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A MATHEMATICAL MODEL FOR WASTEWATER TREATMENT PROCESS OF AN OXIDATION POND

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Abstract. This study presents a mathematical model for wastewater treatment process (WWTP) of an oxidation pond. The model permits investigating the effects of a biological-based product called mPHO on the degradation of contaminants as well as increase the amount of dissolved oxygen (DO) in the pond. At this aim, an ordinary differential equation with coupled equations has been developed to study the correlation between the amount of bacteria (phototrophic and Coliform), chemical oxygen demand (COD), and dissolved oxygen (DO) existing in the pond. The mathematical model is employed to simulate the behaviour of the system where the numerical results demonstrate that the proposed model gives a good approximation of the interaction processes that occur naturally between biological and chemical substances involved in the pond.

Keywords Mathematical model; Wastewater treatment process (WWTP).

1.0 INTRODUCTION

There is a lot of waste being produced every day as a result of human activities. Wastewater can be classified into several types including industrial waste, municipal waste, food waste, sewage from houses and industries. This kind of waste must be carefully treated to ensure that there would be no harm to human being and the environment. Severe pollution has become our main concern of producing a mathematical model that can help the preservation and conservation of environment to run smoothly and to help the development of human capital. The execution of wastewater treatment process depends on symbiotic relationships of biological organism found in the system. Therefore, one needs to carefully understand the ecological system to build a deeper understanding and later to construct the so-called symbiotic relationship and function related to wastewater treatment process [1-3].

Various types of wastewater treatment have been generated to ensure that good quality of water can be provided. One of the most widely used treatment process for a medium size communities is oxidation pond technique [4]. The construction and maintenance cost of this treatment is inexpensive compared to other perceived treatment systems. The core procedure of an oxidation pond treatment process is that the contaminants and organic matter are degraded either in anaerobic or aerobic reaction. At each stage, existing microorganisms are used to breakdown either organic or inorganic substances of influent and reduce organic material to other forms (carbon dioxide, water, and cell biomass).

The wastewater treatment plant in this study is located in Taman Timor oxidation pond (see Fig. 1.1 and Fig. 1.2) near UTM Johor Bahru, estimated about 1,909 square metres and about 1.5 metres in depth, 54 metres length and about 2,864.13 cubic metres of total volume of water. To enhance the effectiveness of oxidation pond technique, a biological-based product mPHO (see Fig. 1.4) has been added regularly within three months period of study. Samples were collected at two points, which are CP1 (influent and application of mPHO) and CP2 (effluent) (see Fig. 1.3). Comparison of data taken at both points CP1 and CP2 have shown that mPHO has a good effect in reducing the concentration of BOD, COD and pollutant while phototrophic bacteria and dissolved oxygen concentration was increased.

Previous studies that have been done in wastewater treatment process are basically on a treatment to improve water in terms of quality and also to minimise the construction cost [5, 6]. However, we want to emphasise the use of mathematical models to build substantial relationship between parameters considered in this study.

Figure 1.1: Physical condition of oxidation pond at Taman Timor, Johor

Figure 1.2: The aerial view of the oxidation pond at Taman Timor, Johor

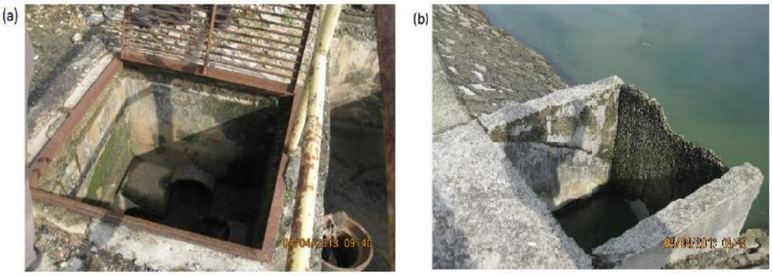

Figure 1.3: The location of sampling points CP1 and CP2 at the pond

Figure 1.4: The biological-based product (mPHO)

2.0 Literature Review

Mathematical modelling in solving actual problems has been developed quite a long time ago. One of the issues of concern is to scrutinise the quality of water accessible in rivers and stabilisation ponds. One of the earliest mathematical modellings of water quality was developed by Streeter and Phelps around 1925 [7] to study the relationship between BOD and DO on the River Cam, Eastern England. This model explains how BOD and DO can vary along with time observed.

Many studies have been conducted to predict the effluent quality of river and stabilisation ponds [7-10]. Model developed by Streeter and Phelps has been frequently used as a reference, while ordinary differential equations became the basis in building initial equation relating to each parameter. Through this model, significant relationship between parameters involved in the ecosystem is obtained. For instance, pollutant against DO, algae against DO, and bacterial against algae.

There are also other mathematical models being developed for a specific problem that occurs in a given locality. For instance, a study on Tha Chin river stream in Thailand that considered the effects of substances contaminated with dissolved oxygen [9]. This study proposed the model of two-dimensional coupled advection-dispersion equations for both state variables, respectively. The relationship between state variables used by taking into account the interactions that occur between materials contaminated with oxygen will produce harmful substances. In this model, contamination and oxygen concentration is just permitted to fluctuate along the length of the stream and they were dealt as homogeneous over the cross-segment of the river subject to Dobbin's criterion [10]. For simplification, the model is reduced to steady state solutions and then solved analytically for simple cases.

Apart from that, there are also studies to develop mathematical model to predict the specific growth rate and biomass concentration of the microbes in wastewater treatment [11]. This finding presents Michaelis-Menten term in the hypothesis of growth model considering the growth rate of enzymatic will take the same form. In biochemistry, Michaelis-Menten term expressions can be considered as one of the best-known models for enzyme kinetics. It is associated with German biochemist Leonor Michaelis and Canadian physician Maud Menten. The model describes the rate of enzymatic reactions by relating the reaction rate v with substrate S by the formula $v = \frac{V_{\max}[S]}{K_m + [S]}$, where $V_{\max}$ is the maximum rate attained by the system that can be called as saturating substrate concentrations. The Michaelis constant K_m is the substrate concentration that take the half value of $V_{\max}$. There are also other ecological study that develops the mathematical model consists of only DO and BOD [5]. A Beck modified Khanna Bhutiani model (BMKB model) has been developed to study the coexisting interaction that occurs between DO and BOD. The study of wastewater took place at river Suswa, India. The results were achieved by computing DO divided by BOD of the same upstream in the past season, which remains a single output solution. The model has been proved by the water quality information of the samples gathered from river Suswa in various seasons.

A more complex model has been also considered that discussed about the variations of COD, DO, ammonia, phosphorus, bacteria and algae concentrations with time and the dimensions of the pond [12-15]. This model predicts the correlation between those aforementioned variables at the effluent to measure the quality of the stabilisation pond (natural pond). A two-dimensional hydraulic model has been employed considering the dispersed flow and diffusion in horizontal and vertical directions, respectively. The pilot scale of this model focused around the accumulated data from a full-scale lake in Turkey. The model can be utilised for redesigning new outline of the lakes, thus enhancing the pro-fluent nature of existing lakes.

Although many studies have been generated related to wastewater and environment, there are still less results stated the comparison between simulation results and fields data [16-18]. This might happened because of the difficulty in having a reliable data to be used in the simulation procedure. This is the purpose of our study, to use the experimental data as the basis, and comparing the results simulated by a mathematical model.

3.0 MATHEMATICAL MODEL

We modelled the wastewater treatment process using a system of ordinary differential equations, which is the first order ODE with coupled-equation [1, 3, 19-22]:

The variables and parameters used in this mathematical model are as follows:

$M(t)$ is the concentration of PSB in the pond (mg/liter) where t varies from initial time up to 70 days.
$P(t)$ is the concentration of microbes (Coliform) in the pond (mg/liter).
$D(t)$ is the concentration of chemical oxygen demand in the pond (mg/liter).

$X(t)$ is the concentration of dissolved oxygen in the pond (mg/liter).
m is the concentration of PSB in one liter of mPHO (mg/liter).
$U(t)$ is the amount of mPHO applied to the pond according to the JBMI schedule per liter in 70 days.
P_0 is the concentration of microbes (Coliform) at CP1 (mg/liter).
D_0 is the concentration of chemical oxygen demand at CP1 (mg/liter).
X_0 is the concentration of dissolved oxygen at CP1 (mg/liter).
X_{atm} is the saturated oxygen concentration=10 mg/liter.
v_s is the average amount of sewage coming in (liter/day).
v_p is the volume of the pond in liter.
c_1 to c_{19} are constants determined by parameter estimations based on the experimental data at CP2.

Our model is composed of four coupled equations. These equations were accounted for the evolution of four state variables concentration (pollutant, DO, COD and PSB) with time dependent. The rates of change of the concentration with time t, $0 \le t \le 70$ are expressed as

$$\frac{dP(t)}{dt} = (c_1 - c_2 P(t) - c_3 M(t))P(t) + \frac{c_4 X(t)P(t)}{X(t) + c_5} + \frac{v_{in}\ P_0(t)}{v_p} \tag{1}$$

$$\frac{dM(t)}{dt} = (c_6 - c_7 M(t) - c_8 P(t))M(t) + \frac{c_9 X(t)M(t)}{X(t) + c_{10}} + \frac{c_{11}\ v_{in}\ mU(t)}{v_p} \tag{2}$$

$$\frac{dD(t)}{dt} = -c_{12} D(t) - c_{13} M(t)D(t) + \frac{c_{14} X(t)D(t)}{D(t) + c_{15}} + \frac{v_{in}\ D_0(t)}{v_p} \tag{3}$$

$$\frac{dX(t)}{dt} = c_{16}(X_{atm} - X(t)) - c_{17} X(t)M(t) - c_{18} X(t)P(t) - c_{19} X(t)D(t) + \frac{v_{in}\ X_0(t)}{v_p}\ . \tag{4}$$

3.1 PARAMETER ESTIMATION

The parameters of the proposed model can be estimated using a set of data collected through sampling from the pond in 70 days. The corresponding graphs of these data shown in Figure 3.1 to 3.4. Based on the given data, we want to determine the unknown parameters in equations (1-4) by the solution of parameter estimation problem. Then, a derivative-free optimisation algorithm is employed to estimate the optimum value of the parameters $c_1, c_2, \ldots, c_{19}$. A random value for each parameter is initially generated, where the cost function of this problem can be formulated as follows,

$$f(c_1,\ldots,c_{19}) = \sum_{i=1}^{12}\left|P(t_i) - P^*(t_i)\right| + \sum_{i=1}^{12}\left|M(t_i) - M^*(t_i)\right| + \sum_{i=1}^{10}\left|D(t_i) - D^*(t_i)\right| \quad (5)$$
$$+ \sum_{i=1}^{10}\left|X(t_i) - X^*(t_i)\right|.$$

Here $P^*(t_i)$ is the amount of pollutant measured at CP2 at time t_i, similar explanation goes for M^*, $D*$ and $X*$.

This cost function has to be a minimised subject to the mathematical model, which has been described in Section 3.1. The current schedule of mPHO gives us the following parameters for the problem. These procedures were iteratively repeated until some acceptable values for the parameters are obtained. After performing the aforementioned optimisation process, the following values for the parameters were obtained:

$c1$ = 0.018335	$c11$= 0.000056
$c2$= 0.021041	$c12$= 0.198528
$c3$=0.024755	$c13$= 0.014884
$c4$ = 0.018643	$c14$ = 0.025081
$c5$ = 0.012740	$c15$ = 0.018056
$c6$ = 0.012418	$c16$ = 0.015532
$c7$ = 0.026238	$c17$ =0.025985
$c8$=0.028729	$c18$ = 0.015218
$c9$ = 0.018214	$c19$= 0.000853
$c10$ = 0.018177	

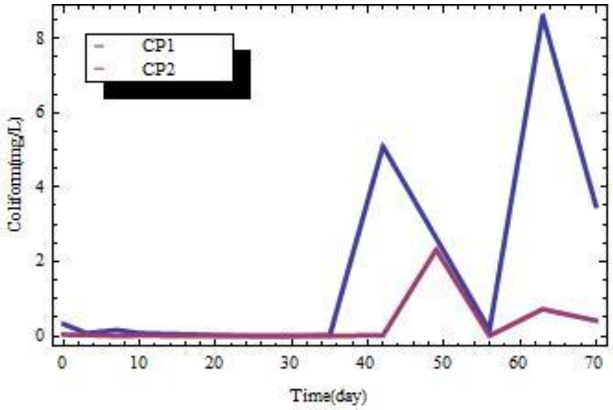

Figure 3.1: The dynamics of pollutant (Coliform) at point CP1 and CP2

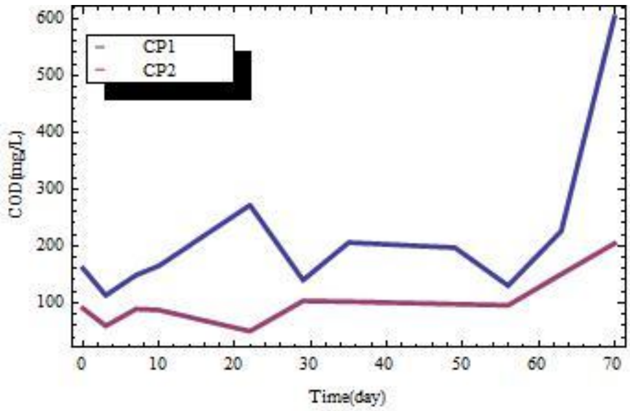

Figure 3.2: The dynamics of COD and point CP1 and CP2

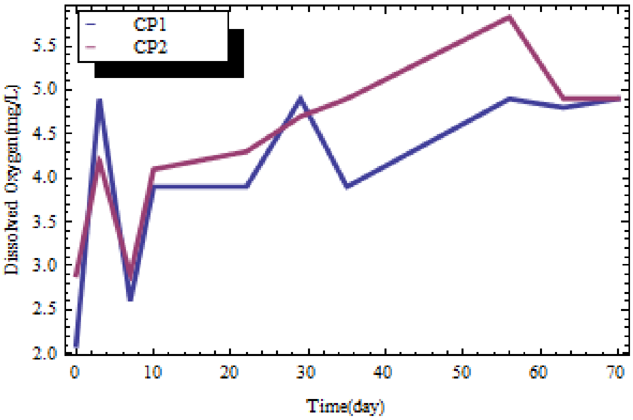

Figure 3.3: The dynamics of DO at point CP1 and CP2

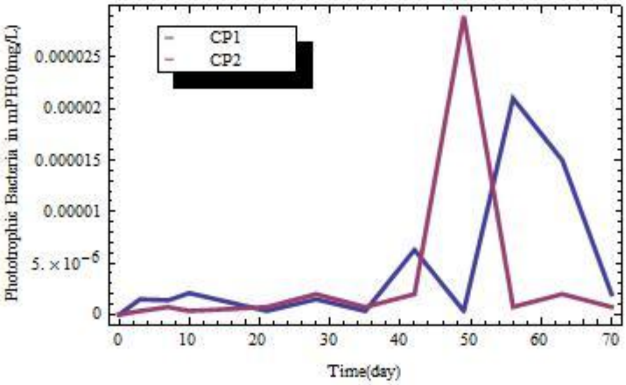

Figure 3.4: The dynamics of PSB at point CP1 and CP2

4.0 NUMERICAL SIMULATION

Using the above parameters, we simulate the mathematical model to obtain the results depicted in Fig. 4.1 to Fig. 4.4. Fig. 4.1 is the graph of concentration of Coliform in the pond, which shows that the solution of pollutant from the model has almost followed every data from CP2 except for the peak data at day 49. Fig. 4.2 describes the variation of PSB in mPHO that shows the concentration of PSB that keep increasing until day 40, when it starts to decrease until the end of treatment period.

Fig. 4.3 is the graph of dissolved oxygen with respect to time *t*. As can be seen, the amount of oxygen in the pond increased as time increases. This figure shows that the amount of oxygen is always higher than the amount of dissolved oxygen at CP2 until day 40 when it starts to decrease. Fig. 4.4 shows the amount of COD in the pond with respect to time *t*. This figure shows that the values of COD are always lower than at the point CP2 except for the time interval between 12 to 29. These values also decreased with some fluctuation along with time.

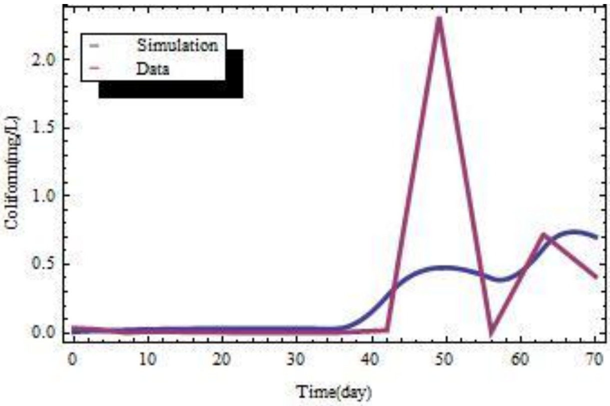

Figure 4.1: The dynamics of pollutant (Coliform) from simulation and point CP2

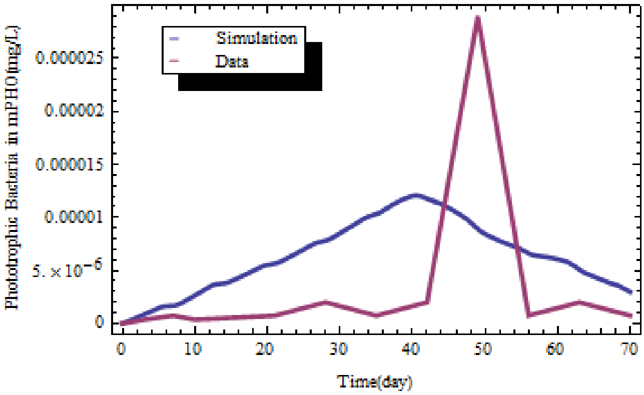

Figure 4.2: The dynamics of PSB from simulation and point CP2

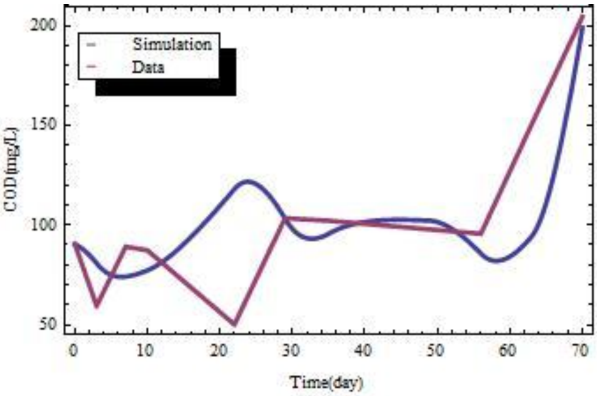

Figure 4.3: The dynamics of COD from simulation and at point CP2

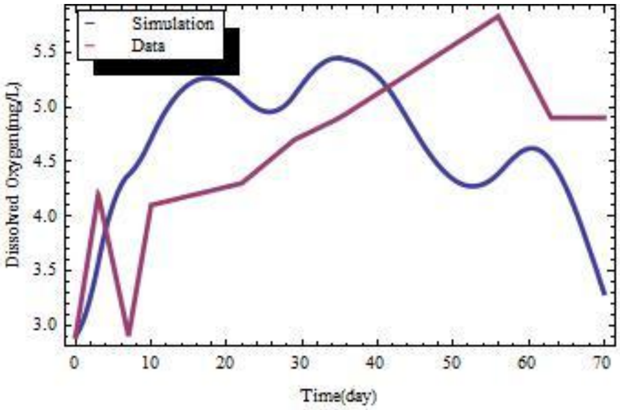

Figure 4.4: The dynamics of DO from simulation and at point CP2

5.0 CONCLUSIONS

In this study, a mathematical model for wastewater treatment process of the oxidation pond has been developed and then using real data a set of optimum parameters were obtained for this model. These parameters were used to simulate the model where the numerical results showed that the model can predict the behaviour of the microorganisms involved in the pond. This mathematical model has showed the effectiveness of mPHO in improving water quality of oxidation pond.

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REFERENCES

[1] Gray, N. F. (2004). Biology Of Waste Water Treatment. Imperial College Press.

[2] Wiesmann, U., Choi, I. S., and Dombrowski, E. M. (2007). Fundamentals Of Biological Wastewater Treatment. John Wiley & Sons.

[3] Ingalls, B. P. (2013). Mathematical Modeling In Systems Biology: An Introduction. MIT Press.

[4] Hussein, I., Abdel-Shafy and Mohammed, A. M. Salem, (2007). Efficiency of oxidation ponds for wastewater treatment in Egypt. Nato Science For Peace And Security Series-C: Environmental Security. 175-184.

[5] Gloyna, E. F. (1971). Waste Stabilization Ponds. Geneva.: World Health Organization.

[6] Von Sperling, M. (2007). Waste Stabilisation Ponds. IWA Publishing.

[7] Beck, M. B., and Young, P. C. (1975). A dynamic model for DO—BOD relationships in a non-tidal stream. Water Researc. 9(9), 769-776.

[8] Bhutiani, R. and Khanna, D. (2007). Ecological study of river Suswa: Modeling DO and BOD. Environmental Monitoring and Assessment. 125, 183–195

[9] Pimpunchat, B., Sweatman, W.L., Wake, G.C., Triampo, W., Parshotam, A. (2009). A mathematical model for pollution in a river and its remediation by aeration. Applied Mathematics Letters. 22, 304–308

[10] Dobbins, W.E. (1965). BOD and oxygen relationships in streams. Department of Civil Engineering, New York University.

[11] Abdulkareem, A. (2004). Modeling of microbial growth in a wastewater treatment plant: a case study of textile industry in Kaduna, Nigeria. AU Journal of Technology, Assumption University, Bangkok. Thailand. 8, 45–54

[12] Beran, B., Kargi, F. (2005). A dynamic mathematical model for wastewater stabilization ponds. Ecological Modelling. 181, 39–57.

[13] May, R.M. and Leonard, W.J. (1975). Nonlinear aspects of competition between three species. SIAM Journal on Applied Mathematics. 29, 243–253.

[14] Morley, D.A. (1979). Mathematical Modelling In Water And Wastewater Treatment. Applied Science Publishers.

[15] Haberman, R. (1977). Mathematical Models. Classics in Applied Mathematics.

[16] Ahuva Barzily and Yehuda Kott. (1989). Survival of pathogenic bacteria in elevated temperature oxidation ponds. Wat. Sci . Tech. 21, 109-114.

[17] Zahir Bakiri, Derradji Chebli and Saci Nacef. (2012). Dynamic modeling of the secondary settler of a wastewater treatment via activated sludge to low-load. Energy Procedia. 18, 1-9.

[18] Olutiola, P. O., Awojobi, K. O., Oyedeji, O., Ayansina, A. D. V., and Cole, O. O. (2010). Relationship between bacterial density and chemical composition of a tropical sewage oxidation pond. African Journal of Environmental Science and Technology. 4 (9), 595-602.

[19] Chen, W., Yao, C., and Lu, X. (2014). Mathematical modeling and modification of an activated sludge benchmark process evaluated by multiple performance criteria. Korean Journal of Chemical Engineering. 1-12.

[20] Dochain, D., Gregoire, S., Pauss, A., and Schaegger, M. (2003). Dynamical modelling of a waste stabilisation pond. Bioprocess and Biosystems Engineering. 26(1), 19-26.

[21] El-Gohary, F., Wahaab, R. A., El-Hawary, S., Shehata, S., Badr, S., and Shalaby, S. (1993). Assessment of the performance of oxidation pond system for wastewater reuse. Water Science & Technology. 27(9), 115-123.

[22] Parker, D. S., Merrill, M. S., and Tetreault, M. J. (1992). Wastewater treatment process theory and practice: the emerging convergence. Water Science & Technology. 25(6), 301-315.