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CONJUGATE EFFECTS OF HEAT AND MASS TRANSFER ON FREE CONVECTION PAST AN INFINITE OSCILLATING VERTICAL FLAT PLATE WITH RADIATION EFFECT

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Abstract. The conjugate effects of heat and mass transfer on natural convection flow of an incompressible viscous fluid past an infinite oscillating vertical plate with radiation effect have been investigated in this paper. The governing equations are transformed into dimensionless forms by using a set of suitable transformations and solved analytically by the Laplace transform method. The exact solutions for velocity, temperature and concentration are obtained. Numerical results are shown graphically for some selected values of embedded flow parameters and discussed.

Keywords: Heat and mass transfer; Radiation effect; Laplace transformation; Exact solutions.

1.0 INTRODUCTION

There has been much interest in the study of laminar incompressible flows of viscous fluids. Stokes' [1] was the first who studied the laminar incompressible flow of viscous fluid past an infinite horizontal plate started impulsively or oscillating harmonically in its own plane. Later on, several other researchers studied the Stokes' problems for different fluids and configurations. Because these problems are important not only from a theoretical point of view but are also encountered in several applied problems [2]. In first problem of Stokes the wall is initially at rest and a transient flow is induced to the fluid by the suddenly application of an impulsive motion whereas in the second problem of Stokes, the motion is generated by an oscillating plate. The solution of the former problem, in closed form and in terms of tabulated functions, was given by Panton [3], who considered the solution to be a summation of transient and steady state parts. Erdogan [4] solved this problem through a Laplace transform technique. Fetecau *et al*. [5] established new exact steady and transient solutions for Stokes' second problem by considering an incompressible hydrodynamic viscous fluid in a non-porous space. Cruz and Linz [6] investigated the Stokes' problems for porous plate with wall transpiration whereas Abdulhameed *et al*. [7] extended Cruz and Linz [6] problem for MHD fluid in a porous medium.

On the other hand, the study of free convection flow and heat transfer under various geometrical configurations has attracted considerable interest of many researchers because of its large number of applications in industry and technology. Few of these applications are found in fiber and granular insulation, geothermal systems, filtration processes, nuclear reactors and design of space ship [8,9]. Having such motivation in mind, Soundalgekar [10] extended the corresponding problem of Stokes' [1] to infinite vertical plate moving impulsively in the upward direction with the effects of free convection currents caused by the temperature difference between the plate and far away fluid temperatures. In the continuation, the free convection flow past an oscillating vertical plate was investigated by Soundalgekar and Patil [11] considering the constant heat flux at the plate.

However, in nature, the free convection current is not only caused due to the temperature difference of the flow but also due to the differences in concentration in the material constitution. For example, in atmospheric flows there exists differences in H_2O concentration and hence the flow is affected by such concentration. Flows in bodies of water are affected by the effects upon the density of temperature, concentration of dissolved material and suspended particulate matter. In the recent years, the conjugate study of heat and mass transfer is of great practical importance for engineers and scientists because of its numerous applications in many branches of science and engineering such as in chemical and hydrometallurgical industries [12].

Motivated by the above investigations, the present analysis is focused on the study of conjugate effects of heat and mass transfer on free convection flow past an oscillating vertical plate with radiation effect. Exact solutions are obtained using Laplace transform technique. Such exact solutions are very useful to assess the accuracy of approximate numerical and theoretical procedures as well as experimental practices.

2.0 MATHEMATICAL FORMULATION

Let us consider the conjugate effect of heat and mass transfer on natural convection flow of an incompressible viscous fluid past an infinite vertical plate. It is assumed that at the initial moment $t = 0$ both the plate and the fluid are at rest at the constant temperature and species concentration T_∞ and C_∞, respectively. At time $t = 0^+$ the plate begins to oscillate in its plane ($y = 0$) according to

$$\mathbf{v} = a\cos(\omega^* t)\,\mathbf{i}, \quad t > 0, \tag{1}$$

where the constant a is the amplitude of the motion, $\mathbf{i}$ is the unit vector in the vertical flow direction and ω^* is the frequency of oscillation. Because the plate is infinite, the velocity vector is $\mathbf{v} = u(y,t)\,\mathbf{i}$, where y is the coordinate measured in the normal direction to the plate. Under Boussinesqs' approximation and using above assumptions we obtain the following set of Partial Differential Equations

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\,\beta_T\,(T - T_\infty) + g\,\beta_C\,(C - C_\infty), \tag{2}$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{3}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2}. \tag{4}$$

Assuming that no slipping occurs between the plate and the fluid, the appropriate initial and boundary conditions are

$$\begin{aligned} t = 0: &\quad u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{for any} \quad y \geq 0, \\ t \geq 0: &\quad u = a\cos(\omega^* t), \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\ &\quad u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \end{aligned} \tag{5}$$

Here T is the fluid temperature, C is the species concentration in the fluid, T_w is the constant plate temperature, C_w is the species concentration at the surface of the plate, q_r is the radiative flux in the $y-$ direction, g is the acceleration due to gravity, k is the thermal conductivity of the fluid, ν is the kinematic viscosity, ρ is the fluid density, D is the chemical molecular diffusivity, β_T is the thermal volume expansion, β_C is the concentration volume expansion and c_p is the specific heat at constant pressure. Following Magyari and Pantokratoras [8], we adopt the Rosseland approximation for radiative flux q_r, namely

$$q_r = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* is the Stefan-Boltzman constant and κ^* is the mean absorbtion coefficient. We assume that the difference between the fluid temperature T and the free stream temperature T_∞ is small, so that expanding in Taylor series T^4 about T_∞ and neglecting the second and higher order terms, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Substituting (7) into Eq. (3), it becomes

$$\rho c_p \frac{\partial T}{\partial t} = k\left(1 + \frac{16\sigma^* T_\infty^3}{3k\kappa^*}\right)\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

Further, we introduce the following dimensionless variables

$$\begin{gathered} \tau = \frac{a^2}{\nu}t, \quad Y = \frac{a}{\nu}y, \quad U = \frac{u}{a}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \omega = \frac{\nu}{a^2}\omega^* \\ Gr_T = \frac{g\,\beta_T\,(T_w - T_\infty)L^3}{\nu^2}, \quad Gr_C = \frac{g\,\beta_C\,(C_w - C_\infty)}{\nu^2}, \quad Gr = Gr_T + Gr_C, \end{gathered} \tag{9}$$

where L is a characteristic length of the plate and Gr is the Grashoff number. Substituting (9) into Equations (2), (4) and (8), we obtain the following dimensionless equations

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial Y^2} + \frac{\theta + N\phi}{1+N}, \tag{10}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1+N_r}{\Pr}\frac{\partial^2 \theta}{\partial Y^2}, \tag{11}$$

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial Y^2}, \tag{12}$$

and the initial and boundary conditions (5) become

$$\begin{aligned} &\tau < 0: \quad U = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for} \quad \text{any} \quad Y \geq 0 \\ &\tau \geq 0: \quad U = \cos(\omega \tau), \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad Y = 0 \\ &\qquad\quad U \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad Y \to \infty. \end{aligned} \tag{13}$$

Here Pr is the Prandtl number, Sc is the Schmidt number, N is the ratio of the buoyancy forces due to the temperature and concentration and N_r is the radiation parameter, which are defined as

$$\Pr = \frac{\mu c_p}{k}, \quad Sc = \frac{\nu}{D}, \quad N = \frac{Gr_C}{Gr_T}, \quad N_r = \frac{16\sigma^* T_\infty^3}{3kk^*}. \tag{14}$$

It is worth mentioning that β_T is a positive quantity, while β_C can take positive or negative values. Therefore, the thermal and concentration buoyancy forces act in the same direction, or they assist each other in driving the flow, in the case $N > 0$ and they oppose each other when $N < 0$. The case $N = 0$ corresponds to the situation when there is no buoyancy force effect from mass diffusion.

3.0 SOLUTIONS

Using Laplace transform technique, the solutions of Eqs. (10) to (12), under the imposed initial and boundary conditions (13) are given as

$$\theta(Y,\tau) = \operatorname{erfc}\left(\frac{Y}{2}\sqrt{\frac{a_0}{\tau}}\right), \tag{15}$$

$$\phi(Y,\tau) = \operatorname{erfc}\left(\frac{Y}{2}\sqrt{\frac{Sc}{\tau}}\right), \tag{16}$$

$$U(Y,\tau) = U_1(Y,\tau) + U_2(Y,\tau) + a_5 U_3(Y,\tau) - a_3 U_4(Y,\tau) - a_4 U_5(Y,\tau), \tag{17}$$

with

$$U_1(Y,\tau)=\frac{1}{4}e^{-i\omega\tau}\left[e^{-y\sqrt{-i\omega}}\operatorname{erf}c\left(\frac{Y}{2\sqrt{\tau}}-\sqrt{-i\omega\tau}\right)+e^{y\sqrt{-i\omega}}\operatorname{erf}c\left(\frac{Y}{2\sqrt{\tau}}+\sqrt{-i\omega\tau}\right)\right],$$

$$U_2(Y,\tau)=\frac{1}{4}e^{i\omega\tau}\left[e^{-y\sqrt{i\omega}}\operatorname{erf}c\left(\frac{Y}{2\sqrt{\tau}}-\sqrt{i\omega\tau}\right)+e^{y\sqrt{i\omega}}\operatorname{erf}c\left(\frac{Y}{2\sqrt{\tau}}+\sqrt{i\omega\tau}\right)\right],$$

$$U_3(Y,\tau)=\left(\frac{Y^2}{2}+\tau\right)\operatorname{erf}c\left(\frac{Y}{2\sqrt{\tau}}\right)-Y\sqrt{\frac{\tau}{\pi}}e^{-\frac{Y^2}{4\tau}},$$

$$U_4(Y,\tau)=\left(\frac{Y^2a_0}{2}+\tau\right)\operatorname{erf}c\left(\frac{Y}{2}\sqrt{\frac{a_0}{\tau}}\right)-Y\sqrt{a_0}\sqrt{\frac{\tau}{\pi}}e^{-\frac{Y^2a_0}{4\tau}},$$

$$U_5(Y,\tau)=\left(\frac{Y^2Sc}{2}+\tau\right)\operatorname{erf}c\left(\frac{Y}{2}\sqrt{\frac{Sc}{\tau}}\right)-Y\sqrt{Sc}\sqrt{\frac{\tau}{\pi}}e^{-\frac{Y^2Sc}{4\tau}},$$

$$U_6(Y,\tau)=\left(\frac{Y^2Sc}{2}+\tau\right)\operatorname{erf}c\left(\frac{Y}{2}\sqrt{\frac{\mathrm{Pr}_{\mathrm{eff}}}{\tau}}\right)-Y\sqrt{\mathrm{Pr}_{\mathrm{eff}}}\sqrt{\frac{\tau}{\pi}}e^{-\frac{Y^2\mathrm{Pr}_{\mathrm{eff}}}{4\tau}},$$

where

$$a_0=\frac{\mathrm{Pr}}{1+N_r},\ a_1=\frac{1}{1+N},\ a_2=\frac{N}{1+N},\ a_3=\frac{a_1}{a_0-1},\ a_4=\frac{a_2}{Sc-1},\ a_5=a_3+a_5.$$

Clearly, the obtained solutions given by Eqs. (15) to (17) satisfy all the imposed initial and boundary conditions. This provides a useful mathematical check to our calculi.

4.0 RESULTS AND DISCUSSION

In order to get physical insight and to understand the effects of embedded parameters in the problem, the numerical computations are carried out for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number for different values of pertinent parameters such as Prandtl number Pr, radiation parameter N_r, buoyancy parameter *N*, Schmidt number *Sc*, oscillating frequency ω ,dimensionless time τ and phase angle $\omega\tau$.

We begin our discussion from the Prandtl number Pr when the other parameters are kept fixed as shown in Fig.1. Two different values of Prandtl Pr = 0.71 and Pr = 1 are chosen such that physically they correspond to air and electrolytic solution respectively. It is observed that velocity of the fluid decreases with increasing Prandtl number. Moreover, this figure clearly indicates that velocity for air is more than that for electrolytic solution which is in consistent with the physical observation that the fluids with high Prandtl number have greater viscosity which makes the fluid thick and hence move slowly. Furthermore, it is seen that for large values of the radiation parameter N_r, the momentum boundary layer becomes thinner and hence velocity increases. Moreover, the buoyancy parameter *N* in the general solution (17) takes negative, zero and positive values. Here, the positive values of *N* correspond to cooling of the plate, negative values correspond to heating of the plate and zero value shows the case when there is no convection current as shown in Fig. 1. Here the different values of buoyancy parameter *N* show that an increase in *N* leads to an increase in the velocity of an air flow due to enhancement in the buoyancy force. The velocity in case of electrolytic flow is the minimum for the negative values of the buoyancy parameter. However, in case of no convection, when *N* = 0, the flow velocity stays in the middle of the boundary layer.

The velocity profiles are shown in Fig. 2 for different values of Schmidt number *Sc*, oscillating frequency ω, and dimensionless time τ. It is observed that velocity increases with increasing time whereas decreases for large values of Schmidt number and oscillating frequency. Two different values of Schmidt number *Sc* = 0.2 and *Sc* = 0.6 are chosen which correspond to hydrogen and water vapour, respectively. The momentum boundary layer falls gradually for hydrogen rather than water.

In Fig. 3 the velocity profiles are shown for different values of phase angle $\omega\tau$. It is observed that the fluid is oscillating between -1 and 1. These fluctuations near the plate are maximum and decrease for further increasing values of the independent variable y. This figure can easily help us to check the accuracy of our results. For illustration of such results we have concentrated more of the values of and $\omega\tau = 0, \frac{\pi}{2}$ and π. We can see that for these values of

$\omega\tau$, the velocity shows its values either 1, 0 or -1 which are identical with the imposed boundary conditions of velocity in Eq. (13). Hence, the graphical and mathematical results are found in excellent agreement.

The temperature profiles for different values of Prandtl number Pr are shown in Fig. 4. Four different values of Prandtl number Pr = 0.015 (mercury), 0.7 (air), 1.0 (electrolytic solution) and 7.0 (water) are chosen. We observed that the temperature decreases on increasing Pr, because lower Pr value has more uniform temperature distribution across the thermal boundary layer as compared to higher Pr value. This phenomenon occurs when the lesser values of Prandtl number are equivalent to increasing thermal conductivity. Therefore, heat is capable to diffuse away from the heated surface more quickly compare to bigger values of Prandtl number. Thus, the temperature of water falls more rapidly compared to air and mercury. Near the plate, the temperature is maximum and approaches zero in the free stream region asymptotically. In free convection problems, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers. For small values of Pr, the heat diffuses very quickly compared to the momentum transfer. This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer. Moreover, Fig. 5 presents the temperature profiles for different values of radiation parameter N_r and dimensionless time τ for air when Pr = -0.71. It is observed that temperature increases with increasing values of N_r and τ.

Fig. 6 is plotted to show the influence of Schmidt number *Sc* on the concentration profiles. The physical values of the Schmidt number are chosen as *Sc* = 0.22 (hydrogen), 0.60 (water vapour), and 0.87 (Methanol). It is noticed that the concentration of the fluid decreases as the Schmidt number increases. The concentration falls gradually and progressively for hydrogen in distinction to other gases. Physically, it is true since increases of *Sc* mean a decrease in the molecular diffusivity, which results in decrease of concentration boundary layer. Hence, the concentration of species is smaller for higher values of *Sc*. On the other hand, the concentration profiles increase with increasing values of the Schmidt number.

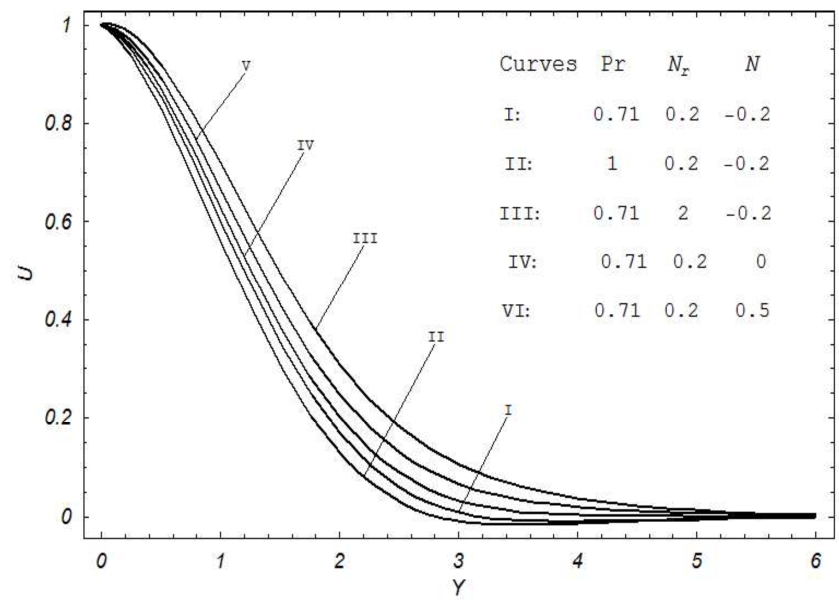

Figure 1. Profiles of velocity for different values of Pr, N_r and N when $\tau = 0.2, Sc = 0.2, \omega = 2.$

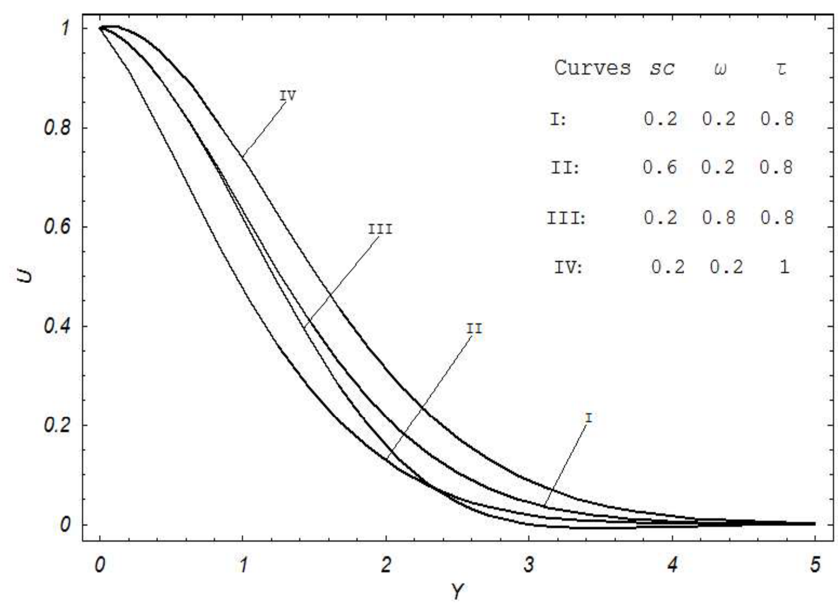

Figure 2. Profiles of velocity for different values of Sc, ω and τ when $\Pr = 0.71, N_r = 0.6, N = 6.$

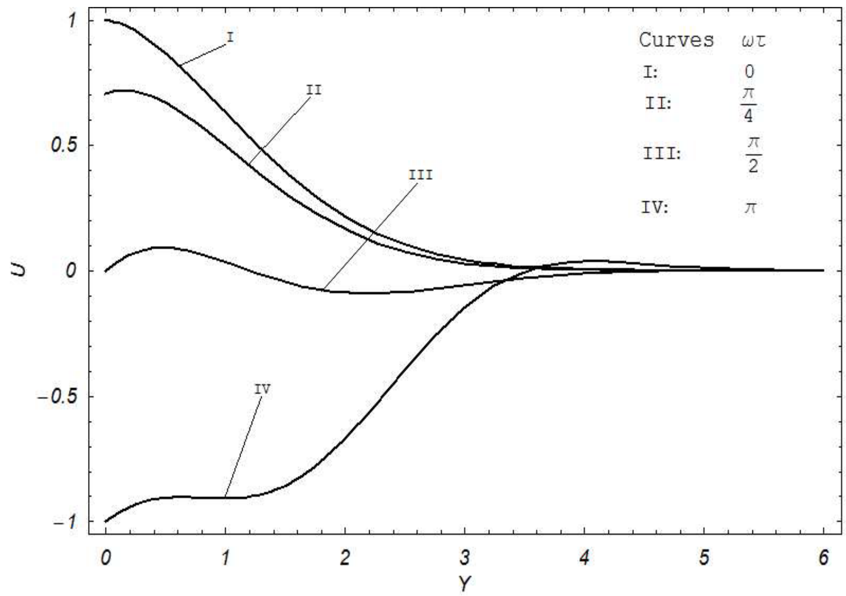

Figure 3. Profiles of velocity for different values of $\omega\tau$ when $\tau = 0.8, N_r = 0.1, N = 6, Sc = 0.6, \Pr = 0.71.$

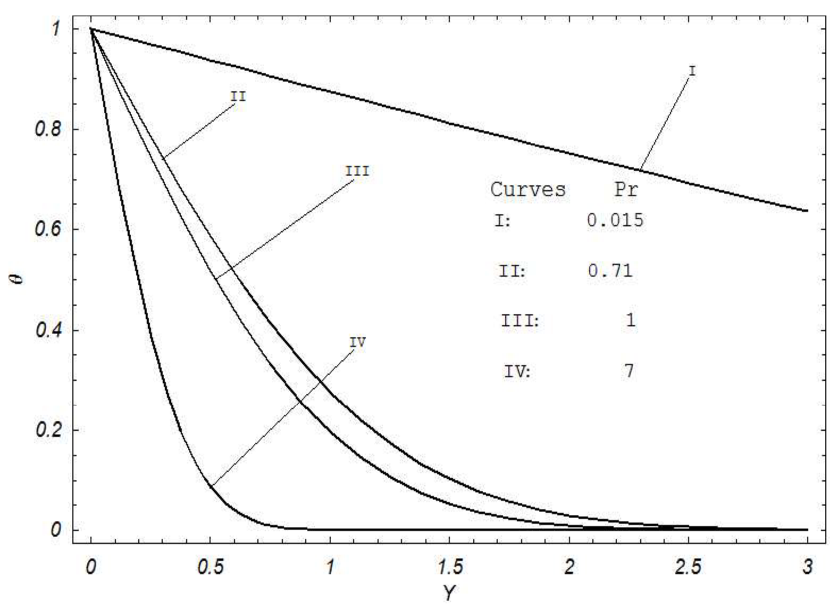

Figure 4. Profiles of temperature for different values of Pr when $\tau = 0.1$ and $N_r = 2$.

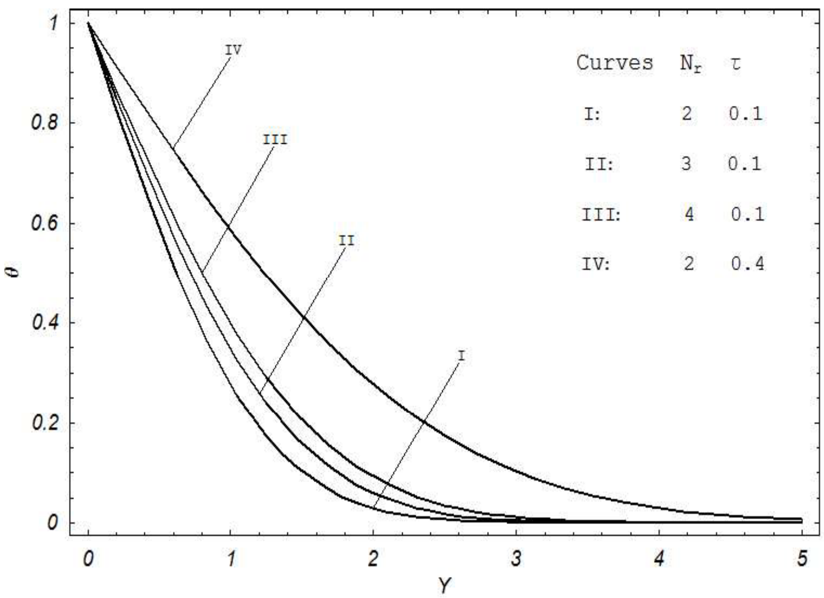

Figure 5. Profiles of temperature for different values of N_r and τ when Pr = 0.71.

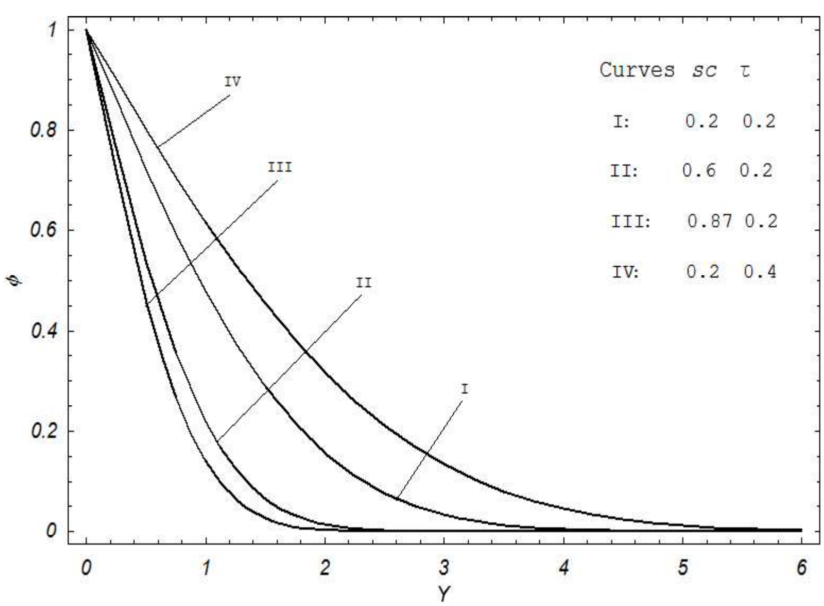

Figure 6. Profiles of concentration for different values of Sc and τ .

5.0 CONCLUSIONS

In this study an exact analysis is performed to study the conjugate effects of heat and mass transfer on natural convection flow of an incompressible viscous fluid past an infinite oscillating vertical plate with radiation effect. The dimensionless governing equations are solved by using the Laplace transform technique. The results for velocity, temperature and concentration are obtained and plotted graphically. The conclusions of this study are as follows:

1). Velocity increases with increasing N_r , N and τ whereas decreases for larger Pr, Sc and ω .
2). Velocity shows a strong fluctuation near the plate for large values of $\omega\tau$.
3). Temperature increases with increasing N_r and τ whereas decreases when Pr is decreased.
4). Concentration increases with increasing τ and decreases for larger Sc.

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