HALL CURRENT EFFECT ON g-JITTER INDUCED MHD MIXED CONVECTION FLOW PAST AN INCLINED STRETCHING SHEET

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Abstract. This paper studies the effect of g-jitter induced magnetohydrodynamic (MHD) mixed convection flow of an incompressible, viscous and electrically conducting fluid past an inclined stretching sheet. The effect of Hall current is also studied. The governing boundary layer equations are transformed into a non-dimensional form and then solved numerically using finite difference scheme known as Keller-box method. The effects of the magnetic parameter, Hall parameter, amplitude of modulation, frequency of oscillation and inclination angle on the axial and transverse velocity profiles, and temperature profile as well as axial and transverse skin frictions and heat transfer coefficient are presented graphically and discussed in detailed.

*Keywords*g-jitter; MHD; Hall current; inclined stretching; Keller-box

1.0 INTRODUCTION

In recent years, progress has been considerably made in the study of heat and mass transfer in magnetohydrodynamic flow due to its application. The magnetohydrodynamics (MHD) of electrically conducting fluid is encountered in many problems in geophysics, astrophysics as well as in engineering applications such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Many works have been reported on flow and heat transfer of electrically conducting fluids over a stretched surface in the presence ofmagnetic field. For example, Uddin and Kumar [1] studied the unsteady MHD free convection heat and mass transfer flow of incompressible and electrically conducting fluid past a semi-infinite inclined porous plate with the effects of chemical reaction and radiation. Besides that, Singh [2] discussed the study of convective MHD boundary layer flow on inclined plate in porous medium with combined buoyancy forces arising due to thermal and mass diffusion. Very recently, Aurangzaib *et al*. [3] studied the unsteady MHD mixed convective heat and mass transfer along an inclined stretching plate in a micropolar fluid. In their paper, the problem is reduced to a system of non-dimensional partial differential equations and solve numerically by using an implicit finite-difference scheme. Their results showed that due to the increased in inclination angle, the velocity and angular velocity profiles decreased, while the temperature and concentration increased. However, for ionized gases, the conventional MHD is not valid under the strong electric field.An ionized gas which has low density and/or strong magnetic field, the conductivity normal to the magnetic field is reduced due the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon is known as Hall current effect [4]. MHD viscous flow with Hall current has important application in problems of Hall accelerators as well as flight MHD. Hall effects also play a significant role in the dynamics of the fluid and the magnetic fields of many astrophysical objects, such as dense molecular clouds, formation of white dwarf or instabilities in accretion disks.

A considerable amount of work has been devoted to the study of MHD flow by mixed convection with the effect of Hall current. Chaudhary and Preeti [5] proposed to study the effect of radiation heat absorption and mass transfer by including Hall effects on the unsteady flow of viscoelastic fluid past an infinite vertical porous plate. Along the stretching sheet, Abo-Eldahab*et al*. [6] provided a boundary layer analysis for the combined effect of Hall current, heat generation and wall blowing or suction on the MHD mixed convection flow. Two cases of the temperature boundary conditions were considered at the surface. This problem has been solved numerically by applying an efficient numerical technique based on shooting method. Later, Sharma *et al*. [7] investigated the Hall effects on the combined heat and mass transfer flow which occur due to buoyancy forces caused by thermal diffusion and mass diffusion. In this problem, unsteady flow was considered and vertical porous plate is immersed in porous medium with a constant magnetic field and heat source or sink applied perpendicular to the plate. Meanwhile, Ali *et al*. [4] has considered the Hall effect on MHD mixed convection boundary layer flow over a stretched vertical flat plate theoretically. An applied magnetic field must be quite strong in order to have a good effect of Hall current. The analysis indicates that, the Hall effect on the temperature is small, and the magnetic field and Hall currents produce opposite effects on the shear stress and the heat transfer at the stretching surface.

Motivated by the work above, this paper investigates the problem of MHD mixed convection flow past an inclined stretching sheet in the presence of the g-jitter and Hall current effect. This problem is followed closely to the study of Sharidan et al. [8] which are studied the effect of g-jitter induced mixed convection on the flow and heat transfer characteristics associated with a stretching vertical surface. To our best knowledge, this problem has not been studied before. Following Rees and Pop [9] and Sharidan *et al*. [8], a simple model problem in which the gravitational field depends on time takes the form

$$\mathbf{g}^{*}(\mathbf{t}) = g(t)\mathbf{k} = g_0\left[1+\varepsilon\cos(\pi\omega t)\right]\mathrm{k} \tag{1}$$

where g_0 is the time-averaged value of the gravitational acceleration, $\mathbf{g}^{*}(\boldsymbol{t})$ acting along the direction on the unit vector $\boldsymbol{k}$ which is oriented in the upward direction**,** ε is a scaling parameter, which gives the magnitude of the gravity modulation relative to g_0, t is the time and ω is the frequency of oscillation of the g-jitter driven flow.

If $\varepsilon << 1$, then the forcing may be seen as a perturbation of the mean gravity. Since the governing equation of the proposed problem are non-linear, this kind of forcing leads to the phenomenon of streaming, where a time-periodic forcing with zero mean produces a periodic response consisting of a steady-state solution with a non-zero mean and time-dependent fluctuations involving higher harmonics (Sharidan *et al*., [8]).

2.0 GOVERNING EQUATIONS

Consider the unsteady MHD mixed convection flow of a viscous, incompressible and electrically conducting fluid over an inclined stretching sheet with an acute angle γ to the vertical with the present of g-jitter and Hall current effect. A stationary frame of reference (*x*, *y*, *z*) that has its origin located at the center of the sheet is chosen, where the *x*-axis is extending along the surface with inclination angle γ to the vertical in the upward direction, *y*-axis is normal to the surface and *z*-axis is transverse to the *xy*-plane. An extra effect is considered, where the uniform magnetic field B_0 is applied in the *y*-direction which is normal to the flow direction.The flow configurations and coordinate system are shown in Figure 1.

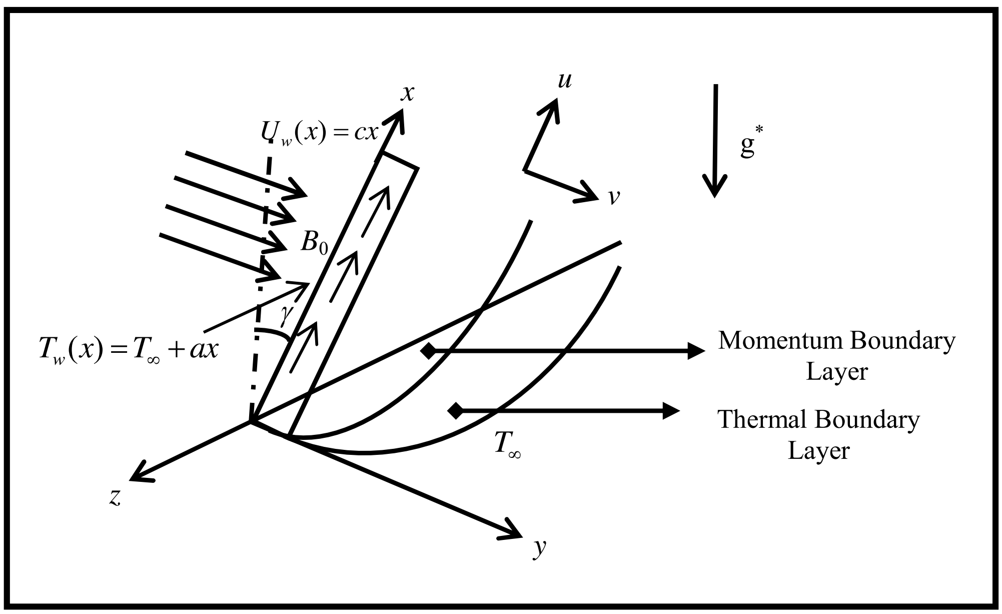


Figure 1 Physical model and coordinate system

In this problem, the plate is assumed has a linear velocity $u_w(x)$ moves in *x*-direction of the flow and temperature of the plate varies linearly with the distance *x* along the plate, where $T_w(x) > T_\infty$ with $T_w(x)$ being the temperature of the plate and T_∞ being theuniform temperature of the ambient fluid.Meanwhile, the velocity and temperature of the continuous stretching surface are assumed to be in the form of $u_w(x) = cx$ and $T_w(x) = T_\infty + ax$ where a, and c are constants and $c > 0$.

The generalized Ohm's Law, taking Hall current into account is given in the form,

$$\mathbf{j} = \frac{\sigma}{1+m^2}\left(\mathbf{E} + \mathbf{V}\times\mathbf{B} - \frac{1}{en_e}\mathbf{j}\times\mathbf{B}\right)$$

where**j** is the electric current density vector, m is the hall parameter, **E** is the intensity vector of the electric field, **V** is the velocity vector, **B** is the induced magnetic vector, e is an electron charge and n_e is the electron number density. In this problem, it is assumed that **E**=0. The effect of Hall current gives rise to a force in the z-direction which produces a transverse velocity in this direction. Assuming the plate to be electrically non-conducting, the generalized Ohm's law gives $j_y = 0$ everywhere on the flow. Then, the current density components, j for *x*- and *z*-axes are given by:

$$\mathrm{j}_x = \frac{\sigma B_0}{1+m^2}(mu - w)$$

$$\mathrm{j}_z = \frac{\sigma B_0}{1+m^2}(u + mw)$$

Then, the governing equation under an assumptions with the Boussinesq and boundary layer approximations are given by

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

Momentum equation in *x*-direction:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + \left[g(t)\beta(T - T_\infty)\right]\cos\gamma - \frac{\sigma B_0^2}{\rho(1+m^2)}(u + mv) \tag{3}$$

Momentum equation in *z*-direction:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \upsilon\frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu - w) \tag{4}$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where u, v and w are the velocity components along x, y and z axes, g is the acceleration due to gravity, B_0 is the magnetic induction, $m=\omega_e t_e$ is the Hall parameter, t_e is the electron collision time, $\omega_e = e\frac{B_0}{m_e}$ is the electron frequency, e is the electron charge, m_e is the mass of the electron, $\sigma = e^2\frac{n_e t_e}{m_e}$ is the electrical conductivity, n_e is the electron number density, and ρ is the density of fluid.

The appropriate initial and boundary conditions for the above boundary layer equations are,

$$\begin{aligned} &t\le 0: u=v=w=0, T=T_\infty \text{ any } x, y, z,\\ &t>0: u_w(x)=cx, v=0,\ w=0\ , T_w(x)=\mathrm{T}_\infty+ax \text{ on } y=0,\\ &u\to 0,\ w\to 0\ , T\to T_\infty \text{ as } y\to\infty. \end{aligned} \tag{6}$$

The complexity of the problem can be reduced by introducing the following non-dimensional variables, (Sharidan *et al*., [8] and Aurangzaib *et al*., [10])

$$\begin{aligned} &\tau=\omega t,\quad \eta=\left(\frac{c}{\upsilon}\right)^{1/2} y,\quad \psi=(c\upsilon)^{1/2} x f(\tau,\eta)\\ &w=cxh(\tau,\eta),\quad \theta(\tau,\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},\quad g(\tau)=\frac{g(t)}{g_0} \end{aligned} \tag{7}$$

where $\psi(x,y)$ is the stream function which is defined as

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \tag{8}$$

and identically satisfied the continuity equation (2). By substituting (7) into equations (3), (4) and (5), the following nonlinear ordinary differential equations are obtained as,

$$\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}-\left(\frac{\partial f}{\partial\eta}\right)^2+\lambda[1+\varepsilon\cos\ (\pi\tau)]\theta\cos\gamma-\frac{M}{1+m^2}(f'+mh)=\Omega\frac{\partial^2 f}{\partial\tau\,\partial\eta} \quad (9)$$

$$\frac{\partial^2 h}{\partial\eta^2}+f\frac{\partial h}{\partial\eta}-\frac{\partial f}{\partial\eta}h-\frac{M}{1+m^2}(mf'-h)=\Omega\frac{\partial h}{\partial\tau} \quad (10)$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}+f\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\theta=\Omega\frac{\partial\theta}{\partial\tau} \quad (11)$$

with the boundary conditions (6) become

$$\begin{gathered} f=0,\ \frac{\partial f}{\partial\eta}=1,\ h=0,\ \theta=1\ on\ \eta=0, \\ \frac{\partial f}{\partial\eta}\to 0,\ h\to 0,\ \theta\to 0\ as\ \eta\to\infty, \end{gathered} \quad (12)$$

wherePr is the Prandtl number , Ω is the non-dimensional frequency, M is the magnetic parameter, and λ is the mixed convection parameter, which are defined as

$$\Pr=\frac{\upsilon}{\alpha},\ \Omega=\frac{\omega}{c},\ M=\frac{\sigma B_0^{\ 2}}{c\rho},\ \lambda=\frac{Gr_x}{\mathrm{Re}_x^2}. \quad (13)$$

Further, $Gr_x=[g_0\beta[T_w(x)-T_\infty]x^3]/\upsilon^2$ being the local Grashof number and $\mathrm{Re}_x=u_w(x)\frac{x}{\upsilon}$ is the local Reynolds number, respectively. We notice that $\lambda>0$ corresponds to assisting flow (heated plate) while $\lambda<0$ for opposing flow (cooled plate) and $\lambda=0$ corresponds to the forced convection flow, respectively.

This problem is aimed at finding the physical quantities of principal interest such as the axial velocity, $f'(\eta)$, transverse velocity profile, $h(\eta)$ and the temperature profiles, $\theta(\eta)$ as well as the skin friction coefficient in the *x*-direction, C_{fx}, skin friction coefficient in the z-direction, C_{fz}, and the local Nusselt number, Nu_x, are defined as

$$C_{fx} = \frac{\tau_{wx}(x)}{\left(\frac{\rho u_w^2}{2}\right)},\ C_{fz} = \frac{\tau_{wz}(x)}{\left(\frac{\rho u_w^2}{2}\right)},\ Nu_x = \frac{q_w(x)x}{k(T_w - T_\infty)} \tag{14}$$

where the $\tau_{wx}(x)$ and $\tau_{wz}(x)$ is the shear stress at the wall in the *x*- and *z*-direction and $q_w(x)$ is the heat flux from the surface of the plate, which are given by (see [8])

$$\tau_{wx} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ \tau_{wz} = \mu\left(\frac{\partial w}{\partial y}\right)_{y=0},\ q_w(x) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{15}$$

*k*is the thermal conductibility and μ is the dynamic viscosity. Using variables (7), we get

$$C_{fx}\,\mathrm{Re}_x^{\frac{1}{2}} = 2\frac{\partial^2 f}{\partial \eta^2}(\tau,0),\ C_{fz}\,\mathrm{Re}_x^{\frac{1}{2}} = 2\frac{\partial h}{\partial \eta}(\tau,0) \quad \frac{Nu_x}{\mathrm{Re}_x^{\frac{1}{2}}} = -\frac{\partial \theta}{\partial \eta}(\tau,0) \tag{16}$$

3.0 RESULTS AND DISCUSSION

The system of the governing equations (9), (10) and (11) together with the boundary conditions (12) is nonlinear differential equations depending on the various values of the parameters such as frequency of the oscillation, Ω, amplitude of modulation, ε, Prandtl number, Pr, magnetic parameter, *M*, hall parameter, *m*, mixed convection parameter, λ and the angle of inclinationparameter γ. These equations are solved numerically by using Keller-Box method. This method has been found to be very suitable in dealing with nonlinear parabolic problems.

Table 1 Comparison of heat transfer rate, $-\theta'(0)$ when $\lambda = 0,\ M = 0,\ m = 0,\ \gamma = 0$ and various value of Pr.

Pr	Sharidan *et al.* [8]	Ali *et al.* [4]	Present Results
0.01	0.0199	0.0198	0.0199
0.72	0.8086	0.8086	0.8086
1.0	1.0000	1.0000	1.0000
3.0	1.9238	1.9237	1.9238
10.0	3.7225	3.7208	3.7225
100.0	12.3953	12.3004	12.3953

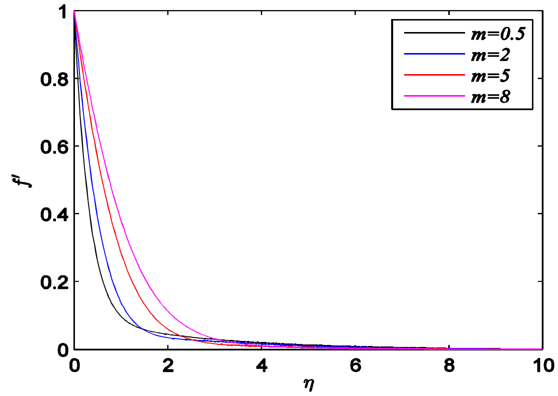
Table 1 represents the comparison of the result for some values of Pr on heat transfer rate, $\theta'(0)$, between Sharidan *et al*. [8] and Ali et al. [4] with the present result by neglecting the buoyancy effect $(\lambda = 0)$, magnetic parameter (*M*= 0), hall parameter (*m*= 0) and inclination angle parameter $(\gamma = 0)$ .The comparison shows that the numerical solutions obtained by the present problem are in very good agreement with Sharidan *et al*. [8] and Ali *et al*. [4]. These agreeable comparisons lend confidence in the numerical results and support well the present results obtained in this problem. From the table, we also noticed that, heat transfer rate increased when the Prandtl number increased.

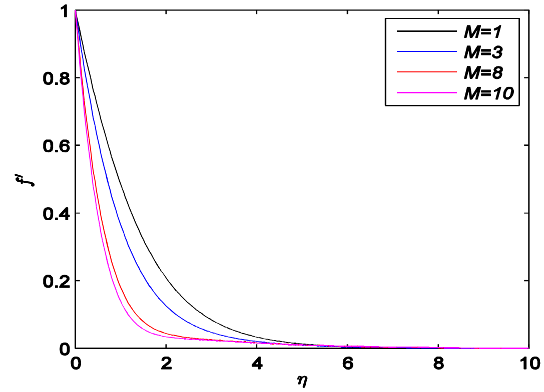
Further, Figures 2 and 3 show the effect of Hall parameter, *m* and magnetic parameter, *M* on the axial velocity, $f'(\eta)$ transverse velocity, $h(\eta)$ and temperature profiles, $\theta(\eta)$ respectively. It is observed that as the Hall parameter increases, the axial velocity profile also increases while the temperature profile decreases. Figure 2 shows that, larger values of the magnetic parameter is considered (*M*=10) in order to give a great influence of Hall parameter. On the other hand, Figure 2(b) shows that the transverse velocity profile increases gradually with *m* but twist the pattern when the values of *m* equal to 5 and 8.

Meanwhile, Figure 3 shows that as the magnetic parameter increases, the axial velocity profile decreases and increases in the temperature profile. These results influenced by the effect of magnetic parameter where the imposition of a magnetic field to an electrically conducting fluid creates a drag like force called Lorentz force. This force has the tendency to slow down the flow along the sheet at the expense of increasing its temperature [11]. In addition, the increasing of M shows an irregular change on transverse velocity profile and normally increases in the profile starting from $\eta = 4$.

Finally, The variation of axial skin friction, $f''(\tau, 0)$, transverse skin friction, $-h'(\tau, 0)$ and rate of heat transfer coefficients, $-\theta'(\tau, 0)$ are shown in Figures 4 and 5 with various values of amplitude of the gravity modulation, ε, frequency of oscillation, $\Omega = 0.2,\ 1 \text{ and } 5$, mixed convection parameter, $\lambda_c = -0.05$, inclination angle parameter, $\gamma = \pi/6$ and $\pi/3$, Pr = 0.72, magnetic parameter, M=1.5 and Hall parameter, m=0.5. It is worth to mention that, the magnitude of the gravity of modulation is vary from 0 to 1 since if $\varepsilon > 1$, then the gravity is perceived to be reverse its direction over part of the g-jitter cycle (see Sharidan *et al*., [8]). All figures show that the effect of increasing ε is give an almost proportional increase and decrease in axial skin friction, transverse skin friction and rate of heat transfer coefficient. It is also observed from the figures, that at any particular inclination angle, as Ω increases, the range values of axial skin friction, transverse skin friction, and heat transfer coefficient are decrease.

Based on Figures 4 and 5, it is also noticed that axial skin friction, transverse skin friction and heat transfer coefficient increased when the value of inclination angle was increased. This phenomena is reflects to the fact that as the plate is inclined from the vertical, the buoyancy force effect due to the thermal diffusion decreases as $\cos\gamma$ decreases.





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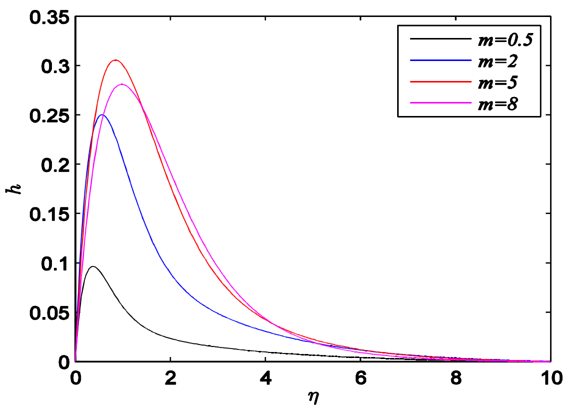

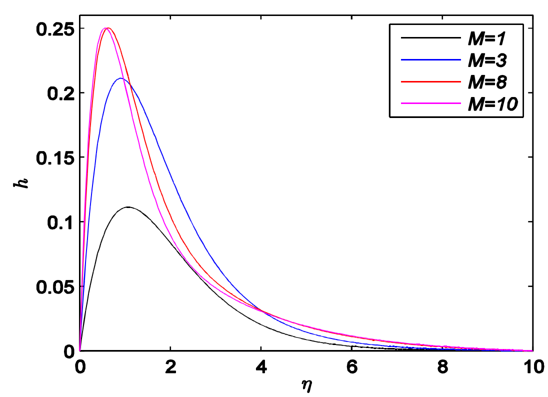

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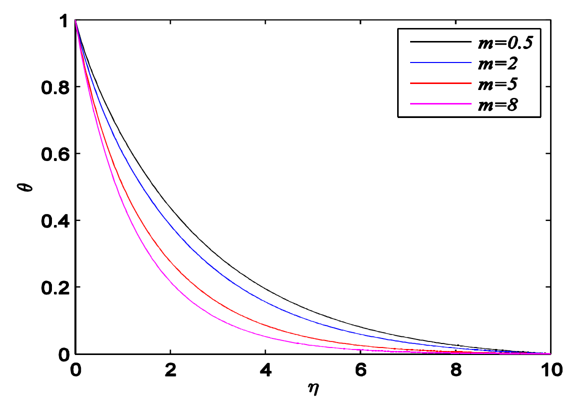
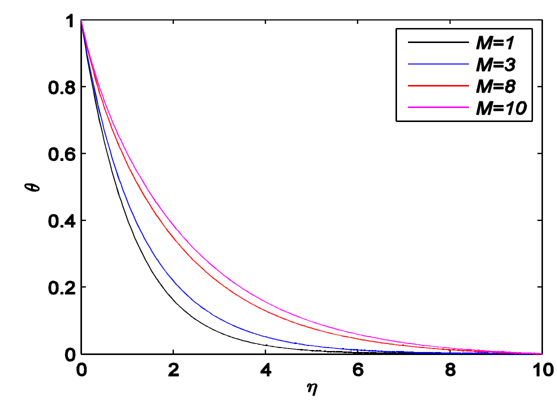
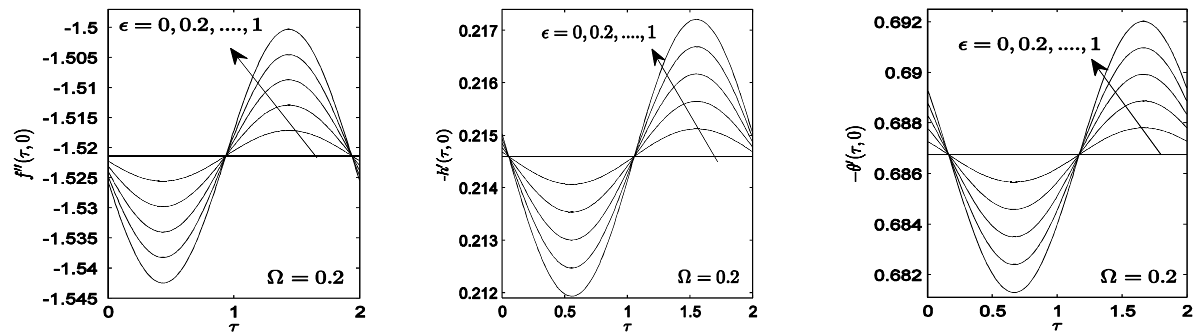




c)

Figure 2 Axial velocity (a), transverse velocity (b) and temperature (c) profiles for different m with Pr=0.72, M=10 and $\gamma = \pi/3$.

Figure 3 Axial velocity (a), transverse velocity (b) and temperature (c) profiles for different M with Pr=0.72, m=0.5 and $\gamma = \pi/3$.



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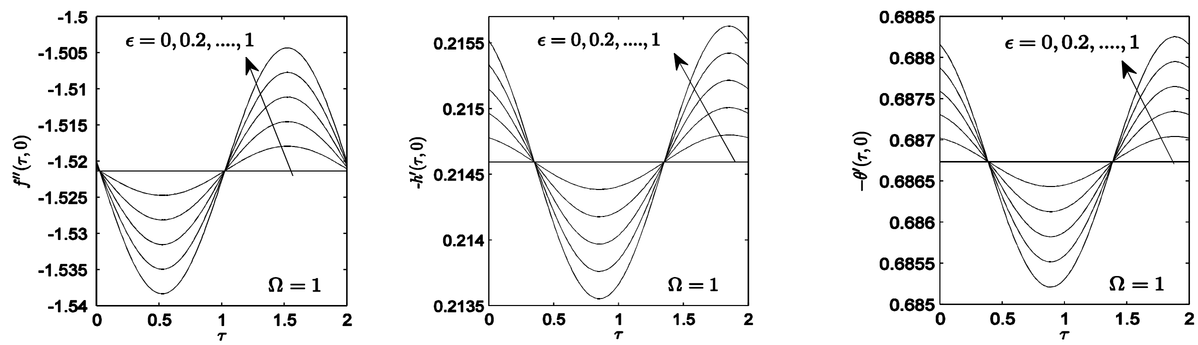

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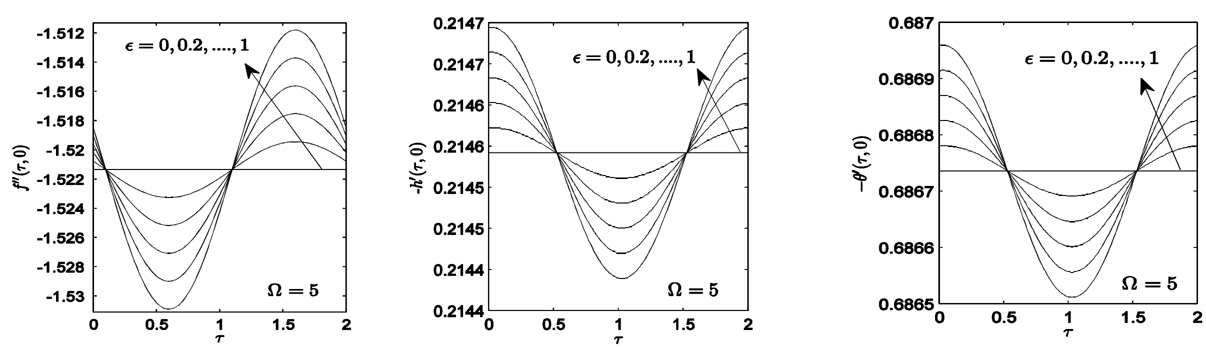
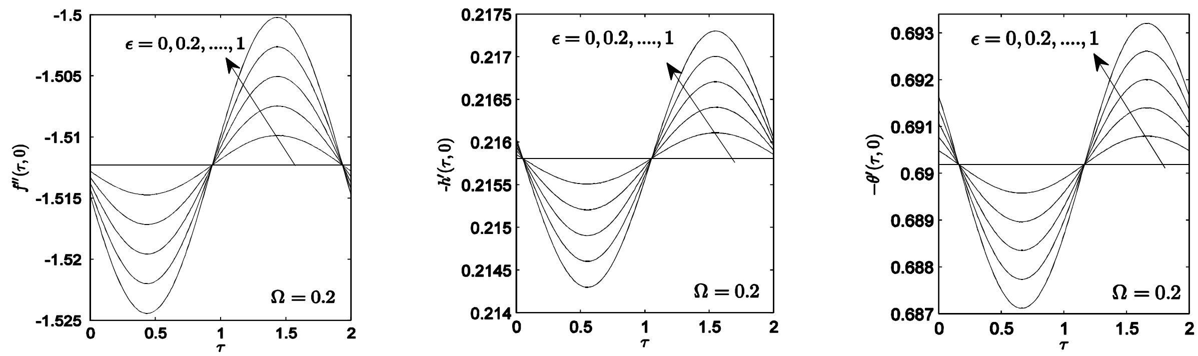


c)

Figure4 Variation of axial skin friction, $f''(\tau,0)$, transverse skin friction, $-h'(\tau,0)$ and heat transfer coefficient, $-\theta'(\tau,0)$ for $\lambda_c = -0.05$, $\Pr = 0.72, M = 1.5$, $\gamma = \pi/6$ with different values of ε for *m*=0.5.



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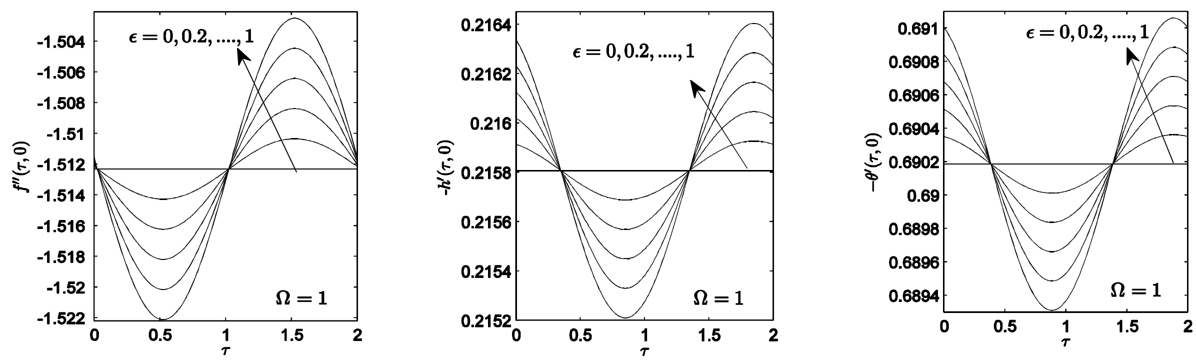

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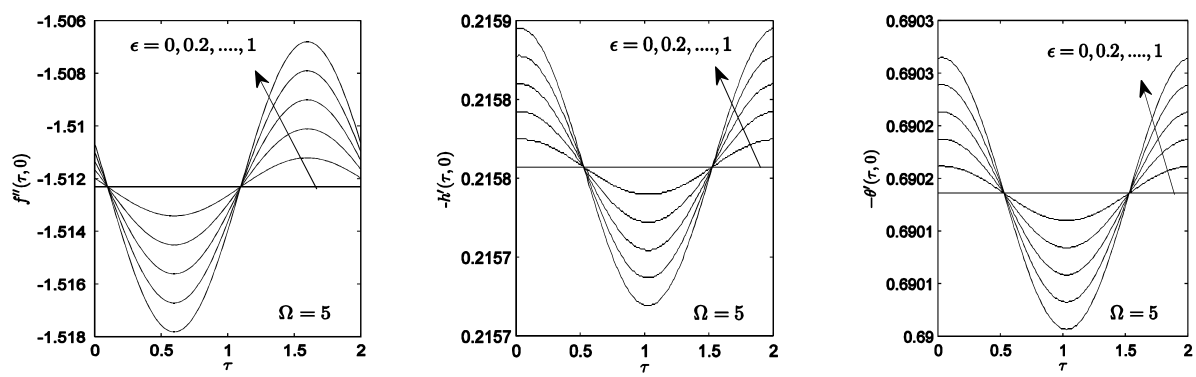

c)

Figure5 Variation of axial skin friction, $f''(\tau,0)$, transverse skin friction, $-h'(\tau,0)$ and heat transfer coefficient, $-\theta'(\tau,0)$ for $\lambda_c = -0.05$, $\Pr = 0.72, M = 1.5$, $\gamma = \pi/3$ with different values of ε for m=0.5.

4.0 SUMMARY AND CONCLUSION

The present study investigates the problem of unsteady MHD mixed convection flow of an incompressible, viscous and electrically conducting fluid past an inclined stretching sheet associated with the effect of g-jitter and Hall current. In this paper, the governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations is solved numerically using Keller-box method. The numerical results when the magnetic parameter, hall parameter, inclination angle parameter, buoyancy effect are absent have been compared with previously published results and it has been found that the agreement is good. From the present investigation, the following conclusions can be drawn:

1. An increase of magnetic parameter leads to a decrease of axial velocity profile, while the opposite effect applies for temperature profile.
2. An increase of Hall parameter, leads to an increase of axial velocity profile and decrease on the temperature profile.
3. The effect of magnetic and hall parameters give an irregular change on the transverse velocity profile.
4. The axial skin friction, transverse skin friction and heat transfer coefficient increase with increasing inclination angle parameter.

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