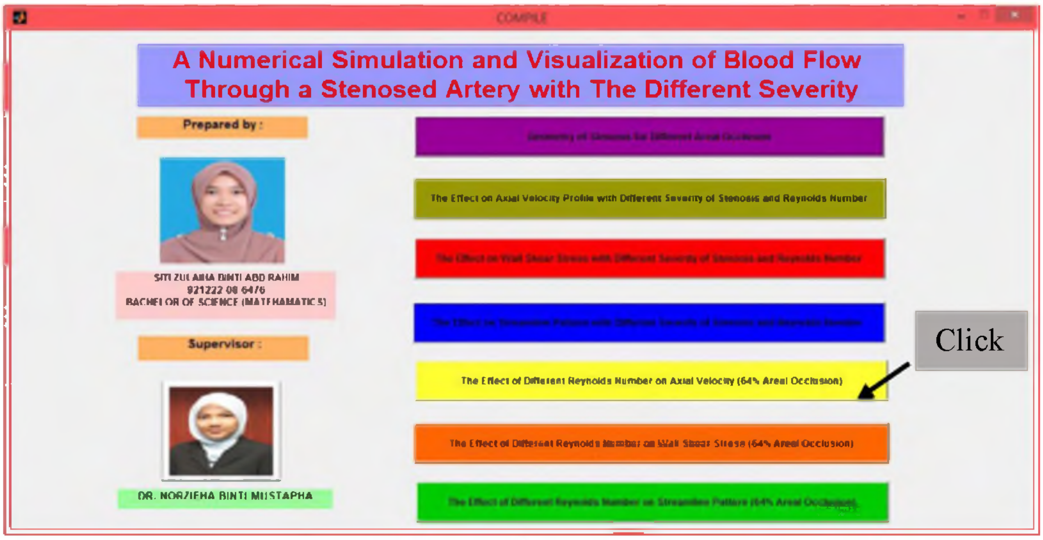
The figures below show the visualization of blood flow through a stenosed artery. All the result on blood flow through a stenosed artery obtained are visualized by using Graphical User Interface (GUI) in Matlab programming.



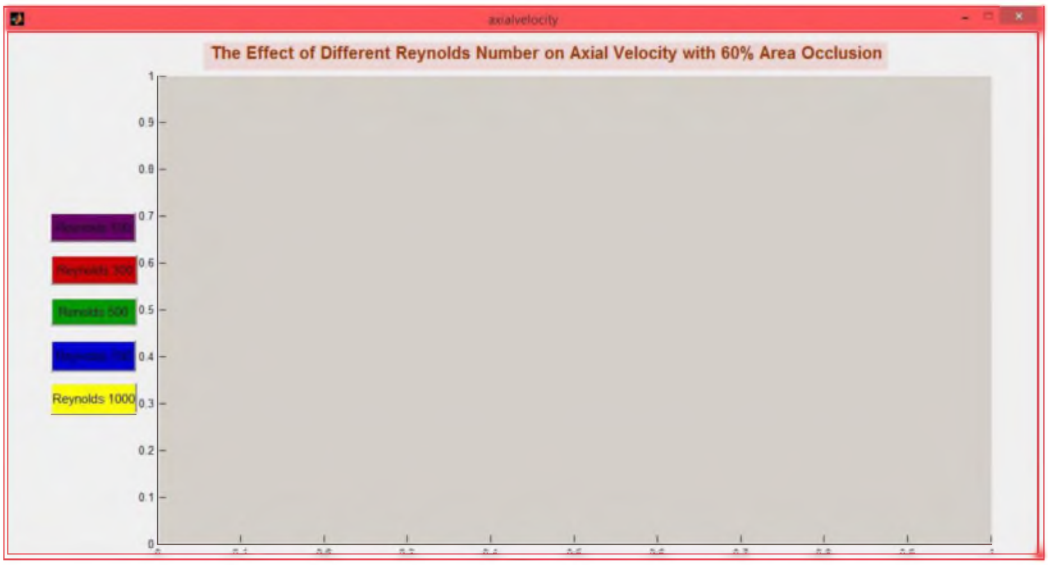
Open the ‘GUIstandalone’ file



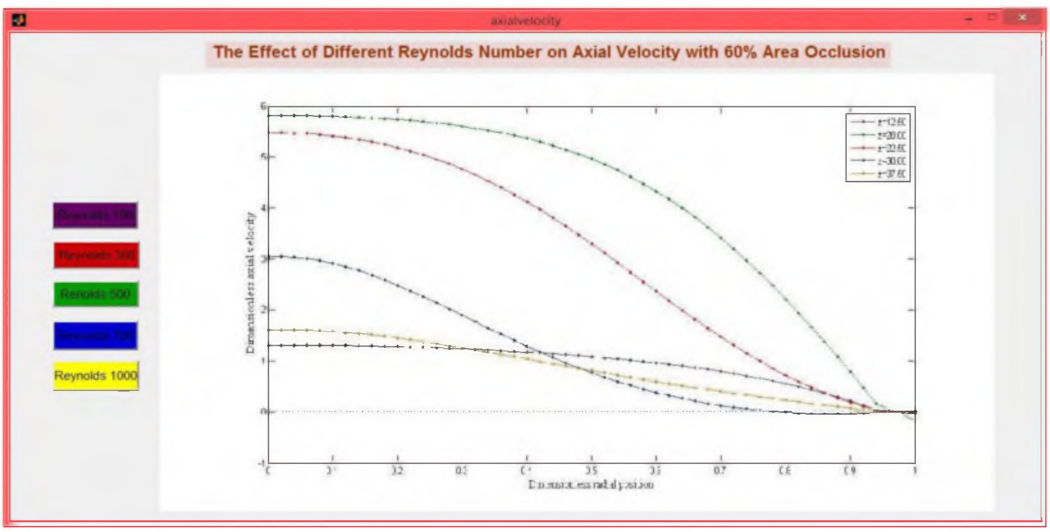
Click on the ‘GUIstandalone’ application



Click on any button



Click on any button



1.5 CONCLUSION

First we have discussed the result for axial velocity profile, wall shear stress and flow rate for 30%, 50% and 70% area of occlusion for lower and higher Reynolds number. From the results obtained, we can conclude that the more severe stenoses will give more effect on blood flow. Secondly, we have investigated that when Reynolds number is high, many flow separation will occurs and effect the flow rate.Lastly, we have visualized all the results obtained by using Graphical User Interface (GUI). GUI are ease of use to visualize the results, higher productivity and better accessibility. GUI is the basic tools use to visualize the results before develop a software.

REFERENCES

Ang, K.C. and Mazumdar, J. (1995). Mathematical modelling of triple arterial strenoses. *Australian Physical and Engineering Sciences in Medicine*. 18(2): 89 – 94.

Dasgupta, K., Chanda, A., Choudhury, A.R., and Nag, D. (2010). Geometry & Hemodynamics of Arterial Stenosis: a Clinical and Computational study. *Proceeding of 2010 International Conference on System in Medicine and Biology*, India.

Layak, G.C., Midya, C. (2007). Effect of Constriction Height on Flow Seperation in a Two-dimensional Channel. *Journal on Cummunications in Nonlinear Science and Numerical Simulation*., 754 – 759.

Minagar, A., Jimenez, J.J. and Alexander, J.S. (2006). Multiple sclerosis as a vascular disease. *Neurological Research*. 3: 230 – 235.

Mishra, B.K., & Verma, N. (2010). Effect of stenosis on Non-Newtonian Flow of Blood in Blood Vessels. *Australian Journal of Basic and Applied Sciences*, 4(4), 588 – 601.

Mustapha, N., Mandal, P.K., Johnston, P.R. and Amin, N. (2010). A Numerical Simulation of Unsteady Blood Flow through Multi-Irregular Arterial Stenoses. *Applied Mathematical Modelling*. 34: 1559 – 1573.

Sidik, N.A (2013). *Introduction to the Computational Fluid Dynamics*. Malaysia: UTM Press Publisher.

Sousa, F.S., Mangiavacchi, N., Nonato, L.G., Castelo, A., Tome', M.F., & Mckee S. (2004).A Front-Tracking/ Front-Capturing Method for the Simulation of 3D Multi-Fluid Flow with Free Surfaces. *J Comput Physics*.