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THE UNSTEADY FREE CONVECTION FLOW OF ROTATING MHD SECOND GRADE FLUID IN POROUS MEDIUM WITH EFFECT OF RAMPED WALL TEMPERATURE

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Abstract. In this paper, the unsteady free convection flow of an incompressible second grade fluid in a vertical plate with ramped wall temperature is investigated. The fluid in magnetohydrodynamics (MHD) flows through a porous medium is also considered. The fluid is electrically conducting under the assumption of a small magnetic Reynolds number applied in a vertically inward direction to the flow. The governing equations are modeled in a rotating plate such that both the fluid and plate in unison with angular velocity. The exact solutions of velocity and temperature are obtained in closed form by using Laplace transform method. These solutions are presented graphically and discussed for second grade parameter α, rotation parameter ω, porosity parameter K, magnetic parameter M, Prandtl number $\Pr$, and Grashof number Gr.

Keywords Second grade fluid; free convection; rotating; MHD; porous medium; ramped wall temperature

1.0 INTRODUCTION

The study on MHD flow and porous medium (porosity) in rotating frame have been stimulated the interest of researcher in fluid mechanic problems due to their wide range of scientific applications in various fields. Asghar *et al*. [1] studied the steady flow of a rotating third grade fluid past a porous plate using analytical method. The results nonlinear boundary value problem has been solved using HAM. They concluded that the rotation causes a reduction in the layer thickness. Tiwari and Ravi [2] investigated the transient rotating flow of an incompressible second grade fluid in a porous medium. They used Laplace transform technique to find mainly two basic flow situations which are a sudden started and a constant acceleration flow, respectively. They found that both steady and unsteady solutions are strongly dependent upon the porosity parameter.

Moreover, Hayat *et al*. [3] worked on the hydromagnetic oscillatory flow of an incompressible second grade fluid bounded by a porous plate, when the entire system rotates about an axis normal to the plate. The magnetic field is applied transversely to the direction of the flow and the problem has been solved analytically for steady and unsteady cases by using Laplace transform technique. The analysis of the obtained results showed that the flow is influenced by the parameters of second grade fluid, rotation and applied magnetic field. Sajid *et al*. [4] discussed the MHD rotating boundary layer flow of viscous fluid caused by the shrinking surface. The similarity transformations have been used for reducing the partial differential equation into a system of two coupled ordinary differential equations. Then, HAM is employed for the analytical solution. They found that hydrodynamic flow is not stable for the shrinking surface and only MHD flow is meaningful in the case of shrinking surface. Moreover, Das *et al*. [5] discussed the unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system. They solved for exact solution of the governing equation by using Laplace transform method. Solution for the velocity distribution as well as shear stress has been obtained for small times and also for large times.

Furthermore, Hayat *et al*. [6] also discussed the unsteady rotating MHD flow of an incompressible second grade fluid but considered in a porous medium. The flow is induced by a suddenly moved plate in its own plane where both the fluid and plate rotate in unison with the same angular velocity. Their analytical solutions are obtained by using Fourier Sine transform. The existing solutions of Newtonian fluid have been deduced as limiting cases by choosing second grade parameter equal to zero. Then, Hayat *et al*. [7] produced a result on MHD flow in a porous medium of Maxwell fluid. They also using a same Fourier Sine transform to obtain the analytical solution of the governing equation. The result of the Newtonian case has been obtained as a special case by taking Maxwell fluid parameter equal to zero.

Recently, Khan *et al*. [8] studied the rotating flow of second grade fluid, where the fluid is electrically conducting and fills the porous region at $z > 0$ for constant and variable accelerations. In this problem, Laplace transform method has been used and expressed the results obtained as a sum steady-state and transient solutions. Also, Salah *et al*. [9] have solved a same problem as Khan *et al*. [8] but they produced the exact solution by using Fourier Sine transform and Laplace method. However, in these results, they are not using steady-state and transient solutions for satisfying the initial and boundary conditions.

The temperature conditions near the wall play an important role in several industrial applications and it was extensively studied by a number of researchers using different sets of thermal conditions at the boundary plate. Chandra *et al*. [10] studied the effect of ramped wall temperature on unsteady free natural convective flow of incompressible viscous fluid near a vertical plate. The dimensionless governing equations have been solved by using Laplace transform method. Then, Rajesh [11] also studied the unsteady free convection incompressible viscous fluid in vertical plate with ramped wall temperature but in presence of thermal radiation and MHD effects. The author used Laplace transform method to find the analytical solution. Deka *et al*. [12] has produced the exact solution for unsteady natural convection flow past an infinite vertical plate with ramped wall temperature passing through a porous medium. Laplace transform also has been used to solve the dimensionless governing equations of the fluid flows.

Then, Seth *et al*. [13] investigated the influence of radiation on unsteady hydromagnetic natural convection transient flow near an impulsively moving flat plate with ramped temperature in a porous medium. In this problem, they used Laplace transform method to solve the governing equations. They found that, the magnetic field tends to decelerate fluid flow whereas thermal buoyancy force, radiation and permeability of porous medium have reverse effect on it for both ramped temperature and isothermal plate. Here, fluid velocity and fluid temperature are found to be smaller in case of ramped temperature plate compared to the case of isothermal plate. The reason is that the ramped wall temperature has difference and various temperatures in every single time.

Motivated by previous studies of rotating MHD flow in porous medium, in this present study we are interested to tackle the problem of unsteady free convection flow of rotating MHD second grade fluid in porous medium with ramped wall temperature. The governing equations for this problem also solved by using Laplace transform technique.

2.0 MATHEMATICAL FORMULATION AND SOLUTION

Let us consider the unsteady free convection flow of an incompressible second grade fluid in the rotating infinite vertical plate with ramped wall temperature. The x-axis is taken along the plate in the upward direction and z-axis is taken normal to plate. Initially, both the fluid and the plate are at rest to a constant temperature T_∞. At $t = 0^+$, both of fluid and plate start to rotate with constant angular velocity Ω parallel to z-axis. A uniform transverse magnetic field of strength B_0 is applied parallel to the axis of rotation. It is assumed that induced magnetic field, the external electric field and the electric field due to polarization of charges are negligible. At the same time, the temperature of the plate is raised or lowered at $T_\infty + (T_w - T_\infty)t/t_0$ when $t \leq 0$, and thereafter, for $t > t_0$, is maintained at the constant temperature T_w. The main objective of this research is to study the heat transfer process of fluid motion in rotating plate. Under the usual assumption of Boussinesq approximation, the governing equations of momentum $F(z,t)$ and energy $T(z,t)$ are given as

$$\frac{\partial F}{\partial t} + 2i\Omega F = \upsilon \frac{\partial^2 F}{\partial y^2} + \frac{\alpha_1}{\rho} \frac{\partial^3 F}{\partial y^2 \partial t} - \frac{\sigma B_0^2}{\rho} F - \frac{\mu \phi}{\rho k_1}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right) F + gB\left(T - T_\infty\right) \tag{1}$$

and

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial z^2}, \tag{2}$$

in which $F = u + iv$ is the complex velocity where u and v are its real and imaginary parts, ρ designates the density of the fluid, υ the kinematic viscosity, α_1 second grade parameter, σ is electrical conductivity, $\phi(0 < \phi < 1)$ the porosity and $k_1 > 0$ the permeability of the porous medium, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, T is the temperature of the fluid, k is thermal conductivity and c_p is the specific heat capacity of the fluid at constant pressure. The appropriate initial and boundary conditions are

$$F(z,0) = 0\ ;\ z > 0 \tag{3}$$

$$F(0,t) = 0\ ;\ t > 0 \tag{4}$$

$$F(z,t) \to 0, \text{ as } z \to \infty,\ t \geq 0, \tag{5}$$

and

$$T(z,0) = T_\infty\ ;\ z > 0 \tag{6}$$

$$T(0,t) = \begin{cases} T_\infty + (T_w - T_\infty)\dfrac{t}{t_0}\ ;\ 0 < t \leq t_0 \\ T_w\ ;\ t > 0 \end{cases} \tag{7}$$

$$T(z,t) \to 0 \quad \text{as } z \to \infty,\ t \geq 0, \tag{8}$$

Introducing the following non-dimensional variables

$$F^* = \frac{F}{v_0},\ z^* = \frac{v_0}{\upsilon}z,\ t^* = \frac{t}{t_0},\ T^* = \frac{T - T_\infty}{T_w - T_\infty},\ \Pr = \frac{\mu c_p}{k}. \tag{9}$$

Substitute dimensionless variables (9) into equations (1) and (2) and dropping out the star "*" notation, we get

$$\frac{\partial^2 F}{\partial z^2} = a_1\frac{\partial F}{\partial t} + b_0 F - \alpha\frac{\partial^3 F}{\partial z^2 \partial t} - GrT \tag{10}$$

and

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2}, \tag{11}$$

which are

$$a = \frac{\upsilon}{v_0^2 t_0}, \omega = \frac{\upsilon\Omega}{v_0^2}, M^2 = \frac{\sigma\upsilon B_0^2}{v_0^2\rho}, \frac{1}{K} = \frac{\phi\upsilon^2}{k_1 v_0^2}, \alpha = \frac{\alpha_1 v_0^2}{\rho\upsilon^2},$$

$$a_1 = a + \frac{\alpha}{K}, b_0 = 2i\omega + M^2 + \frac{1}{K}, Gr = \frac{t_0}{v_0}g\beta(T_w - T_\infty).$$

According to the above non-dimensionalisation process, the characteristic time t_0 can be defined as $t_0 = \dfrac{\upsilon}{v_0^2}$ [Seth *et al*. [13]].

Then, the dimensionless initial and boundary conditions (3-8) are

$$F(z,0) = 0, \tag{12}$$

$$F(0,t) = 0\ ;\ \ t > 0, \tag{13}$$

$$F(z,t) \to 0,\ as\ z \to \infty,\ t \ge 0, \tag{14}$$

and

$$T(z,0) = 0;\ z \ge 0, \tag{15}$$

$$T(0,t) = \begin{cases} t\ ,\ 0 < t \le 1 \\ \\ 1\ ,\ t > 1 \end{cases} = tH(t) - (t-1)H(t-1) \tag{16}$$

$$T(z,t) \to 0\ \ as\ z \to \infty,\ t \ge 0, \tag{17}$$

where *H*(.) is the Heaviside step function.

The governing equations (10)-(11) subject to initial conditions (12) and (15) are solved by using the Laplace transform technique. Hence, by taking Laplace transform we get

$$\frac{d^2\bar{F}(z,q)}{dz^2} - \left(\frac{a_1 q + b_0}{1 + \alpha q}\right)\bar{F}(z,q) = -\frac{Gr\bar{T}(z,q)}{1+\alpha q}, \tag{18}$$

and

$$\frac{d^2\bar{T}(z,q)}{dz^2} - q\,\mathrm{Pr}\,\bar{T}(z,q) = 0;\ z,q > 0, \tag{19}$$

together with the boundary conditions in the transformed q-domain

$$\bar{F}(0,q) = 0, \tag{20}$$

$$\bar{F}(z,q) \to 0\ as\ z \to \infty,\ t \ge 0 \tag{21}$$

and

$$\bar{T}(0,q) = \frac{1 - e^{-q}}{q^2}, \tag{22}$$

$$\bar{T}\left(z,q\right)\to 0 \;\; as\; z\to\infty,\; t\ge 0. \tag{23}$$

The solution for equation (19) by using the boundary conditions (22-23) is

$$\bar{T}\left(z,q\right)=\frac{1-e^{-q}}{q^{2}}\left(e^{-z\sqrt{\Pr}\sqrt{q}}\right). \tag{24}$$

In order to solve the solution of equation (18), substitute equation (24) into equation (18) and Laplace transform with corresponding conditions (20-21) is given by

$$\bar{F}\left(z,q\right)=\left(1-e^{-q}\right)\bar{F}_{1}\left(q\right)\bar{F}_{2}\left(z,q\right), \tag{25}$$

where

$$\bar{F}_{1}\left(q\right)=\frac{1}{q\left[\left(q+m_{1}\right)^{2}-m_{2}^{2}\right]}\cdot\frac{Gr}{\alpha\Pr} \tag{26}$$

and

$$\bar{F}_{2}\left(z,q\right)=\frac{1}{q}e^{-z\sqrt{\frac{a_{1}q+b_{0}}{\alpha q+1}}}-\frac{1}{q}e^{-z\sqrt{q}\sqrt{\Pr}} \tag{27}$$

which are $m_{1}=\dfrac{\Pr-a_{1}}{2\alpha\Pr}$ and $m_{2}=\dfrac{\sqrt{\left(\Pr-1\right)^{2}+4\alpha\Pr b_{0}}}{2\alpha\Pr}$.

The inverse Laplace transform for equation (26) can be directly obtained by using partial fraction. Then, we have

$$F_{1}\left(t\right)=\frac{Gr}{b_{0}m_{2}}\left[m_{1}\sinh\left(m_{2}t\right)+m_{2}\cosh\left(m_{2}t\right)\right]e^{-m_{1}t}-\frac{Gr}{b_{0}}. \tag{28}$$

According to Khan *et al*. [8], we need to use inversion compound function for first term of right hand side of equation (27). Hence we get

$$F_{2}\left(z,t\right)=\frac{a_{1}}{\alpha}e^{\frac{-1}{\alpha}t}\int_{0}^{\infty}erfc\left(\frac{z}{2\sqrt{u}}\right)e^{\frac{-ua_{1}}{\alpha}}I_{0}\left(\frac{2}{\alpha}\sqrt{ua_{2}t}\right)du+ \frac{b_{0}}{\alpha}\int_{0}^{\infty}\int_{0}^{t}erfc\left(\frac{z}{2\sqrt{u}}\right)e^{\frac{\left(-ua_{1}-s\right)}{\alpha}}I_{0}\left(\frac{2}{\alpha}\sqrt{ua_{2}s}\right)dsdu-erfc\left(\frac{1}{2}\sqrt{\frac{\Pr}{t}}z\right). \tag{29}$$

Therefore, the inverse Laplace transform of equations (24) and (25) will be obtained by using the second shift property

$$L^{-1}\left\{e^{-aq}F(q)\right\} = f(t-a)H(t-a) \text{ if } f(t)=L^{-1}\left\{F(q)\right\}. \tag{30}$$

Hence, by introducing equations (28) and (29), solution for equation (25) is

$$F(z,t)=U(z,t)H(t)-U(z,t-1)H(t-1), \tag{31}$$

and by convolution theorem

$$U(z,t)=(F_1 * F_2)(t)=\int_0^t F_1(t-s)F_2(z,s)\,ds. \tag{32}$$

From equation (24), we write

$$T_1(z,t)=L^{-1}\left\{\frac{1}{q}\left[\frac{1}{q}e^{-z\sqrt{\Pr}\sqrt{q}}\right]\right\}=\left(\frac{z^2\Pr}{2}+t\right)erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}\right)-\frac{z\sqrt{\Pr t}}{2\sqrt{\pi}}e^{-\frac{z^2\Pr}{4t}}, \tag{33}$$

we obtain the temperature distribution as

$$T(z,t)=T_1(z,t)-T_1(z,t-1)H(t-1). \tag{34}$$

In this paper, we are also discussed the solution for isothermal temperature at the plate. This is because, we want to compare the results of fluid flow near a plate with ramped wall temperature. In this case, the initial and boundary conditions are the same excepting equation (7) that becomes $T(0,t)=1$ for $t \ge 0$. Therefore, the expression for dimensionless temperature for isothermal plate in equation (19) is obtained as

$$\bar{T}(z,q)=\frac{1}{q}\left(e^{-z\sqrt{\Pr}\sqrt{q}}\right), \tag{35}$$

and by using Laplace transform method, we have

$$T(z,t)=erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}\right). \tag{36}$$

The expression of momentum equation for this case is

$$\bar{F}(z,q)=\bar{F}_3(q)\bar{F}_2(z,q), \tag{37}$$

where

$$F_3(q) = \frac{Gr}{\alpha \Pr q^2 + (\Pr - a_1) q - b_0}. \tag{38}$$

Hence, the inverse Laplace transform for equation (38) is

$$F_1(t) = \frac{Gr}{\alpha \Pr m_2} \sinh(m_2 t) e^{-m_1 t}. \tag{39}$$

Again, by using convolution theorem, we get

$$F(z,t) = (F_3 * F_2)(t) = \int_0^t F_3(t-s) F_2(z,s) ds. \tag{40}$$

3.0 RESULTS AND DISCUSSION

In order to insight of this ramped wall temperature problem, the velocity profiles of parameters second grade α, rotation ω, magnetic M, porosity K, Prandtl number $\Pr$ and Grashof number Gr are plotted through graphs. The velocity profiles in Figures (1-12) are shown the comparison between isothermal and ramped wall temperatures for all parameters involved in real and imaginary parts. Figures 1-2 elaborate the effect of second grade parameter α. The velocity decreases and then increases on increasing second grade parameter in real and imaginary parts. Figure 3 described the effect of rotation parameter ω in real part. The velocity is decreases when the value of ω is increases. But, in Figure 4 for imaginary part, the velocity is increases with increasing the values of ω. The effect of magnetic parameter M is discussed in Figures 5 and 6. These two Figures give a same behavior of velocity in real and imaginary parts. The velocity is decrease when the value of magnetic is increase. This is because due to the fact that magnetic force acts against the direction of flow and causes the velocity to slow down. It is obvious to see that for larger values of K, the velocity is increase for both cases of real and imaginary parts in Figure (7-8). The porosity of fluid will reduce the drag force and hence causes the velocity to increase. Figures 9 and 10 showed the comparison of velocity on different values of Prandtl number $\Pr$ in real and imaginary part. These two graphs give same results where the velocity is decreases on increasing the value of $\Pr$. The thermal boundary layer thickness decreases by increasing the value of $\Pr$. The effect of Grashof number is plotted in Figures 11 and 12. When the value of Gr is increases in real and imaginary parts, the velocity is also increases due to the fact an increase in

Gr gives rise to buoyancy effects which results in more induced flow. Moreover, it is observed that velocity of the fluid is lower for ramped wall temperature compared to isothermal temperature.

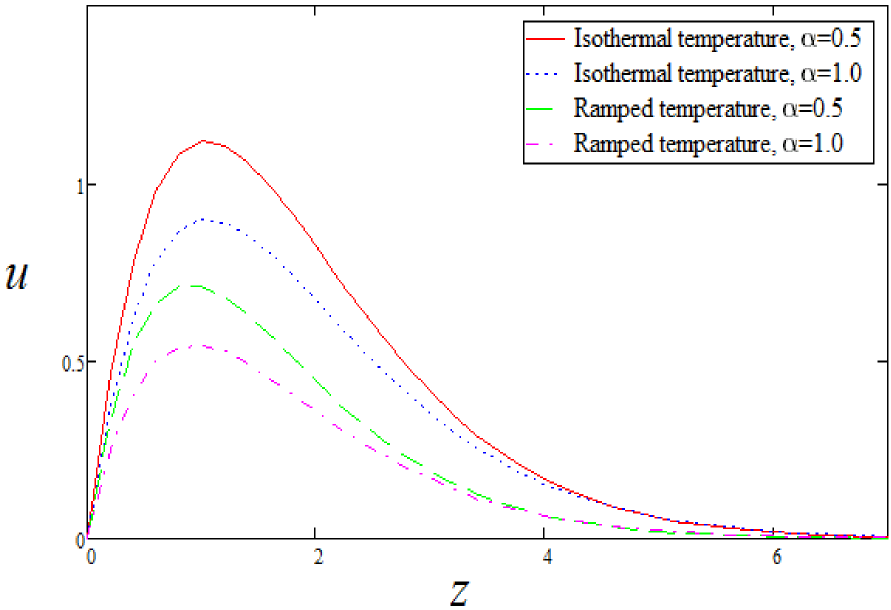

Figure 1: Velocity profiles for different values of α with $\Pr = 0.71$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in real part.

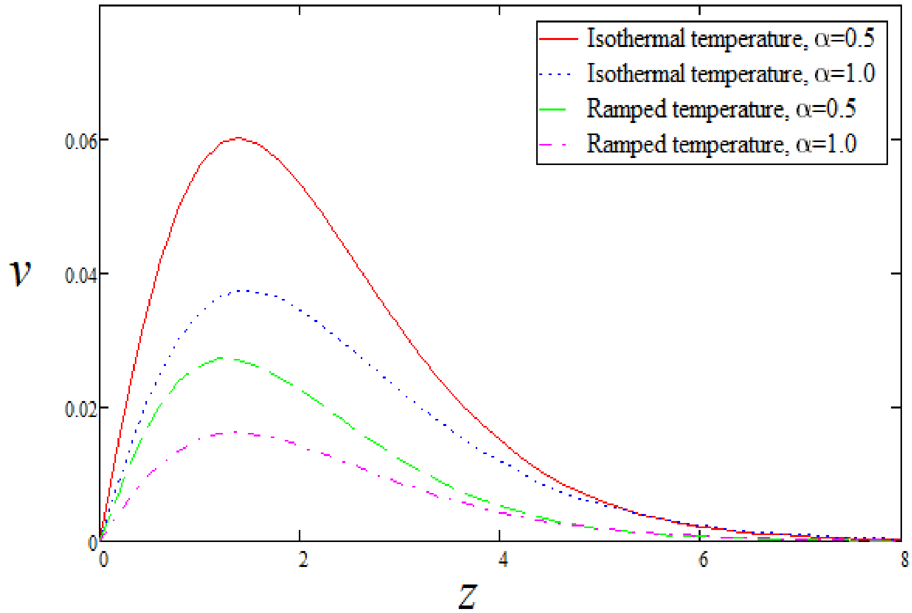

Figure 2: Velocity profiles for different values of α with $\mathrm{Pr} = 0.71$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in imaginary part.

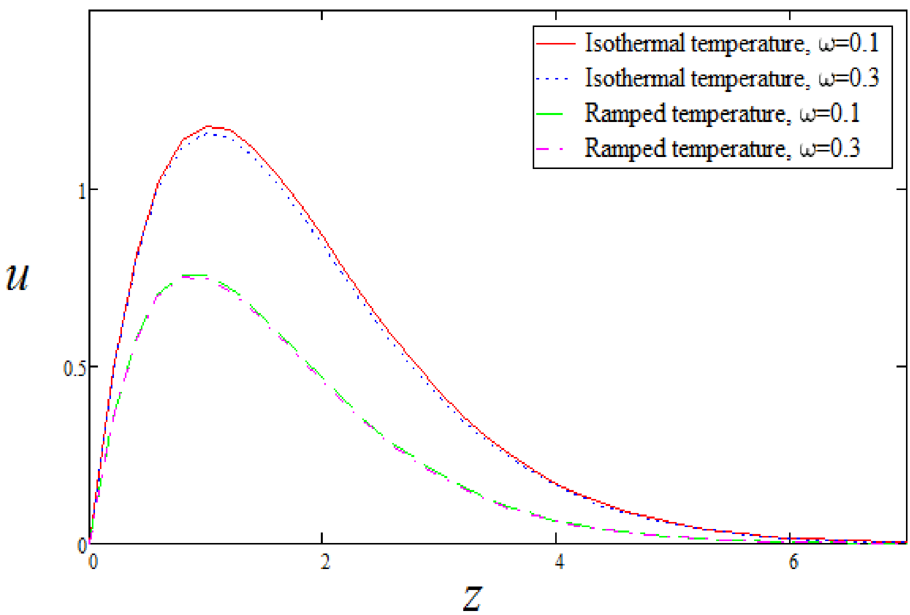

Figure 3: Velocity profiles for different values of ω with $\alpha = 0.4$, $\mathrm{Pr} = 0.71$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in real part.

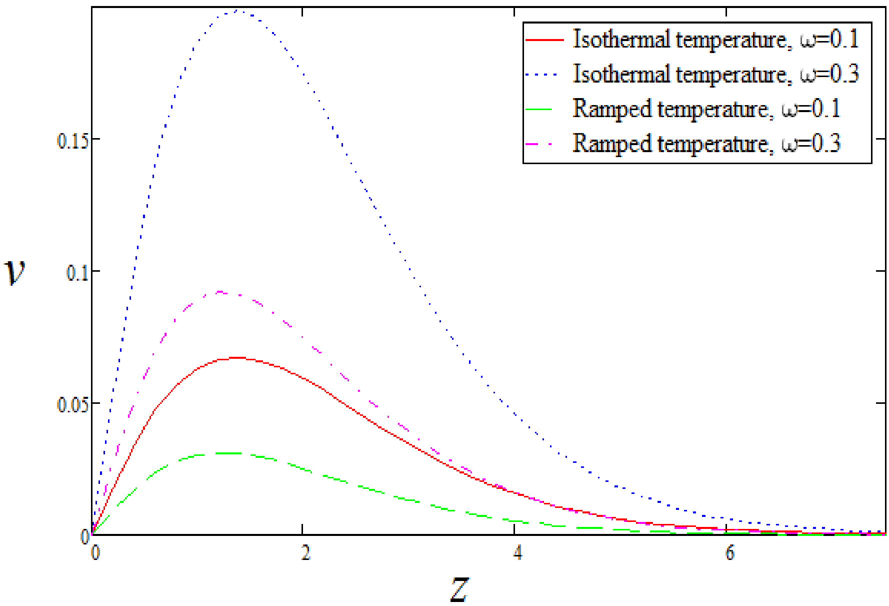

Figure 4: Velocity profiles for different values of ω with $\alpha = 0.4$, $\Pr = 0.71$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in imaginary part.

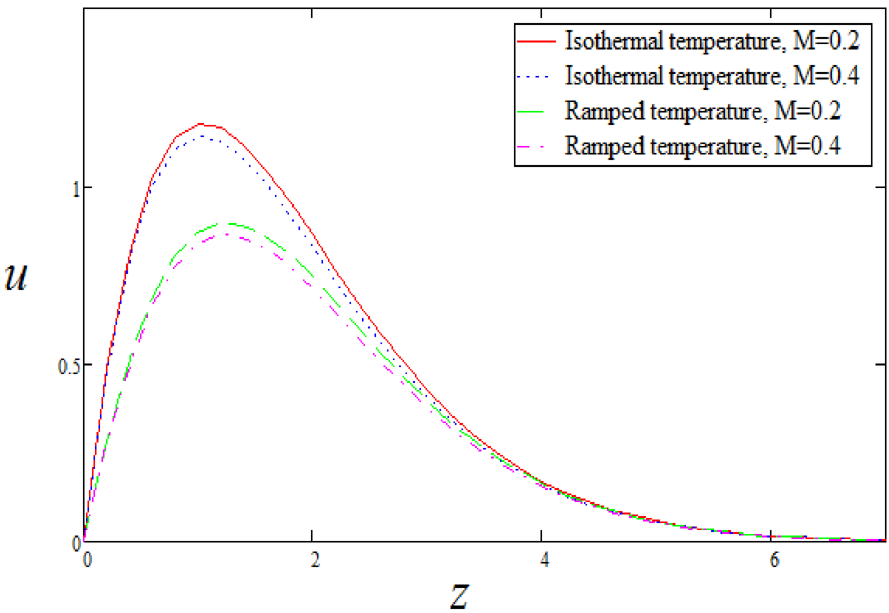

Figure 5: Velocity profiles for different values of M with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $Gr = 5$, $K = 2$ and $t = 1.5$ in real part.

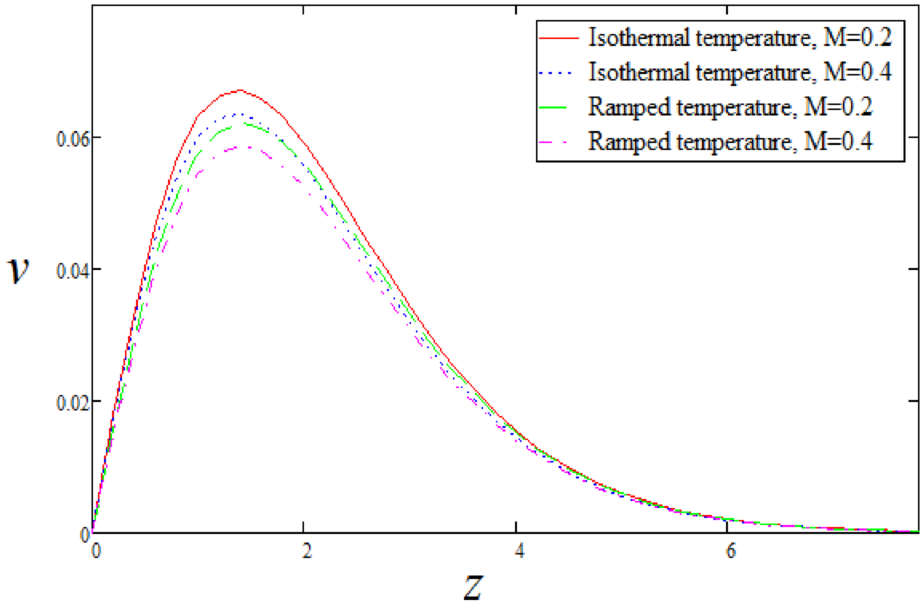

Figure 6: Velocity profiles for different values of M with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $Gr = 5$, $K = 2$ and $t = 1.5$ in imaginary part.

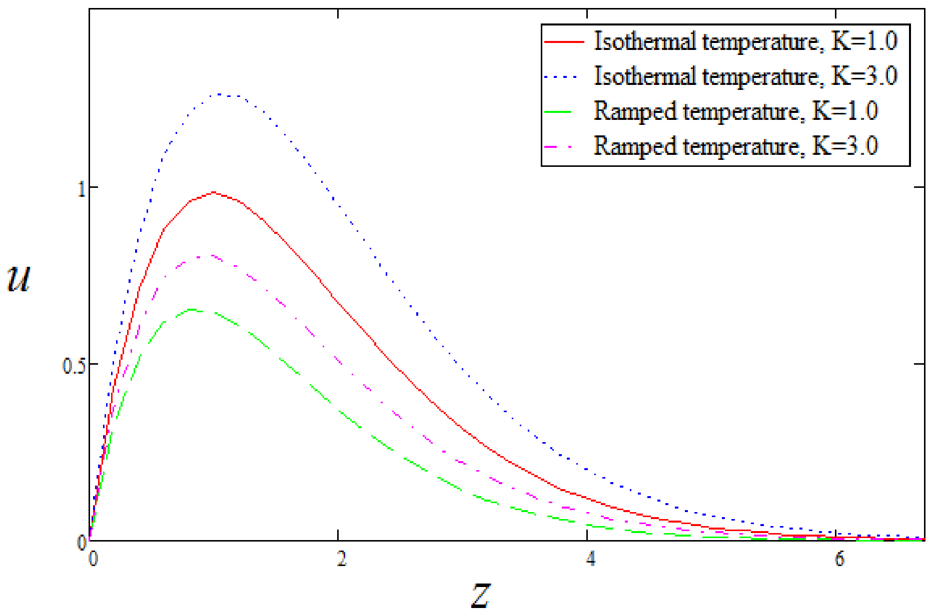

Figure 7: Velocity profiles for different values of K with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, and $t = 1.5$ in real part.

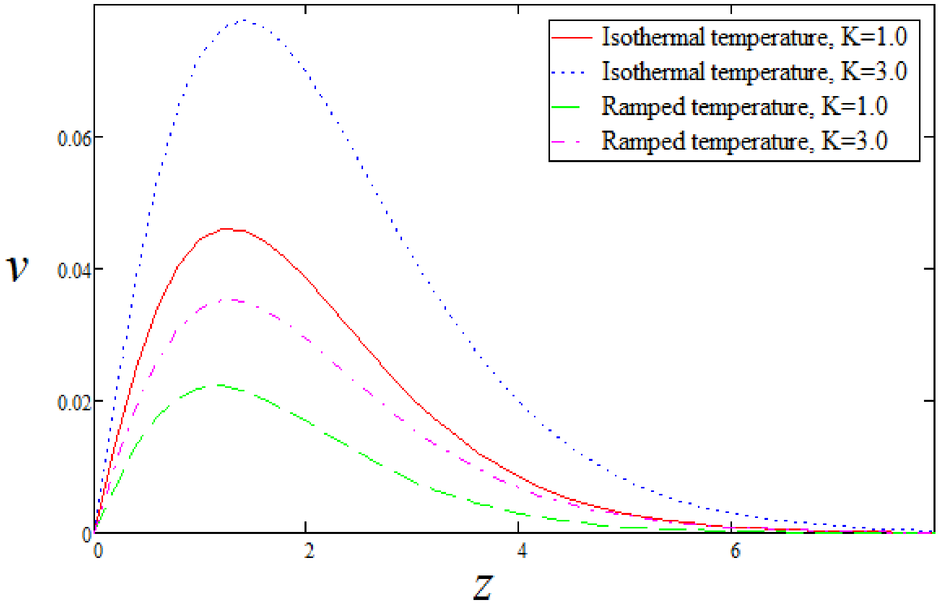

Figure 8: Velocity profiles for different values of K with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, and $t = 1.5$ in imaginary part.

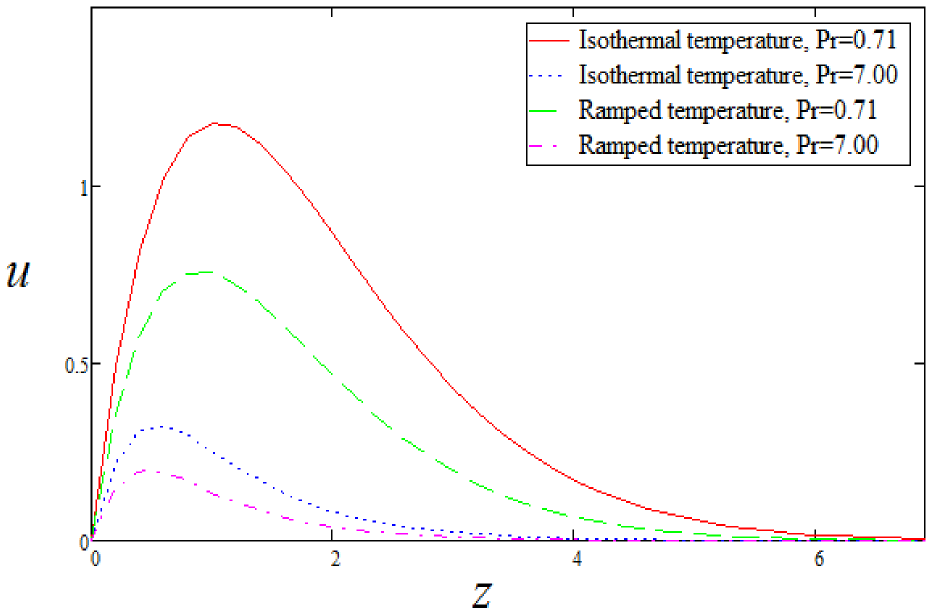

Figure 9: Velocity profiles for different values of Pr with $\alpha = 0.4$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in real part.

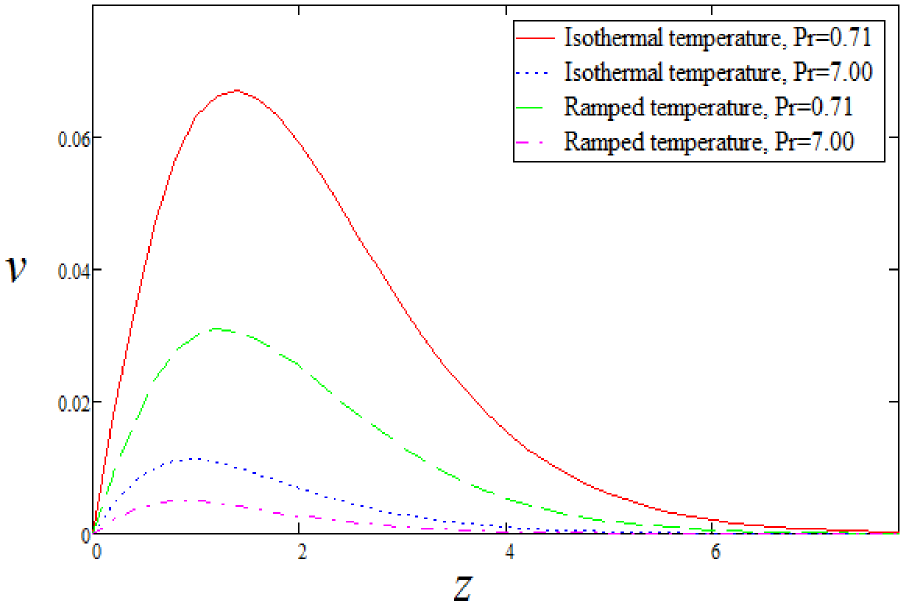

Figure 10: Velocity profiles for different values of Pr with $\alpha = 0.4$, $\omega = 0.1$, $Gr = 5$, $M = 0.2$, $K = 2$ and $t = 1.5$ in imaginary part.

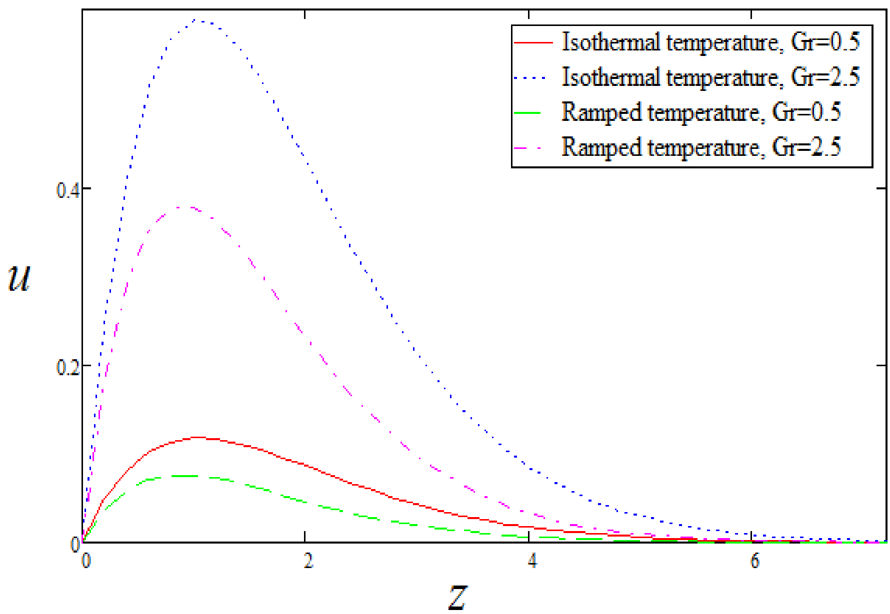

Figure 11: Velocity profiles for different values of Gr with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $M = 0.2$, $K = 2$ and $t = 1.5$ in real part.

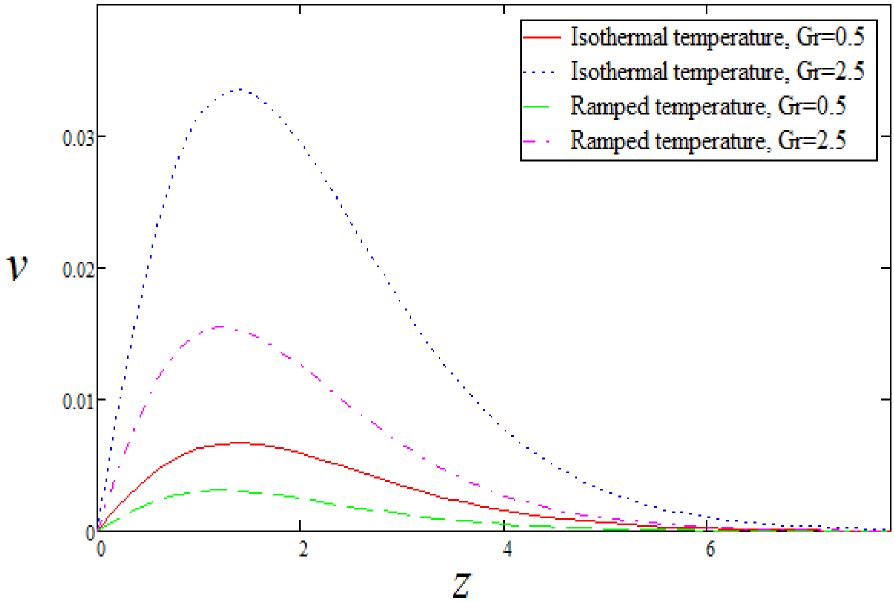

Figure 12: Velocity profiles for different values of Gr with $\alpha = 0.4$, $\Pr = 0.71$, $\omega = 0.1$, $M = 0.2$, $K = 2$ and $t = 1.5$ in imaginary part.

4.0 SUMMARY AND CONCLUSION

In this paper, a mathematical model is presented to investigate the free convection effect on the unsteady MHD rotating flow of a second grade fluid in porous medium with ramped wall temperature at the wall. The equations of velocity and temperature are transformed into dimensionless forms and then solved analytically by using the Laplace transform technique. The graphical results are prepared to observe the effects of various parameters such as second grade parameter α , rotation parameter ω , magnetic parameter M , porosity parameter K , Prandtl number $\Pr$, and Grashof number Gr . The effect of parameters α , M and $\Pr$ are shown the same behavior. The velocity is decrease with increasing the value of α , M and $\Pr$. But, the velocity will increase when the value of parameters K and Gr is increase. As expected, the effect of parameter ω on the velocity shows opposite behavior. The velocity is decrease in real part but increase in imaginary part when the value of ω is increase.

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