IMPROVED TWO-PHASE SOLUTION STRATEGY FOR MULTIOBJECTIVE FUZZY STOCHASTIC LINEAR PROGRAMMING PROBLEMS WITH UNCERTAIN PROBABILITY DISTRIBUTION

ABDULQADER OTHMAN HAMAD AMEEN

UNIVERSITI TEKNOLOGI MALAYSIA

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ABDULQADER OTHMAN HAMAD AMEEN

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To my beloved wife and children

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ABSTRACT

Multiobjective Fuzzy Stochastic Linear Programming (MFSLP) problem where the linear inequalities on the probability are fuzzy is called a Multiobjective Fuzzy Stochastic Linear Programming problem with Fuzzy Linear Partial Information on Probability Distribution (MFSLPPFI). The uncertainty presents unique difficulties in constrained optimization problems owing to the presence of conflicting goals and randomness surrounding the data. Most existing solution techniques for MFSLPPFI problems rely heavily on the expectation optimization model, the variance minimization model, the probability maximization model, pessimistic/optimistic values and compromise solution under partial uncertainty of random parameters. Although these approaches recognize the fact that the interval values for probability distribution have important significance, nevertheless they are restricted by the upper and lower limitations of probability distribution and neglected the interior values. This limitation motivated us to search for more efficient strategies for MFSLPPFI which address both the fuzziness of the probability distributions, and the fuzziness and randomness of the parameters. The proposed strategy consists two phases: fuzzy transformation and stochastic transformation. First, ranking function is used to transform the MFSLPPFI to Multiobjective Stochastic Linear Programming Problem with Fuzzy Linear Partial Information on Probability Distribution (MSLPPFI). The problem is then transformed to its corresponding Multiobjective Linear Programming (MLP) problem by using α-cut technique of uncertain probability distribution and linguistic hedges. In addition, Chance Constraint Programming (CCP), and expectation of random coefficients are applied to the constraints and the objectives respectively. Finally, the MLP problem is converted to a single-objective Linear Programming (LP) problem via an Adaptive Arithmetic Average Method (AAAM), and then solved by using simplex method. The algorithm used to obtain the solution requires fewer iterations and faster generation of results compared to existing solutions. Three realistic examples are tested which show that the approach used in this study is efficient in solving the MFSLPPFI.

ABSTRAK

Masalah Pengaturcaraan Linear Kabur Stokastik Multiobjektif (MFSLP) di mana ketaksamaan linear pada kebarangkalian adalah kabur dikenali sebagai Masalah Pengaturcaraan Linear Kabur Stokastik Multiobjektif dengan Maklumat Separa Linear atas Taburan Kebarangkalian (MFSLPPFI). Kewujudan ketidakpastian menyebabkan kesukaran yang unik dalam masalah pengoptimuman terkekang kerana kehadiran matlamat yang bercanggah dan data sekitaran yang rawak. Kebanyakan kaedah penyelesaian MFSLPPFI bergantung pada model jangkaan pengoptimuman, model peminimuman varians, model pemaksimuman kebarangkalian, nilai pesimistik/optimistik, dan penyelesaian kompromi di bawah ketidakpastian separa parameter rawak yang terlibat. Walaupun pendekatan itu mengiktiraf pentingnya nilai selang bagi taburan kebarangkalian, namun taburan kebarangkalian hanya menjurus kepada had atas dan bawah taburan kebarangkalian dan mengabaikan nilai-nilai dalaman. Kekangan tersebut memberi motivasi bagi mencari strategi penyelesaian yang lebih efisien bagi masalah MFSLPPFI yang mengambilkira kedua-dua kekaburan taburan kebarangkalian dan kekaburan dan kerawakan parameter. Konsep penyelesaian bagi penyelidikan ini berasaskan strategi penyelesaian dua fasa, terdiri daripada transformasi kabur dan transformasi stokastik. Pertama, fungsi kedudukan digunakan untuk mentransformasi MFSLPPFI kepada masalah Pengaturcaraan Linear Stokastik Multiobjektif dengan maklumat separa linear kabur pada taburan kebarangkalian (MSLPPFI). Masalah yang diperoleh kemudiannya ditransformasi kepada masalah Pengaturcaraan Linear Multiobjektif (MLP) yang setara dengan teknik potongan-α bagi taburan kebarangkalian tidak pasti dan lindung nilai linguistik. Selain itu, Pengaturcaraan Kekangan Peluang (CCP) dan jangkaan pekali rawak masing-masing diaplikasikan kepada kekangan dan objektif. Akhirnya, masalah MLP ditukar kepada masalah Pengaturcaraan Linear berobjektif tunggal (LP) menerusi satu Kaedah Penyesuaian Purata Aritmetik (AAAM) dan masalah LP tersebut diselesaikan dengan menggunakan kaedah simpleks. Algoritma yang digunakan untuk mendapatkan penyelesaian memerlukan bilangan lelaran yang kurang dan penjanaan keputusan yang lebih pantas berbanding penyelesaian sedia ada dalam literatur. Tiga contoh realistik diuji yang hasilnya menunjukkan pendekatan yang digunakan dalam kajian ini efisien dalam menyelesaikan MFSLPPFI.

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LIST OF ABBREVIATIONS

AAAM	-	Adaptive Arithmetic Average Method
BLP	-	Bilevel Linear Programming
BP	-	Bilevel Programming
CC	-	Chance Constraint
CCA	-	Chance Constrained Approach
CCCP	-	Compromise Chance Constrained Programming
CCP	-	Chance Constrained Programming
CP	-	Compromise Programming
DE	-	Differential Evolution
DLP	-	Deterministic Linear Programming
DM	-	Decision Maker
EMO	-	Evolutionary Multiobjective Optimization
EMP	-	Evolutionary Multiobjective Programming
FDSA	-	Fuzzy Dual Simplex Algorithm
FFLP	-	Fully Fuzzy Linear Programming
FLP	-	Fuzzy Linear Programming
FLPF	-	Fuzzy Linear Programming with Fuzzy Linear Partial Information on Probability Distribution
FMF	-	Fuzzy Membership Function
FMO	-	Fuzzy Multiobjective Optimization
FMP	-	Fuzzy Multiobjective Programming
FNLP	-	Fuzzy Number Linear Programming
FP	-	Fuzzy Programming
FPSA	-	Fuzzy Primal Simplex Algorithm

FRC	-	Fuzzy Random Coefficient
FRMLP	-	Fuzzy Random Multiobjective Linear Programming
FRP	-	Fuzzy Random Programming
FRSMP	-	Fuzzy Robust Stochastic Multiobjective Programming
FSGP	-	Fuzzy Stochastic Goal Programming
FSLP	-	Fuzzy Stochastic Linear Programming
FSLPPFI	-	Fuzzy Stochastic Linear Programming Problems with Fuzzy Linear Partial Information on Probability Distribution
FSOP	-	Fuzzy Stochastic Optimization Programming
FSP	-	Fuzzy Stochastic Programming
FT_pMF	-	Fuzzy Trapezoidal Membership Function
FT_rMF	-	Fuzzy Triangular Membership Function
FULPAL	-	Fuzzy Linear Programming based on Aspiration Levels
FV	-	Fuzzy Variable
FVLP	-	Linear Programming Problem with Trapezoidal Fuzzy Variables
GA	-	Genetic Algorithm
GP	-	Goal Programming
i.e.	-	That Is To Say
IFRLP	-	Integrate Fuzzy Robust Linear Programming
IMSLP	-	Interval-Parameter Multistage Stochastic Linear Programming
KKT	-	Karush-Kuhn-Tuker
LHS	-	Left Hand Side
LMOP	-	Linear Multiple Objective Problems
LP	-	Linear Programming
LSP	-	Linear Stochastic Programming
MCA	-	Multicriteria Approach
MCDM	-	Multicriteria Decision Making
MDLP	-	Multiobjective Deterministic Linear Programming
MFLP	-	Multiobjective Fuzzy Linear Programming

MFP	-	Multiobjective Fuzzy Programming
MFRP	-	Multiobjective Fuzzy Random Programming
MFSLP	-	Multiobjective Fuzzy Stochastic Linear Programming
MFSLPPFI	-	Multiobjective Fuzzy Stochastic Linear Programming Problems with Linear Partial Information on Probability Distribution
MFSLPPI	-	Multiobjective Fuzzy Stochastic Linear Programming Problems with Incomplement Information on Probability Distribution
MFSNLPPFI	-	Multiobjective Fuzzy Stochastic Non Linear Programming Problems with Linear Partial Information on Probability Distribution
MFSP	-	Multiobjective Fuzzy Stochastic Programming
MILP	-	Multiobjective Integer Linear Programs
MIP	-	Multiobjective Integer Programming
MLDPP	-	Multilevel Decentralized Programming Problems
MLLP	-	Multilevel Linear Programming
MLP	-	Multiobjective Linear Programming
MMP	-	Multiobjective Mathematical Programming
MNLP	-	Multiobjective Nonlinear Programming
MO	-	Multiobjective Optimization
MOLP	-	Multiobjective Optimization Linear Programming
MP	-	Multiobjective Programming
MPOS	-	M-Pareto Optimal Solution
MSFLP	-	Multiobjective Stochastic Fuzzy Linear Programming
MSIP	-	Multiobjective Stochastic Integer Programming
MSLP	-	Multiobjective Stochastic Linear Programs
MSLPF	-	Multiobjective Stochastic Linear Programming with Fuzzy Linear Partial Information on Probability Distribution
MSLPI	-	Multiobjective Stochastic Linear Programming with Incomplement Information on Probability Distribution

MSLPPFI	-	Multiobjective Stochastic Linear Programming Problems with Fuzzy Linear Partial Information on Probability Distribution
MSLPPI	-	Multiobjective Stochastic Linear Programming Problems with Linear Partial Information on Probability Distribution
MSP	-	Multiobjective Stochastic Programming
MSPI	-	Multiobjective Stochastic Program With Incomplete Probability Distribution
NLP	-	Nonlinear Programming
p^{LEP}	-	P-Level Efficient Points
PP	-	Possibilistic Programming
RA	-	Recourse Approach
RFV	-	Random Fuzzy Variable
RHS	-	Right Hand Side
RVC	-	Random Variable Coefficient
SDLP	-	Single-objective Deterministic Linear Programming
SFGP	-	Stochastic Fuzzy Goal Programming
SFLP	-	Stochastic Fuzzy Linear Programming
SGP	-	Stochastic Goal Programming
SLP	-	Stochastic Linear Programming
SLPF	-	Stochastic Linear Programming with Fuzzy Linear Partial Information on Probability Distribution
SLPFI	-	Stochastic Linear Programming with Fuzzy Linear Partial Information on Probability Distribution
SLPI	-	Stochastic Linear Programming with Linear Partial Information on Probability Distribution
SMP	-	Stochastic Multiobjective Programming
SNLP	-	Stochastic Nonlinear Programming
SP	-	Stochastic Programming
SPI	-	Stochastic Program With Linear Partial Information on Probability Distribution
$\mathrm{ST}_p\mathrm{FN}$	-	Symmetric Trapezoidal Fuzzy Numbers

ST_rFN	-	Symmetric Triangular Fuzzy Numbers
T_pFN	-	Trapezoidal Fuzzy Number
T_rFN	-	Triangular Fuzzy Number
UTM	-	Universiti Teknologi Malaysia
VOP	-	Vector Optimization Programming
WPOS	-	Weak Pareto Optimal Solution
X^{CO}	-	Complete Optimal Solution Set
X^{P}	-	Pareto Optimal Solution Set
X^{WP}	-	Weak Pareto Optimal Solution Set

LIST OF SYMBOLS

$\square$	-	End of Proof
$(\Omega, 2^{\Omega}, P)$	-	Probability space
$\widetilde{A}$	-	Fuzzy Set
ϕ	-	The Value of The Objective Function
ϕA_i	-	The Value of The Maximum Objective Function
ϕL_i	-	The value of the Minimum Objective Function
$\lesssim$	-	Essentially Less Than or Equal To
$\preceq$	-	Almost Less Than or Equal To
F	-	Lower Probability of Event $A \in w$
$F(\mathbb{R})(w)$	-	The Set of All Trapezoidal Fuzzy Stochastic Discrete Number Events
$F(\mathbb{R})$	-	The Set of All Trapezoidal Fuzzy Numbers
$F(R)$	-	The Set of All Fuzzy Numbers
F(**S**)	-	The Set of Symmetric Trapezoidal Fuzzy Numbers
inf	-	Infimum
$\mathbb{N}$	-	Natural Numbers
P_F	-	Problem Formulation
P	-	Probability Distribution
p^{LFP}	-	P- Level Efficient Points
$\mathbb{R}$	-	Real Numbers
$\mathbb{R}^n$	-	N- Dimensional of Real Number
$R(F)$	-	Ranking Function
S	-	The Set of All Feasible Solution
SM	-	Summation of All Values of Maximum Objective Functions

SN	-	Summation of All Values of Minimum Objective
T	-	Transpose
$\boldsymbol{T}$	-	The Set of The Interval [0,1]
$\triangleq$	-	Vector Minimization Problem
X	-	A Universal Set
γ	-	Maximum Expectation Function of Compromise Function
π	-	Determinate Polyhedral Set
$\tilde{\pi}$	-	Fuzzy Polyhedral Set
δ	-	An Adaptive Arithmetic Average

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CHAPTER 1

INTRODUCTION

1.1 Introduction

A Linear Programming (LP) problem is a mathematical programming problem with a single linear objective function subject to a linear constraint set and the assumption that parameters are known with certainty. LPs involving more than one conflicting objective functions are called Multiobjective Linear Programming (MLP) problems.

An MLP problem is called a Multiobjective Stochastic Linear Programming (MSLP) problem when the parameters in the MLP problems are random and represented by probability distributions. MLP problems are called Multiobjective Fuzzy Stochastic Linear Programming (MFSLP) problems when the parameters in the MLP problems are fuzzy random and represented by probability distributions.

In all MSLP/MFSLP problems, the probability distributions are supposed to be known. But in many situations, the probability distributions cannot be specified. Such problems are studied under partial uncertainties and described by crisp or fuzzy linear inequalities to find optimal solutions. They are called Multiobjective Stochastic Linear Programming with Incomplement on probability distribution (MSLPI) problems, or Multiobjective Fuzzy Stochastic Linear Programming with

Incomplement on probability distribution (MFSLPI) problems. When the linear inequalities on the probabilities of the states are fuzzy, the problem is called Multiobjective Stochastic Linear Programming Problem with Fuzzy Linear Partial Information on probability distribution (MSLPPFI), or Multiobjective Fuzzy Stochastic Linear Programming Problem with Fuzzy Linear Partial Information on probability distribution (MFSLPPFI) problems.

1.2 Motivation

Real life problems are complicated and are subject to change. And this is just as true in LP problems where the assumption that all the coefficients of an LP model are known with certainty rarely holds in practice. In addition, whether in normal living or in professional settings, there may be various conflicting objectives that need to be considered in making decisions. Therefore, it is more appropriate to extend an LP problem to either a Fuzzy Linear Programming (FLP) problem, a Stochastic Linear Programming (SLP) problem, or a Fuzzy Stochastic Linear Programming (FSLP) problem.

An LP problem could have fuzziness and randomness occur separately or concurrently. Due to two different types of uncertainties, a fuzzy number is assigned to incomplete, inaccurate information in the LP problem. On the other hand, the stochastic variable represents arbitrariness or possibility of events. There are random changes in fuzzy numbers in factual life. For instance, the assessment of tolerance of machining products could be estimated as a fuzzy number. The production lot could vary from time to time, cycle-by-cycle in values. Moreover, random variable could be modeled as fuzzy tolerance values.

FSLP problems appear in numerous real-life situations. The required tools for LP problems such as the right-hand-sides (RHSs) and coefficients of the objectives and constraints could be fuzzy random variables. However, it is complex to resolve accurately the values of these parameters, especially those factors which are unpredictable due to uncertainties in the environment which results in varying parameters. These conditions happen frequently in long-term planning, advance strategies, engineering design and financial modeling, in which the described surroundings (objectives, constraints, coefficients) cannot be evaluated specifically and with certainty (Luhandjula and Gupta, 1996; Hop, 2007b).

An explanatory example of FSLP problem is in production planning. A large reduction in total cost could be considered as an objective that can be represented as a fuzzy stochastic variable since the cost components such as cost of inventory holding, materials, manpower and operation time and machine maintenance. Production output may depend on variables such as speeds, feed rates, and machine running time. Machine running time goes up-and-down and is usually difficult to assess accurately. Fuzzy random variables can be used to model available resources, demand, and other constraint coefficients. Such supposedly statistical data depend on environmental conditions including seasonal changes, market price fluctuation, suppliers' efficiency and cost and benefit of defensive maintenance.

Unstable state of equipment results in loss in production output. To reduce untimely breakdowns, defensive maintenance is the way to active sustainability. Regular inspection, repair, and component replacement as scheduled are preventive measures. Such measures are usually cost effective in terms of materials, wages, and loss of production due to down-time for preventive works. The uncertainty in length of the down-time is caused by the complexity of inspection, repair and/or replacement jobs and the maintenance culture of the staff.

It is desirable to develop a strategy to reduce total down time as a result of breakdowns and for preventive maintenance. An additional factor to consider is the effective life of the machine. All these times and their related costs should be modeled as fuzzy random variables (Hop, 2007b). These instances motivate us to suggest a new model for resolving MFSLPPFI.

By and large real life problems usually include some levels of uncertainties in the values of various parameters. Quoting the philosopher Nietzche: “No one is gifted with immaculate perception”.

Counterfeit certainty is awful science which carries the risks of making wrong enunciation of critical choices. Just assuming values to unknown variables will not result in much loss of values if they do not play important roles. But, in many real situations, building the model based on such assumed values runs the risk of pointing to wrong directions in the analysis. Assuming that the parameters are exactly prescribed may lead to an oversimplified and inflated picture of the certainty.

The principle of “ garbage in, garbage out ” shows clearly what disasters can happen when inaccurate data are falsely given preset values. This is more likely to creating the model which churns out meaningless outcomes. It is imperative that when a probabilistic explanation of unidentified elements is used, it should be cast as an SLP problem (Charnes and Cooper, 1963; Kall, 1976; Luhandjula, 2006). The presence of intrinsic or informational ambiguity, should be transformed to result in FLP problems (Liu and Liu, 2002; Liu, 2003; Liu, 2002; Sakawa, 1993; Verdegay, 1984).

Given everchanging complexities of real life, real world problems are frequently based on information that is vague and probabilistically uncertain (Luhandjula and Gupta, 1996). For instance, consider a production situation that is set in an LP situation, when it is assumed that the member components of constraints are demands which are random variables. If the coefficients in the matrix as given by experts who used fuzzy numbers to relate vague perceptions with data are presented statistically, the result will be an FSLP problem (Luhandjula, 2006).

When there are multiple parameters in an FSLP problem, we obtain an MFSLP problem. Since we cannot avoid complexities in real life, such problems demand to be on center stage.

In many SLP/FSLP problems, the probability distribution is supposed to be identified. But in some cases, we only have partial information on the way the probability distribution behaves. For example selecting a portfolio based on various criteria in the financial market, finding an optimal multiobjective/ multiattribute model for products, the demand on a new product of a client, the amount time to prepare raw materials, production time of a new product and profit of it...etc, are all random phenomena which have to be modeled stochastically.

1.3 Background of the Problem

This section is divided into four subsections that cover relevant information on the previous studies and issues surrounding the area being researched. It begins with attempts to understand and apply MSLPPFI, Multiobjective Fuzzy Linear Programming (MFLP), and MFSLP problems.

1.3.1 Introduction to Background of the Problem

Because the real world is continually changing, its components are in constant motion and are unstable. Frequently the effects of these components are superimposed, at least partially. Many researchers felt the need to incorporate both randomness and fuzziness into multiobjective programming problems (Katagiri and Ishii, 2000; Bector and Chandra, 2005a; Hop, 2007c; Chou *et al.*, 2009). Both fuzziness and randomness co-occurred in the LP problems related to FSLP when coefficients of objective, constraints and goals are fuzzy random variables (Luhandjula, 2006). Hop (2007a, 2007b, 2007c) has proposed a few models to measure attainment of such problems.

Iskander (2005) considered the FSLP problem as:

$$\begin{aligned} &\text{Max } \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \\ &\text{s.t. } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i \text{ , } i = 1, ..., m, \\ &x_j \geq 0 \text{ , } j = 1, ..., n. \end{aligned} \tag{1.1}$$

where $x_j, j = 1, ..., n$ are nonnegative decision variables, $\tilde{c}_j, j = 1, ..., n$ are fuzzy coefficients in the objective function, $b_i, i = 1, ..., m$ are random variables with known distribution functions, while $\tilde{a}_{ij}$ represents the fuzzy coefficient of the j^{th} decision variable in the i^{th} stochastic constraint. The author suggested an approach for solving the FSLP problem by utilizing two possibility as well as two necessity dominance indices previously used by Dubois and Prade (1983). The Chance Constraint (CC) approach, where feasible solutions satisfying uncertain constraints under certain probability are selected, and the α-cut technique are used to obtain the deterministic crisp LP problem. The researchers did not consider MFSLP problems, and did not take into consideration the case where the coefficients in the objective, the Left Hand Side (LHS) of the constraints, as well as its Right Hand Side (RHS) are Fuzzy Stochastic Variables (FSVs). They dealt only with the LHS variables with known distribution functions.

Luhandjula (2006) formulated an FSLP problem as:

$$\begin{aligned} &\widetilde{\text{Min}}\; c(w)x \\ &\text{s.t. } A_i(w)x \lessapprox b_i(w); \; i = 1, ..., m \\ &x \in X = \{x \in \mathbb{R}^n | x \geq 0\} \end{aligned} \tag{1.2}$$

where $c(w), A_i(w)$ and $b_i(w)$ are random variables on $(\Omega, 2^{\Omega}, P)$; in which $\Omega = \{w_1, ..., w_N\}$ a finite set of possible states of nature, 2^{Ω} is the power set of Ω and P the vector of probabilities $p_i = P(\{w = w_i\})$. The symbol $\sim$ expresses the fact that some flexibility are allowed in satisfying the objective and constraints in the linear flexible programs and Linear Stochastic Programs (LSPs) (Zimmermann, 1976; Kall, 1976). The solution of the problem needs combining symmetrical solution techniques (Luhandjula, 1983; Luhandjula *et al.*, 1997) and asymmetrical solution techniques (Chakraborty *et al.*, 1994; Chakraborty, 2002; Luhandjula, 1983). It is noted that Luhadjula did not consider certain FSVs for the coefficients, but applied flexible programs to the system. This lead to a bargaining between the objective function and the constraints, thus weakening the results.

Hop (2007a) considered a Fuzzy Stochastic Goal Programming (FSGP) as;

$$
\begin{aligned}
&(\widetilde{c}_k)_w x \; ; (\widetilde{g}_k)_w, k = 1, l \\
&\text{s.t.} \sum_{j=1}^{n} (\widetilde{a}_{ij})_w x_j \leq (\widetilde{b}_i)_w \qquad (1.3) \\
&x_j \geq 0;\; w \in \Omega;\; i = 1, 2, ..., m;\; j = 1, 2, ..., n;\; k = 1, 2, ..., l
\end{aligned}
$$

where a, b are (m, n) and $(m, 1)$ matrices of constraint coefficients, $(\widetilde{c}_k)_w$ is $(1, n)$ matrix of Fuzzy Random Coefficients (FRCs), and $(\widetilde{g}_k)_w$ are given fuzzy random goals required to maximally satisfy both sides. In other words; if $(\widetilde{c}_k)_w x \leq (\widetilde{g}_k)_w$ the lower attainment values should be maximized. Otherwise, if $(\widetilde{c}_k)_w x \geq (\widetilde{g}_k)_w$ the upper attainment values should be maximized. The author suggested a model to measure attainment value of the FSGP, and a new measure was used to derandomize and defuzzify the FSGP problem to obtain the standard form LP problem.

Another FSLP problem:

Max cx

$$\text{s.t.} \sum_{j=1}^{n} (\widetilde{a}_{ij})_w x_j \leq (\widetilde{b}_i)_w; \;\; i=1,...,m \tag{1.4}$$

$$x_j \geq 0; \; w \in \Omega; \; i=1,...,m; \; j=1,...,n; \; k=1,...,l$$

has been considered (Hop, 2007b), where c is $(1,n)$ matrix. A, b are (m,n), and $(m,1)$ matrices of fuzzy random variable constraint coefficients defined on a probability space $(\Omega, 2^{\Omega}, P)$. The problem was reformulated into its corresponding deterministic LP problem by the restrictions on the superiority and inferiority degrees, as the penalty for the violation to fuzziness and randomness. The author did not consider MFSLP problem but only the single optimization problem. On the other hand Recourse Approach (RA) was utilized to convert the problem from fuzziness and randomness to its corresponding deterministic form. This study focused on the FSLPPFI and highlighted on three aspects which are fuzziness, randomness, and whether a function is deterministic, from the beginning of the problem until solution is found, in solving the Singleobjective Deterministic Linear Programming (SDLP) problem:

1.3.2 Multiobjective Fuzzy Linear Programming Problem

Maleki *et al.* (2000) noted that the possibilistic programming or multiobjective programming methods have shortcomings in solving problems in which all decision parameters are fuzzy numbers. They considered an LP with FVs as:

$$
\begin{aligned}
&\text{Max } \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j, \\
&\text{s.t. } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \,, i = 1, 2, ..., m_0, \\
&\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i \,, i = m_0 + 1, ..., m, \\
&x_j \geq 0, \ \ j = 1, ..., n,
\end{aligned}
\tag{1.5}
$$

where $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}), \tilde{b}_i = (b_i^L, b_i^U, \alpha_i, \beta_i),$ and $\tilde{c}_j = (c_j^L, c_j^U, \omega_j, \eta_j)$ are in the set of all Trapezoidal Fuzzy Numbers ($F(\mathbb{R})$), $i = 1, ..., m,\ j = 1, ..., n$. They modified the FLP problem using a comparison of fuzzy numbers by using areas determined by the membership functions, and introduced an effective method to solve this kind of problems. In addition, they proposed a new method for solving LP problems with fuzzy unknowns via an auxiliary program to the original LP and connecting these LPs via relationships between them.

Cadenas and Verdegay (2000) used Ranking Function ($R(F)$) in MFLP problems, Multiobjective Mathematical Programming (MMP) problems, Vector Optimization Programming (VOP) problems, and Fuzzy Multiobjective Optimization (FMO) problems. MMP problems in their conventional cases were transformed into uni-objective mathematical programming problems either by using the weighted approach or the constant approach (k^{th}-objective λ-constraint), then finding the non-inferior solutions. For the FMO problem, which was the extension of the VOP problem in the fuzzy environment with fuzziness in the constraints and in the objective functions, the following formulation had been developed and studied;

$$
\begin{aligned}
&\text{Min } [c_1^f x, c_2^f x, ..., c_n^f x] \\
&\text{s.t. } Ax \leq b \\
&x \geq 0
\end{aligned}
\tag{1.6}
$$

where each $c_j^f, j = 1, 2, ..., n$ is a N- vector of fuzzy numbers. The existence of the fuzzy goals had been considered and assumed as follows:

$$\begin{aligned}
&\text{Find } x \in \mathbb{R}^N \\
&\text{s.t. } c_j x \underset{g}{\leq} z_j; j = 1, ..., n \\
&Ax \underset{g}{\leq} b \\
&x \geq 0
\end{aligned} \tag{1.7}$$

where $\underset{g}{\leq}$ means that there was a membership function; $\mu_i : \mathbb{R} \rightarrow [0, 1], i = 1, ..., m$, $\forall x \in \mathbb{R}^N$.

The obtained problem was:

$$\begin{aligned}
&\text{Min } [c_1 x, c_2 x, ..., c_n x] \\
&\text{s.t. } Ax \leq \mu^{-1}(\alpha) \\
&x \geq 0, \alpha \in [0, 1]
\end{aligned} \tag{1.8}$$

where μ^{-1} was an m-vector constraint inverse of the membership function $\mu_i; i = 1, ..., m$, $\forall \alpha \in [0, 1]$.

A Fuzzy Number Linear Programming (FNLP) problem in the form of

$$\begin{aligned}
&\text{Max } \tilde{z} \underset{R(F)}{=} \tilde{c} x \\
&\text{s.t. } Ax = b, \\
&x \geq 0
\end{aligned} \tag{1.9}$$

where $b \in \mathbb{R}^m, x \in \mathbb{R}^n, A \in \mathbb{R}^{m \times n}, \tilde{c}^T \in (\mathrm{F}(\mathbb{R}))^n$ is n-dimension of the set of all Trapezoidal Fuzzy Numbers (T_pFNs), had been considered by Nasseri *et al.* (2005), and a linear ranking function was used in solving the problem for comparing fuzzy numbers.

Ganesan and Veeramani (2006) introduced and defined a kind of FLP problem with T$_p$FNs in its symmetric form as:

$$\begin{aligned}
&\max \tilde{z} \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \\
&\text{s.t. } \sum_{j=1}^{n} a_{ij}\tilde{x}_j \preceq \tilde{b}_i, i = 1, 2, ..., m_0, \\
&\sum_{j=1}^{n} a_{ij}\tilde{x}_j \succeq \tilde{b}_i, i = m_0 + 1, m_0 + 2, ..., m, \\
&\tilde{x}_j \succeq 0 \;\forall j = 1, 2, ..., n
\end{aligned} \tag{1.10}$$

where $a_{ij} \in \mathbb{R}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in$ F($\mathbf{S}$)s (The Set of Symmetric Trapezoidal Fuzzy Numbers). The solution for the problem was obtained without converting it to a crisp LP problem.

Some properties in FNLP problems have been explored by Mahdavi-Amiri and Nasseri (2006) when they used a linear ranking function to introduce the dual of the following FNLP problem, where several duality results were presented:

$$\begin{aligned}
&\max \tilde{z} \underset{R(F)}{=} \tilde{c}x \\
&\text{s.t. } \tilde{A}x \underset{R(F)}{\leq} \tilde{b}, \\
&x \geq 0
\end{aligned} \tag{1.11}$$

where $\underset{R(F)}{=}, \underset{R(F)}{\leq}$ were equality and inequality with respect to the $R(F)$ respectively, $\tilde{A} = (\tilde{a}_{ij})_{(m,n)}, \tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n), \tilde{b} = (\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_m)^T$ and $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_i \in$ F($\mathbb{R}$).

Based on Maleki *et al.* (2000) and Maleki (2002), a Linear Programming Problem with Trapezoidal Fuzzy Variable (FVLP):

$$
\begin{aligned}
&\max \tilde{z} \underset{R(F)}{=} \tilde{c}x \\
&\text{s.t. } \tilde{A}x \underset{R(F)}{\leq} \tilde{b}, \\
&x \underset{R(F)}{\geq} \tilde{0},
\end{aligned}
\tag{1.12}
$$

had been considered by Mahdavi-Amiri and Nasseri (2007), where $\tilde{b} \in (\mathrm{F}(\mathbb{R}))^m, A \in \mathbb{R}^{(m,n)}, c^T \in \mathbb{R}^n$ are given and $\tilde{x} \in (\mathrm{F}(\mathbb{R}))^n$ is to be determined , and a linear ranking function on T$_p$FN; $\tilde{a} = (a^L, a^U, \alpha, \beta)$ defined as: $R(F)(\tilde{a}) = \frac{a^L+a^U}{2} + \frac{\beta-\alpha}{4}$. The dual problem on FVLP was established to deduce duality results, those results are then used to develop a dual algorithm to solve the problem by using the primal simplex tableau. It should be noted all these studies did not consider the MFLP problems in the optimization problems but contented themselves with single FLP problem. They also did not use the linear ranking function $R(F)$ as a tool to transform the FLP problem to its corresponding deterministic LP problem. $R(F)$s were used only as a tool to compare FVs.

1.3.3 Multiobjective Stochastic Linear Programming Problem

As related to the stochastic part of the MFSLPPFI, the first work on SLPPFI was presented by Ben Abdelaziz and Masri (2005a). The problem was modeled as:

$$
\begin{aligned}
&\text{Min } c^T(w)x \\
&\text{s.t. } T(w)x - h(w) \geq 0 \\
&x \in X
\end{aligned}
\tag{1.13}
$$

where $c(w)$, $T(w)$ and $h(w)$ were respectively $(n, 1), (m, n)$ and $(m, 1)$ random matrices defined on some probability space $(\Omega, 2^\Omega, P)$ with $\Omega = \{w_1, ..., w_N\}$ a finite set of possible states of nature, 2^Ω was the power set of Ω and P the vector of probabilities $p_i = P(\{w = w_i\})$. The set X is a polyhedral set of feasible solutions that includes the deterministic constraints of the problem on the probability space $(\Omega, 2^{\Omega}, P)$.

In this work, the partial information on the probability distribution P was with in two ways:

- The probability was generated by stochastic inequalities on π

$$\pi = \left\{ p = (p_1, ..., p_N)^t : Ap \leq b, \sum_{i=1}^{N} p_i = 1, p_i \geq 0, i = 1, ..., N \right\} \tag{1.14}$$

  where $A = (a_{ij})$ and $b = (b_i)$ were respectively (s, N) and $(s, 1)$ fixed matrices.

- Either the probability was generated by fuzzy inequalities on π, or the probability distribution was approximated on $\tilde{\pi}$ by

$$\tilde{\pi} = \left\{ p = (p_1, ..., p_N)^t : Ap \preceq b, \sum_{i=1}^{N} p_i = 1, p_i \geq 0, i = 1, ..., N \right\} \tag{1.15}$$

  where A and b were as defined above, and $\preceq$ was a fuzzy inequality which meant that Ap was almost equal or less than b.

The solution was obtained by first applying the fuzziness on the Stochastic Linear Programming with Fuzzy Linear Partial Information on probability distribution (SLPF), then through Stochastic Programming (SP) using Chance Constrained Approach (CCA), for minimizing expected value of the random objective functions on π, and RA after the solution when the deviation or shortage had been obtained.

Ben Abdelaziz and Masri (2010) also studied the Multiobjective Stochastic Linear Programming with Incomplement on probability distribution (MSLPI) problems:

$$\begin{aligned} &\text{Min } Z = C(w)x = [c_1(w)x, ..., c_n(w)x] \\ &\text{s.t. } T(w)x - h(w) \geq 0 \\ &x \in X \end{aligned} \tag{1.16}$$

They addressed the SP, and used the Chance Constrained Programming (CCP) approach as possible resolutions thus sustaining the indeterminate restraint probability level. They extended this method to the MSLPI as follows:

$$\begin{aligned} &\text{Min } Z = C(w)x \\ &\text{s.t. } P[T(w)x - h(w) \geq 0] \geq \alpha, \forall P \in \pi \\ &x \in X \end{aligned} \tag{1.17}$$

by using stochastic inequalities on π as in (1.4), and denoting F as the inferior likelihood purpose interrelated to the set as:

$$F(A) = \inf \{P(A)/P \in \pi\}; \forall A \in \Omega \tag{1.18}$$

This problem is equivalent to:

$$F[T(w)x - h(w) \geq 0] \geq \alpha. \tag{1.19}$$

By utilizing the Compromise Programming (CP) approach which was presented by Zeleny (1982) aimed at multi-objective problems, reducing the distances of the sum from objective- functions to their ideal values, and using the following: CCP approach, CP approach, and Chance Constrained Compromise Programming (CCCP) approach, the CP was addressed as:

$$\begin{aligned}&\text{Min } C(x,w)\\&\text{s.t. } F[(T(w)x - h(w) \geq 0)] \geq \alpha\\&x \in X\end{aligned} \tag{1.20}$$

Next, to minimize the value of;

$$\gamma(x) = \text{Max}_{P\in\pi} E_P[C(x,w)] \tag{1.21}$$

with some extra hypotheses by the Decision Maker (DM) the CCCP problem became:

$$\begin{aligned}&\text{Min Max}_{P\in\pi} E_P[C(x,w)]\\&\text{s.t. } F[(T(w)x - h(w) \geq 0)] \geq \alpha\\&x \in X,\ \delta(w) \geq 0\end{aligned} \tag{1.22}$$

They solved this optimization problem in Singleobjective Deterministic Linear Programming (SDLP) problem under two basic conditions:

(i) detail the form of the lower probability function F and,

(ii) the compromise function $C(x,w)$, or a weighted sum of the gap between the stochastic objective functions values $C(w)x$ and the ideal values c^* for x under event w.

For the value of F, the notion of P-Level Efficient Points (p^{LEP}) was used, and for the CP some hypotheses had been used in addition to the modified L-shaped method.

It must be mentioned that the limitations of the study by Ben Abdelaziz and Masri (2010) are the incomplete information in a linear way and the low value of the minimax solutions. They did not consider the situation where a feasible decision for the deterministic constraints did not satisfy the uncertainty constraints (i.e. deviation or shortage occurs). There are implicit loops of iterations in their solution algorithm when using L-shaped method. Finally, they did not consider fuzzy probability distributions and only depended on a CP approach to the MSLPI. In addition only maximum extreme points of P had been used. Also the probability distributions had been used on the maximum extreme points for the discrete events in the continuous interval of P.

1.3.4 Deterministic Multiobjective Linear Programming Problem

In Deterministic Linear Programming (DLP) problems of the MFSLPPFI; the following mixed deterministic multiobjectives Max/Min problem

$$\begin{aligned}
&\text{Max } E_{P\in\pi} Z_i = E_{P\in\pi} \sum_{j=1}^{n} C_{ij}(w) x_j \; ; i = 1, ..., r, \\
&\text{Min } E_{P\in\pi} Z_i = E_{P\in\pi} \sum_{j=1}^{n} C_{ij}(w) x_j \; ; i = r+1, ..., s, \\
&\text{s.t. } P(L(w)x - l(w)) \geq 0) \geq \alpha, \forall P \in \pi, \\
&x \in X
\end{aligned} \tag{1.23}$$

with conflict in the same constraints can be solved by using an appropriate technique to get an optimal solution for the original problem.

Sen (1983) provided a solution to this kind of problem when he obtained a single value corresponding to each of the objective functions being optimized individually, subjected to constraints as follows:

$$\text{Max } Z_i = \theta_i \; ; i = 1, ..., r, \quad \text{Min } Z_i = \theta_i \; ; i = r+1, ..., s. \tag{1.24}$$

where $\theta_i,\ i = 1, ..., r, r+1, ..., s$ the decision variable may not necessarily be common to all optimal solutions in the presence of conflicts among objectives. But the common sets of decision variables among objective functions were necessary in order to select the best compromised solution. After that, he determined the common set of decision variables from the combined objective function formulated below:

$$\begin{aligned} &\text{Max } Z = \sum_{i=1}^{r} \frac{Z_i}{\theta_i} - \sum_{i=r+1}^{s} \frac{Z_i}{\theta_i} \\ &\text{s.t. } P(L(w)x - l(w)) \geq 0) \geq \alpha, \forall P \in \pi, \\ &x \in X \\ &\forall Z_i \text{ and } \theta_i > 0, i = 1, ..., r, r+1, ...s. \end{aligned} \tag{1.25}$$

This methodology was highly efficient in its application in operation research to get optimal solutions and was thus considered one of the most appropriate methods in finding solutions. However the author did not mention whether the values of the objective functions were or were not positives. Figure 1.1 presents the scenario leading to the problem considered in this study.

1.4 Problem Statement

This study will focus on the development of an improved two-phase solution strategy of the MFSLPPFI. The phases are the defuzzification of the problem to its stochastic counterpart and the conversion of the stochastic problem to deterministic problem. Specifically we transform MFSLPPFI into its corresponding MSLPPFI through Fuzzy Trapezoidal Membership Function (FT$_p$MF) on the Trapezoidal Fuzzy Numbers (T$_p$FNs), then convert the resulting MSLPPFI into MLP problems through stochastic transformations via CCP approach in stochastic constraints, and using stochastic transformations in the objective functions by implementing expectation of the random events after employing the linguistic hedges of P to get the certain

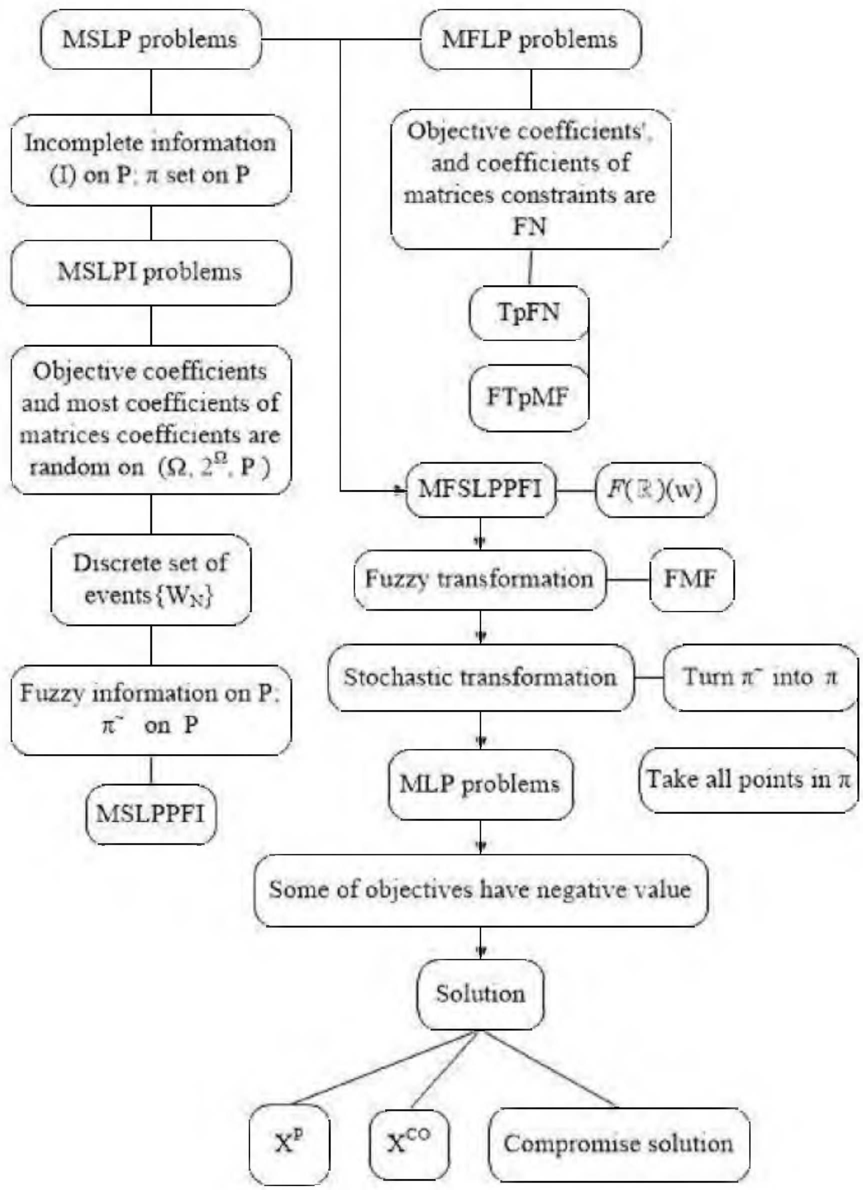

Figure 1.1 Scenario leading to the statement of the problem

probability distributions over intervals of P. In addition, an efficient solution method to the resulting deterministic problem will also be developed.

1.5 Research Questions

The problem statement raises several research challenges. These challenges will be addressed by providing answers to the following questions:

(i)
- How to address the fuzziness in the probability distribution?
- What membership function to use?

(ii)
- How to convert the FSP problem to SP problem?
- What fuzzy transformation technique to be used?

(iii) How to convert the SP problem to deterministic problem?

- What stochastic transformation technique to be used?
- What to do if the deterministic constraints do not fully satisfy the uncertain constraints?

(iv) How to solve the deterministic problem efficiently?

- How to avoid the implicit loops in L-shaped method? (The L-Shaped method is a decomposition method that is useful for solving problems that have the problem of a master problem and several sub problems represented by the side model, and it is an outer linearization procedure that approximates the convex objective term in the stochastic program by successively appending supporting hyper planes (Birge, 1988)).
- What criteria should be considered in evaluating the performance of the proposed solution method?.

1.6 Research Objectives

The following are the objectives in dealing with MFSLPPFI:

(i) To address the MFSLPPFI problem through membership function.

(ii) To transform the MFSLPPFI problem into MSLPPFI problem by using the ranking function as the fuzzy transformation technique. To use linguistic hedges in the probability distribution.

(iii) To transform the MSLPPFI problem into MLP problem by using the CCP Approach via fuzzy transformation in the probability distribution by using the α-cut technique.

(iv) To introduce recourse function in the stochastic transformation technique.

(v) To find a pareto optimal solution for the deterministic MLP.

(vi) To solve the deterministic MLP problem by using Big-M method instead of L-Shaped method to avoid implicit loops in the solution algorithm. And using the cutting-plane method.

(vii) To find a relation between the pareto optimal solution set and the compromise solution set, and compare between them.

1.7 Scope of the Study

This study focuses on the MFSLPPFI. Both objective function and constraints are in fuzzy stochastic forms. The solution of the original problem will be obtained from solving the associated deterministic problem.

1.8 Significance of the Study

This research focuses on developing the transformation techniques for the MFSLPPFI. The proposed techniques will convert the problem to deterministic problem which can be solved easily. The main contributions of this research on the advancement of knowledge are summarized as follows:

(i) Development of a fuzzy transformation technique that transforms an MFSLPPFI into an MSLPPFI problem through a ranking function. This is proposed to soften the rigid requirements of the DM to considering the fuzziness and/or randomness of the DMs judgment in real life optimization problems when he/she dealing with the optimization problem by considering the fuzziness and/or the randomness in objectives, goals and constraints in the problem. Moreover, the DM would be able to use the ranking function as a defuzzifying tool to transform the MFSLPPFI problem into an MSLPPFI problem in various real life situations.

(ii) Development of a stochastic transformation technique that transforms an MSLPPFI problem into an MLP problem using CCP approaches, Linguistic hedge and α-cut technique. The stochastic technique allows the DM to consider the expectation optimization model, the variance minimization model, and the probability optimization model with optimistic/pessimistic values under partial uncertainty of the probability distribution for real life problems. In addition, the DM will be able to recognize the interval values of the probability distribution by dealing with these interval values as linguistic hedges via α-cut technique on fuzzily imprecise variables with probabilistic uncertainty. Furthermore, the development of the stochastic transformation technique supports the DM to detail the form of the lower probability function for the stochastic constraints via the CCP approach. This will lead to the compromise function for the objective functions in the MSLPPFI problem.

(iii) Introduction of a recourse function in the stochastic transformation technique to provide a feasible decision for deterministic constraints which do not satisfy the uncertainty constraints. The significance of the recourse function approach is to provide a tool to help DM in industrial or productive optimization problems to create efficient models where continuous sufficient supplies of the raw materials and the basic resources for production are required.

(iv) Development of an Adaptive Arithmetic Average Method (AAAM) which transforms an MLP problems into its corresponding SDLP problem, instead of using Sen's method and others we found those in literature review. Sen's method is restricted to positive values of objective functions only, whereas AAAM is more general since it is valid for all real numbers. In addition, this approach leads to the compromise solution in less iterations and elapsed time in the solution algorithm.

(v) Development of pareto optimal solution to the deterministic MLP problems. Significantly, this solution finds the complete optimal solution as a compromise solution among conflicting objective functions in the MLP problems.

(vi) Development of the cutting-plane method as the solution technique to the associated MLP problem. Significantly, the cutting-plane method is to help the DM to reduce the obtained solution as an optimum solution for the objectives at the extreme point of the feasible constraints in the optimization problem.

Figure 1.2 shows the significance of the study in the advancement of the knowledge.

Beside academic contributions, the work also has practical contributions. The work will be able to help solve real life and industrial problems which are usually complicated, uncertain and continuously subject to changes, by considering both the fuzziness and randomness in the formulation of the model. In addition, the proposed solution procedure will provide an efficient and fast approach to solution generation which is important when dealing with real life problems which usually involve many variables and need to obtain optimum solutions quickly. The findings of the study

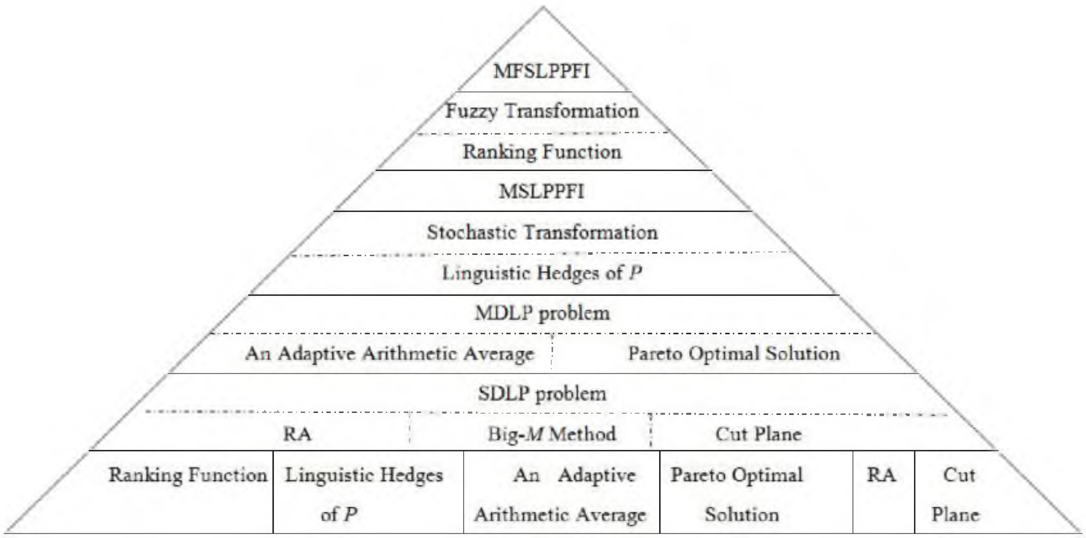

Figure 1.2 Summary of academic contributions

are believed to be able to have positive impacts on organization productivity and competitiveness in many industries.

1.9 Research Framework

Figure 1.3 and 1.4 shows the research framework and the relative knowledge areas related to each component of the MFSLPPFI. In what follows, some major components of the conceptual framework will be briefly described.

We consider MFSLPPFI and use ranking function technique to transform it into MSLPPFI. Then we use α-cut technique to defuzzify the probability distribution from fuzzy assertion into deterministic form to get MSLP problems. After that, by utilizing linguistic hedges of the probability distribution through stochastic transformation, and in addition CCP approach, we convert MSLP problems into its corresponding MDLP problems.

An AAAM will be used to transform the resulted MDLP problems into its corresponding SDLP problem. The research uses Big-M method to solve the SDLP problem. If the solution does not satisfy the uncertain constraints it should be penalized by using RA, and if is it not an extreme point of the feasible constraints, then cut-plane method should be used to introduce it as an extreme point of the feasible constraints.

We also find pareto optimal solution for the MDLP problems and comparing between it and the compromise solution.

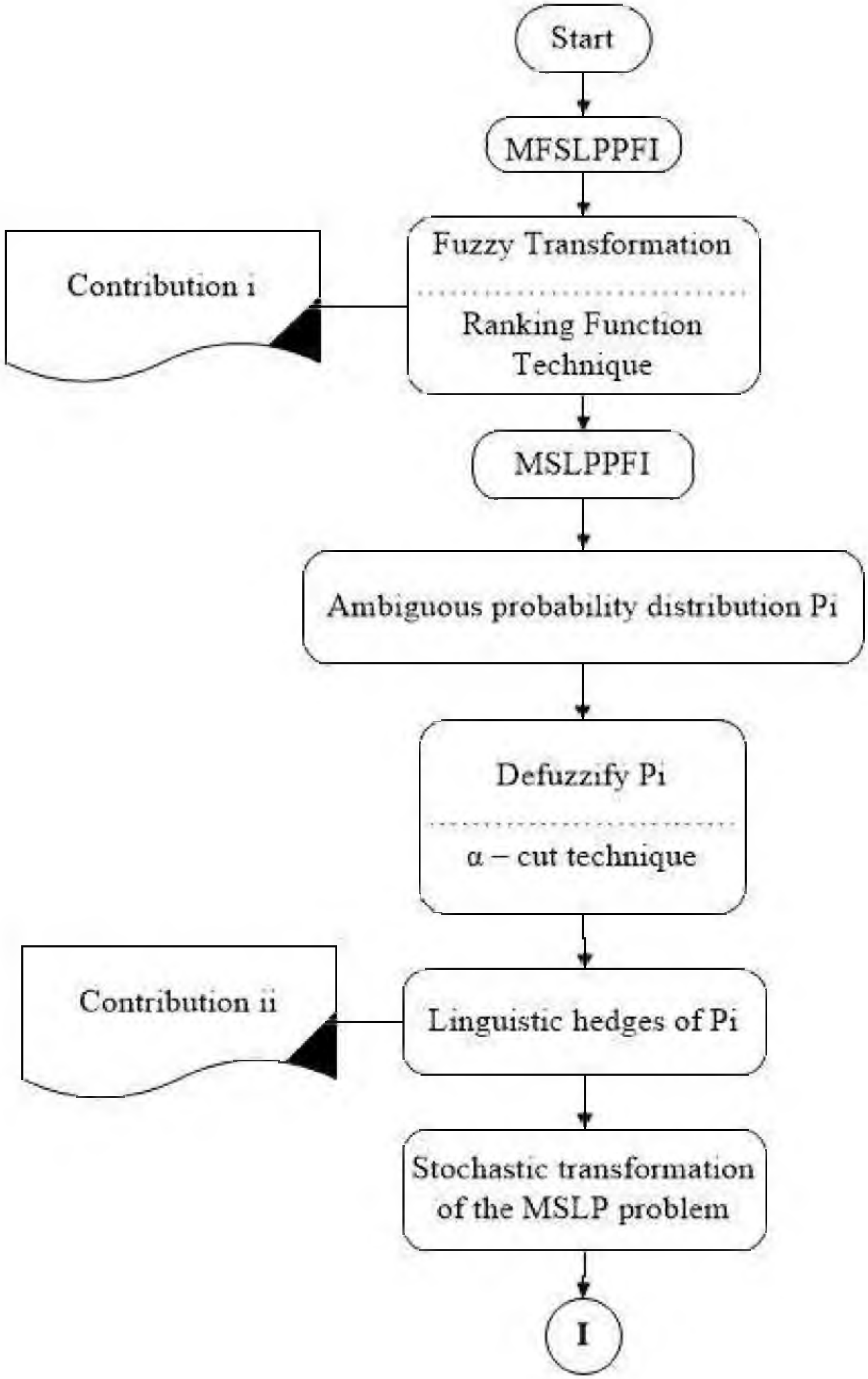

Figure 1.3 Conceptual framework: Part - I

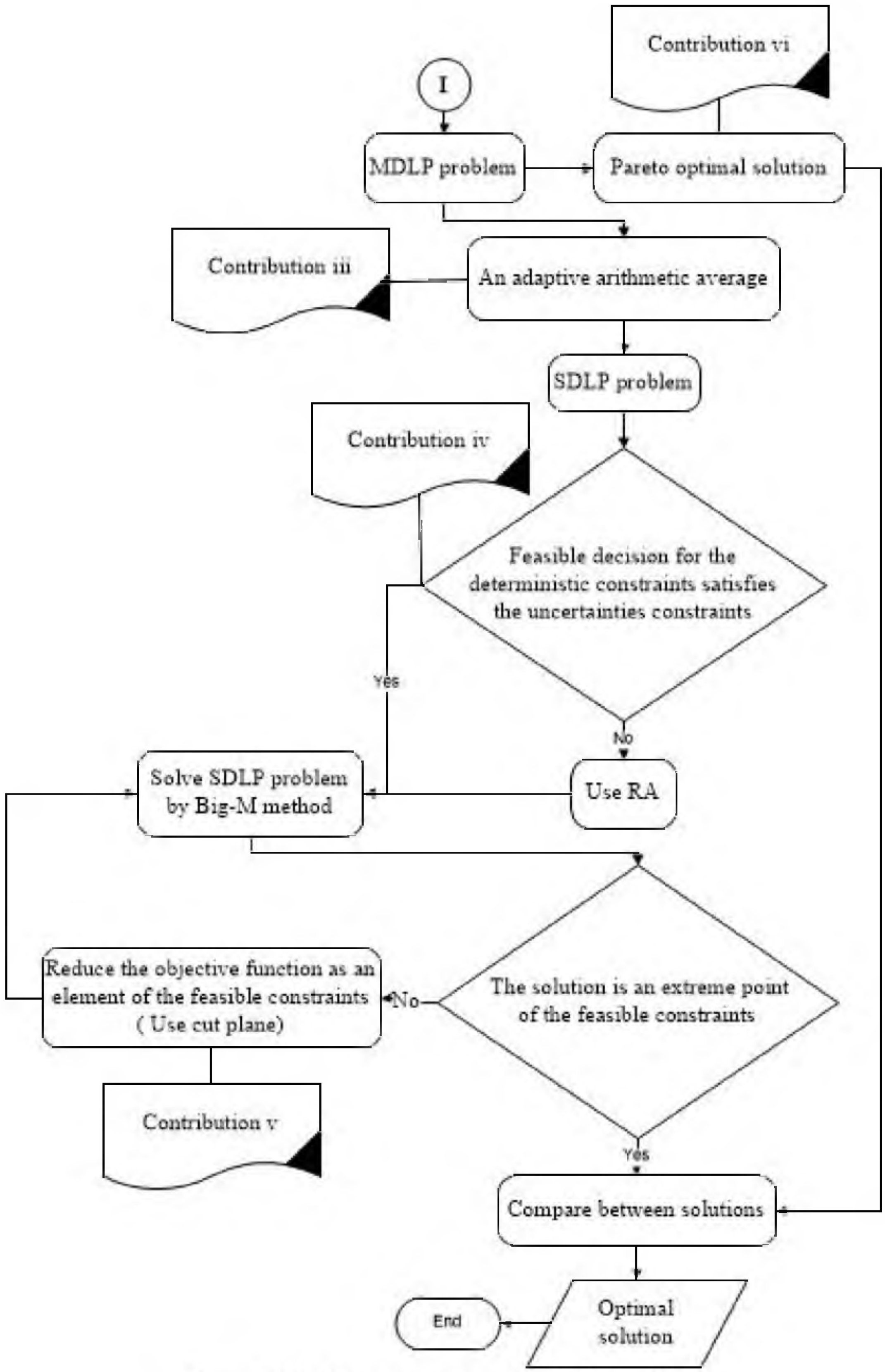

Figure 1.4 Conceptual framework: Part - II

1.10 Outline of the Thesis

The organization of the thesis is as follows:

Chapter 1 provides the introduction to the study domain of this thesis that is MFSLPPFI. The chapter later discusses the background of the problem, statement of the problem, research objectives and contributions.

Chapter 2 provides the literature review of the study areas. Related background works on the study domain are also discussed here.

Chapter 3 provides research methodology where the research activities that will be carried out towards achieving the objectives of this research are presented.

Chapter 4 provides the development of the improved two-phase solution strategy for the MFSLPPFI.

Chapter 5 provides the applications to support the problem statements, research methodology and development of the improved two-phase solution strategy for the MFSLPPFI, through numerical examples.

Chapter 6 presents the results and conclusion of the present work, contributions and further works.

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