# IMPROVED PERFORMANCE OF MCUSUM CONTROL CHART WITH AUTOCORRELATION

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**Abstract.**: In recent years, the importance of quality has become increasingly apparent, and quality control in manufacturing has moved from detecting nonconforming products through inspection to detecting quality abnormalities in the process using statistical process control [1]. where it is used effectively, SPC plays an important role in reducing variation in manufactured items and in increasing the competitiveness of the manufacturer by improving product quality while at the same time decreasing production costs. Charts like the Shewhart X and R charts have found wide use in industry because of their ease of use for technicians and others with minimal training in statistics, since the calculations and plotting can be done by hand. An MCUSUM control chart was constructed with autocorrelated data at different levels of autocorrelation and found to be ineffective in detecting shifts as it occurs. In this article, we have proposed new techniques that can improve the performance of the MCUSUM with autocorrelation using run rule schemes. The techniques was evaluated using ARL measures of performance with 10000 iterations to simulate. The results showed that the performance of MCUSUM with autocorrelation has improved significantly with the new technique which was compared to the existing conventional MCUSUM control chart.

*Keywords* **:** MCUSUM; Autocorrelation; Run Rule; Average run length; SPC

## 1.0 INTRODUCTION

Control charts were originally developed at Bell laboratories by Dr. Walter Shewhart in 1924 specifically to help detect statistical changes in process quality, control charts have since become one of several primary tools of quality control process improvement. A basic assumption in traditional application of statistical process control techniques is that the observations from the processes under investigation are normally and independently distributed (i.i.d). when these assumptions are satisfied, conventional control charts may be applied. however, the independence assumption is often violated in practice.

Autocorrelation is present in the data generated by most continuous and batch process operations as the value of the particular parameter under monitoring is dependent on the previous value of the parameter. It is more apparent for data collected with frequent sampling but can also be due to the dynamics of the processes. for example, observations from automated test and inspection procedures where every quality characteristics is measured on every unit in time order of production, or measurement of process variables. The effect of autocorrelation in the process data within SPC schemes is that it produces control limits that are tighter than desired. which causes an increase in the average false alarm rate and decrease in the ability of detecting changes on the process. control charts acts as the most important statistical process monitoring tool, widely used for the purpose of identifying unusual variations in process parameters.

Even small levels of autocorrelation between successive observations can have big effects on the statistical properties of conventional control charts.[2] and [3] derived approximate run length distribution for the cumulative sum control chart [4] when the process follows an autoregressive process AR(1) or a moving average process MA(1) model.

Since the introduction of CUSUM charts by [5], many researchers have examined these charts from different perspective: for example, [6], [7],[8], [9]. Cusum charts are widely used for the efficient monitoring of internal quality control parameters and their use in analytical laboratories has been emphasized by many researchers including [10] and [11].

Multivariate statistical process control methods are applicable when several process variables are simultaneously monitored. These methods use the relationship between variables to generate powerful control algorithms which are sensitive to assignable causes that are poorly detected by univariate control charts on individual observations. [8] described how a p-directional multivariate normal process can be monitored by using *p* univariate CUSUM charts for the p original variables by using p univariate CUSUM charts for the *p* principal components, which is called the MCUSUM. The MCUSUM gives an out-of-control signal whenever any of the univariate CUSUM charts does same. Also they found that the MCUSUM and MEWMA charts with adjusted control limits are more sensitive in detecting small shifts than the multivariate Shewhart chart. However, their findings is only limited to detecting small shifts with correlated data, when they investigated the impacts of autocorrelated data on the ARL of multivariable Shewhart, MCUSUM and MEWMA charts.

Several different approaches of dealing with autocorrelation has been given in the literature, among the few is [12].

In this paper, we investigated the effect of autocorrelation to the performance of MCUSUM control chart when observations are autocorrelated and reviewed some related literatures. In section 2, the methods and materials used in the study was discussed. While in section 3 the results of the study was discussed and the conclusion in section 4.

## 2.0 EXPERIMENTAL

### 2.1 Materials and Methods

[13] and [14] used Hotelling's $T^2$ statistic to form a monitoring scheme. Denoting $x_i$ a measurement from a sequence of autocorrelated observations which is Gaussian with mean μ and auto covariance function $\gamma_i = E[(x_i - \mu)(x_{i+k} - \mu)]$ and consider $p$-dimensional vectors $x_i$ formed from observations of the univariate process. If the autocorrelated process follows an ARMA time series model $X_i$ is multivariate normal and its covariance matrix is given by

$$= \begin{bmatrix} \gamma_0\, \gamma_1 \cdots \gamma_{p-1} \\ \gamma_1\, \gamma_0 \cdots \cdots \cdots \\ \cdots \cdots \cdots \cdots \cdots .. \\ \cdots \cdots \cdots \cdots \cdots .. \\ \gamma_{p-1} \cdots \gamma_1, \gamma_0 \end{bmatrix} \quad (1)$$

When the process is in control, the mean vector is $\mu_0 = [\mu_0, \dots, \mu_0]$ where $\mu_0$ denotes the in control process mean and assumed known. If $\Sigma$ is known the $T^2$ statistic is given as $T_i^2 = [X_i - \mu_0]\ \Sigma^{-1}\ [X_i - \mu_0]'$, follows a chi square distribution with p degrees of freedom when the process is in control.

MCUSUM chart proposed by [15] is based on the transformation $\sqrt{n}\mathbf{B}\boldsymbol{P_0^{-1}S_i}$. The plotted statistic is given by $Y_i$,

$S_i^* = \sum_{j=i-l_i+1}(\Lambda Z_i + \sqrt{n} de_1 - \frac{Z_0}{\sqrt{m}})$ and

$$Y_i = Max\left\{0, \sqrt{v(S_i^*)^T W_0^{-1}(S_i^*)} - kl_i\right\} \quad (2)$$

where $k$>0 and $l_i = l_{i-1}+1$, if $Y_i$ > 0 and 1. which is used to compute the control limits.
The proposed schemes is to be used as recommended below:

i. Choose the scheme of either 4/4 or 4/5 for quick detection of shift.
ii. Decide on the in-control limits of the MCUSUM control chart.
iii. Compute the control limits of the MCUSUM control chart proposed by Pignatiello and Runger (1990).
iv. Use the sensitivity analysis for comparison of the in-control ARL.`

In this article we are proposing new run rules that can improve the performance of MCUSUM control chart with autocorrelation, this has been observed in the study of multivariate control charts' performance with autocorrelation reported by [4], where the Hotelling's $T^2$ control was constructed with autocorrelated data and found that the charts' performance wasn't as expected as a result of autocorrelation present in the data. Also [16], reported that really autocorrelation affects the performance of MCUSUM control charts and they concludes that if the residuals from a time series model are used instead of the original data, then the ARL properties can be improved considerably. However, their study investigated the effects of the autocorrelation, while we are going to introduce the run rule schemes 4 out of 4 and 4 out 5 that can improve the performance of the MCUSUM control chart with autocorrelation.

## 2.1 The Run Rule Schemes

According to [17], proposed two more supplementary run rules schemes that will enhance the performance of Shewhart control chart; they are 3/3 and ¾ schemes. However, their schemes were only limited to linear trend random values, so, we intend to introduce the run rule schemes that will improve the performance of MCUSUM with autocorrelation, based on their designed methods for the 4/4 and 4/5 runs rules schemes with autocorrelated data. A source code in R package was developed to simulate the ARL values of the run rule schemes based on these conditions below.

i. 4 out of 4 run rule: A process is said to be out of control if 4 consecutive points/observations are below the lower control limits or above the upper control limits.

ii. 4 out of 5 run rule: A process is said to be out of control if any 4 out of 5 consecutive points /observations are below the lower control limit or above the upper control limits.

## 3.0 RESULTS AND DISCUSSION

Tables 1-3 shows the results of the simulated ARL values for the introduced 4 out of 4 and 4 out of5 run rules scheme that will enhance the performance of the MCUSUM control chart with autocorrelation. the results was obtained a Markov Chain method at 10,000 simulations at 0.75 autocorrelation value with 200, 500 and 1000 in-control average run length ($ARL_0$). The results indicates that the 4/4 scheme detects shift quickly than the 4/5 and the MCUSUM proposed by [15] in Tables 1-3. To further buttress the results of the introduced run rules schemes a comparison of the ARL values were displayed on graphs which clearly shows the quickly detection ability of the MCUSUM control chart with autocorrelation can be enhance by introducing the 4/4 and 4/5 schemes as compared with the traditional MCUSUM proposed by [15]. The ARL curves are shown in Figures 1-3 below:

**Table 1: The ARL values for the 4/4, 4/5 and MCUSUM with $ARL_0$=200**

| H=0.75, H=2.5, ARL=200 | | | |
|---|---|---|---|
| Shift | 4/4 run rule | 4/5 run rule | MCUSUM AR(1) |
| 0 | 200 | 199.5 | 205.1 |
| 0.05 | 160.3 | 150.4 | 178.5 |
| 0.1 | 140.6 | 130.5 | 155.6 |
| 0.15 | 120.5 | 110.7 | 141.8 |
| 0.2 | 100.3 | 95.6 | 123.5 |
| 0.25 | 78.6 | 70.5 | 80.7 |
| 0.5 | 28.7 | 25.7 | 32 |
| 1 | 14.5 | 15.8 | 16.9 |
| 1.5 | 4.5 | 4.6 | 4.7 |
| 2 | 1.8 | 2.5 | 2.9 |
| 3 | 1.4 | 1.45 | 1.5 |
| 4 | 1.2 | 1.3 | 1.4 |
| 5 | 1 | 1.05 | 1.1 |

**Table 2: The ARL values for the 4/4,4/5 and MCUSUM with $ARL_0$=500**

| H=0.75, H=2.5, ARL=500 | | | |
|---|---|---|---|
| Shift | 4/4 run rule | 4/5 run rule | MCUSUM AR(1) |
| 0 | 499.5 | 487.5 | 500 |
| 0.05 | 459.4 | 460.7 | 450.6 |
| 0.1 | 350.8 | 400.7 | 430.7 |
| 0.15 | 200.7 | 350.7 | 390.6 |
| 0.2 | 190.8 | 240.5 | 250.7 |
| 0.25 | 180.5 | 210.6 | 200.4 |
| 0.5 | 150.7 | 190.3 | 180.5 |
| 1 | 60.3 | 70.6 | 80.5 |
| 1.5 | 20.7 | 25.5 | 30.5 |
| 2 | 12.5 | 18.7 | 15.4 |
| 3 | 2.5 | 6.5 | 4.5 |
| 4 | 1.2 | 2 | 1.5 |
| 5 | 1 | 1.05 | 1.4 |

**Table 3: The ARL values for 4/4,4/5 and MCUSUM with $ARL_0$=1000**

| H=0.75, H=2.5, ARL=1000 | | | |
|---|---|---|---|
| Shift | 4/4 run rule | 4/5 run rule | MCUSUM AR(1) |
| 0 | 1000 | 1001 | 1000 |
| 0.05 | 800.5 | 750 | 850.7 |
| 0.1 | 760.7 | 720.5 | 740.8 |
| 0.15 | 650.2 | 530.5 | 700.6 |
| 0.2 | 500.3 | 430.6 | 560.8 |
| 0.25 | 430.6 | 420.5 | 450.6 |
| 0.5 | 310.7 | 400.6 | 420.5 |
| 1 | 150.6 | 300.7 | 380.2 |
| 1.5 | 85.5 | 200.8 | 220.5 |
| 2 | 20.7 | 80.5 | 110.5 |
| 3 | 6.8 | 15.7 | 85.7 |
| 4 | 4.5 | 5.1 | 5.6 |
| 5 | 1 | 2 | 3 |

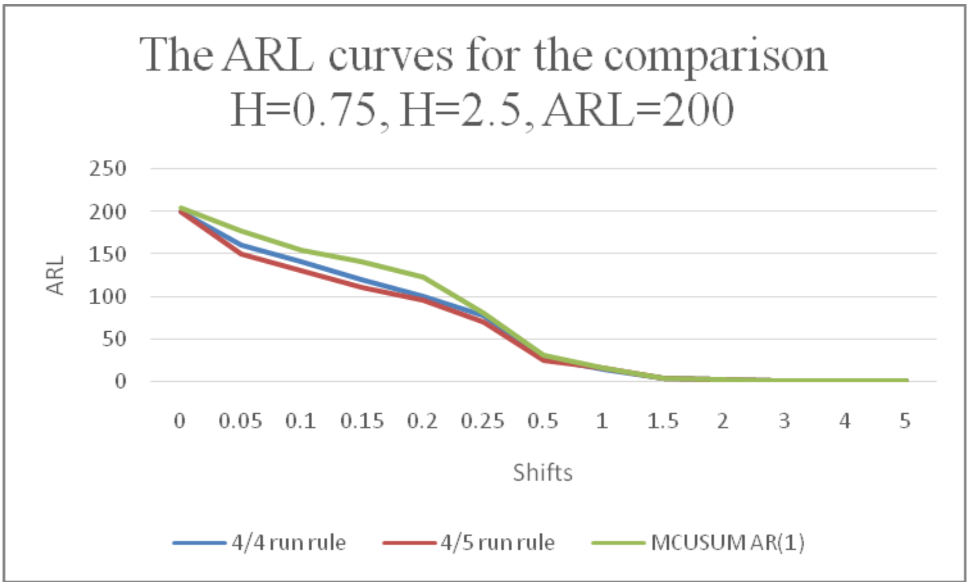


**Figure 1: The ARL curves showing the comparison between them with $ARL_0$=200**

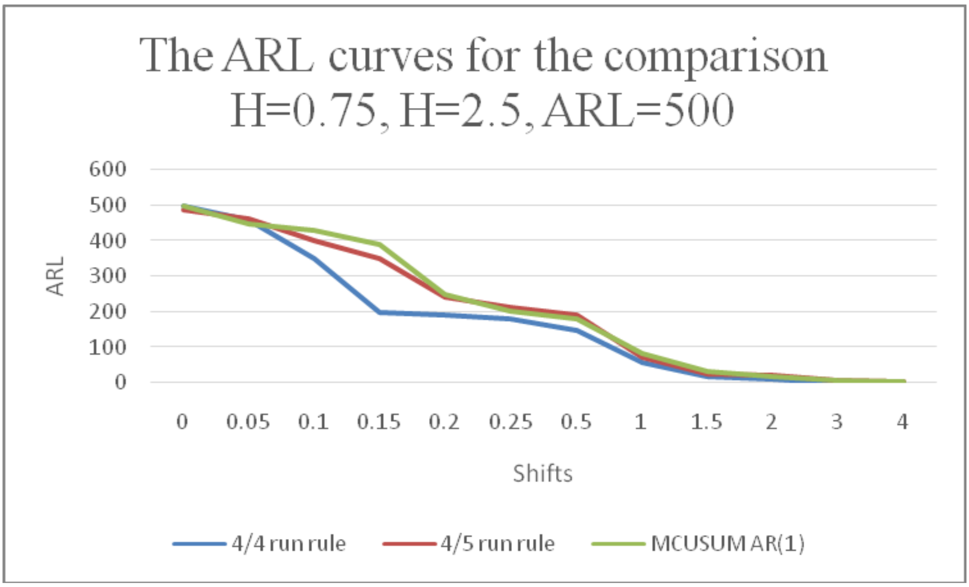


**Figure 2: The ARL curves showing the comparison between them with $ARL_0$=500**

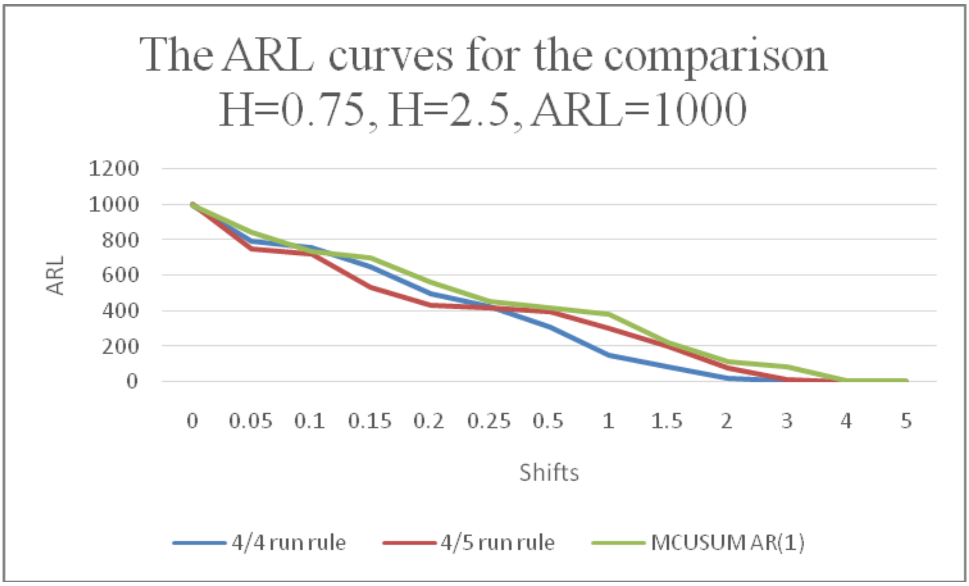


**Figure 3: The ARL curves showing the comparison between them with $ARL_0$=1000**

## 4.0 CONCLUSIONS

In this paper, a multivariate AR(1) process was used to investigate the performance of MCUSUM control chart with autocorrelation, of which we introduced a run rules techniques to enhance the performance of the MCUSSUM with autocorrelation. A simulation study was carried out to generate the ARL values at 10000 simulations with 0.75 correlation value. From the results the 4/4 and 4/5 run rules schemes detect shift quickly better than the traditional MCUSUM control chart. Hence, this gives them a good chance to outperforms the MCUSUM in detecting shift as at when it occurs. based on this study we can conclude that the 4/4 scheme is better than the 4/5 in quick detection of shift in a given process and its easier for the practitioners to implement for improve production as well as quality services.

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