ANALYSIS OF HEAT TRANSFER IN JEFFREY FLUID OVER AN INFINITE VERTICAL PLATE

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Abstract. This paper studies the heat transfer analysis in Jeffrey fluid over an infinite vertical plate. Using the constitutive equations of Jeffery fluid and thermal radiation in the energy equation, the governing equations are modeled. These equations are first simplified by using appropriate dimensionless variables and then solved analytically by using the Laplace transform technique. Closed form solutions of velocity and temperature are obtained. It is found that they satisfy the governing equations and imposed conditions.

Keywords: Heat transfer; Jeffrey fluid; Exact solutions.

1.0 INTRODUCTION

It is well-known from the literature that the traditional viscous fluids (also known as Newtonian fluids) cannot precisely describe the characteristics of many physiological fluids. Examples of such fluids include certain oils, lubricants, mud, shampoo, ketchup, cosmetic products, polymers and blood at low shear rate. Such fluids are known as non-Newtonian fluids. The study of non-Newtonian fluids has gained much importance in view of its promising applications in engineering and industry. However, these fluids do not exhibits the linear relationship between stress and the rate of strain. Due to this non-linear dependence, the analysis of the behavior of fluid motion of the non-Newtonian fluids tends to be much more complicated and subtle in comparison with that of the Newtonian fluids. Therefore, the study of non-Newtonian fluids has attracted the attention of many researchers and have proposed several non-Newtonian fluid models, see for example [1-6] and the references therein.

Amongst them, one of the most popular recently which accounts the rheological effects of viscoelastic fluids is called the Jeffery model. The Jeffrey model is a relatively simple linear model using the time derivatives instead of convective derivatives. Some recent works on Jeffrey model can be found in [7-11]. However, in most of these studies either the convection phenomenon is not included or they are solved using any numerical or approximate technique. On the other hand, convection flows are important in the problems of heat rejection and removal in many devices, processes and systems. In addition, the interest of researchers to study the interaction of convection phenomenon with thermal radiation has been increased greatly during the last few decades due to its importance in many practical involvements such as space technology and processes involving high temperatures. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, power plants for inter planetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these processes [12-14]. Having this motivation, Kavita *et al*. [15] analysed the influence of heat transfer on MHD oscillatory flow of Jeffery fluid in a channel. They used the constitutive equations of Jeffery fluid and derived the unsteady governing equations of momentum and energy under the Boussinesq approximation. We have found that the momentum equation (2.2) in their paper is missing the third order term multiple of the retardation parameter λ_2. Hence they found some erroneous results given by equation (3.10). In another paper [16], the same authors studied the effects of magnetic field on steady free convection flow of Jeffery fluid past an infinite vertical porous plate with constant heat flux. However, due to the assumption of steady flow, the third order term of velocity in the momentum equation automatically vanishes.

Based on the above motivation, the objectives of the present paper are two-fold. First to derive the correct equations for the free convection flow of Jeffery fluid and secondly to establish exact solutions using Laplace transform technique. The importance of this paper is not only due to the fact that exact solutions can be used for accuracy purposes by scientists to compare with their numerical, approximate or experimental results but also because of the accurate form of the governing equations. This problems can be considered as one of the fundamental problem for the unsteady free convection flow of Jeffery fluid and is highly recommended for future research.

2.0 MATHEMATICAL FORMULATION

Let us consider the unsteady free convection flow of a Jeffrey fluid over an infinite vertical plate. The x'-axis is taken parallel to the plate in vertical upward direction and y'-axis is chosen normal to the plate. Initially for both the fluid and plate are at rest and maintained at same uniform temperature T_∞. At time $t' = 0^+$, the plate temperature is raised to T_w which is thereafter maintained constant. Since the plate is considered infinite in the x'-direction, therefore all the physical variables are functions of y' and t' only. The physical model and coordinate system are shown in Figure 1.

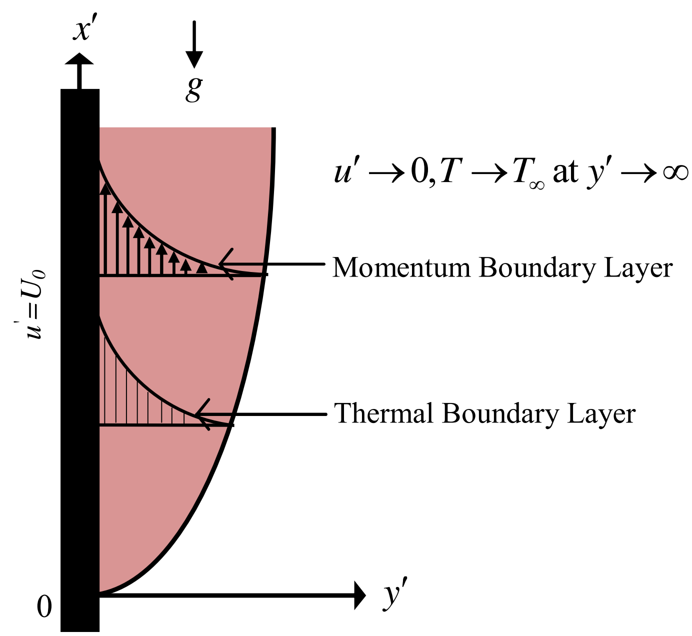

Figure 1. Physical model and coordinate system.

The constitutive equations for a Jeffrey fluid are given by

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{1}$$

where $\mathbf{T}$ is the Cauchy stress tensor and $\mathbf{S}$ is the extra stress tensor defined as

$$\mathbf{S} = \frac{\mu}{1+\lambda_1}\left[\mathbf{A_1} + \lambda_2\left(\frac{\partial \mathbf{A_1}}{\partial t'} + \mathbf{V}.\nabla\right)\mathbf{A_1}\right]. \tag{2}$$

Here μ is the dynamic viscosity, λ_1 and λ_2 are the material parameters of Jeffrey fluid and $\mathbf{A_1}$ is the Rivlin-Ericksen tensor defined by

$$\mathbf{A_1} = \nabla\mathbf{V} + \left(\nabla\mathbf{V}\right)^{\mathrm{T}}. \tag{3}$$

Note that the constitutive equation for extra stress tensor given by equation (1) in Khan *et al*. [7] contains a typo mistake.

Let us assume the velocity of the following form

$$\mathbf{V} = u'(y,t)\mathbf{i}, \tag{4}$$

where $\mathbf{i}$ is the unit vector in the x'-direction and using the Boussinesq's approximation, we get the following equations for velocity and temperature

$$\frac{\partial u'}{\partial t'} = \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u'}{\partial y'^2} + \lambda_2\frac{\partial^3 u'}{\partial t'\partial y'^2}\right] + g\beta(T - T_\infty), \tag{5}$$

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y'^2} - \frac{\partial q_r}{\partial y'}, \tag{6}$$

with initial and boundary conditions

$$t' \le 0 \ : \ u' = 0,\ T = T_\infty \text{ for all } y' \ge 0, \tag{7}$$

$$t' > 0 \ : \ u' = U_0,\ T = T_w \text{ at } y' = 0, \tag{8}$$

$$u' \to 0,\ T \to T_\infty \text{ as } y' \to \infty, \tag{9}$$

where u' is the axial velocity, t' is time, ν is kinematic viscosity, λ_1 is ratio of relaxation to retardation times, λ_2 is the relaxation time, g is acceleration due to gravity, β is volumetric coefficient of thermal expansion, ρ is fluid density, C_p is heat capacity at constant pressure, T is temperature of the fluid, k is thermal conductivity, q_r is radiative flux along the y'-axis, T_∞ is ambient temperature and T_w wall temperature.

Using the Rosseland approximation [14], equation (6) modifies to

$$\rho C_p \frac{\partial T}{\partial t'} = k\left(1+\frac{16\sigma^* T_\infty^3}{3kk^*}\right)\frac{\partial^2 T}{\partial y'^2}, \tag{10}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption co-efficient.

Introducing the following non-dimensional variables

$$y=\frac{y'U_0}{\nu},\ t=\frac{t'U_0^2}{\nu},\ u=\frac{u'}{U_0},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{11}$$

into equations (5) and (10), we obtain

$$\frac{\partial u}{\partial t}=\frac{1}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial t\partial y^2}\right]+Gr\theta, \tag{12}$$

$$\Pr\frac{\partial\theta}{\partial t}=(1+R)\frac{\partial^2\theta}{\partial y^2}, \tag{13}$$

where

$$Gr=\frac{\nu g\beta\left(T_w-T_\infty\right)}{U_0^3},\ \Pr=\frac{\mu C_p}{k},\ R=\frac{16\sigma^* T_\infty^3}{3kk^*},\alpha=\frac{\lambda_2 U_0^2}{\nu},$$

are the Grashof number, Prandtl number, radiation parameter and Deborah number.

The corresponding initial and boundary conditions in non-dimensional form become

$$t\le 0\ :\ u=0,\ \theta=0\ \text{ for all } y\ge 0, \tag{14}$$

$$t>0\ :\ u=1,\ \theta=1\ \text{ at } y=0, \tag{15}$$

$$u\to 0,\ \theta\to 0\ \text{ as } y\to\infty. \tag{16}$$

METHOD OF SOLUTION

In order to obtain the exact solution of the present problem we shall use the Laplace transform technique. Applying the Laplace transform with respect to time t to the system of equations (12) to (16) we obtain

$$\bar{u}(y,q)=\frac{1}{q}e^{-y\sqrt{q\left(\frac{1+\lambda_1}{1+\alpha q}\right)}}+b\left(\frac{1+\lambda_1}{1+\alpha q}\right)\frac{1}{q^2}e^{-y\sqrt{q\left(\frac{1+\lambda_1}{1+\alpha q}\right)}} \\ -b\left(\frac{1+\lambda_1}{1+\alpha q}\right)\frac{1}{q^2}e^{-y\sqrt{q\,\mathrm{Pr}_{\mathrm{eff}}}}, \tag{17}$$

$$\bar{\theta}(y,q)=\frac{1}{q}e^{-y\sqrt{q\,\mathrm{Pr}_{\mathrm{eff}}}}, \tag{18}$$

where $b=\frac{Gr}{\mathrm{Pr}_{\mathrm{eff}}-1}$ and $\mathrm{Pr}_{\mathrm{eff}}=\frac{\mathrm{Pr}}{1+R}$ is the effective Prandtl number defined by Magyari and Pantokratoras [14]. The inverse Laplace transform of equations (17) and (18) yields

$$\theta(y,t)=\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}_{\mathrm{eff}}}{t}}\right), \tag{19}$$

$$u(y,t)=H(t)\left[1-\frac{1}{\pi}\int_0^{\alpha_1}\frac{e^{-xt}}{x}\sin\left(y\sqrt{\alpha_1 x\left(\frac{1+\lambda_1}{\alpha_1-x}\right)}\right)dx\right]$$
$$+b(1+\lambda_1)H(t)\int_0^t\left[1-\frac{1}{\pi}\int_0^{\alpha_1}\frac{e^{-xs}}{x}\sin\left(y\sqrt{\alpha_1 x\left(\frac{1+\lambda_1}{\alpha_1-x}\right)}\right)dx\right]ds$$
$$-b\left(\frac{1+\lambda_1}{\alpha_1}\right)H(t)\left[1-\frac{1}{\pi}\int_0^{\alpha_1}\frac{e^{-xt}}{x}\sin\left(y\sqrt{\alpha_1 x\left(\frac{1+\lambda_1}{\alpha_1-x}\right)}\right)dx\right]$$
$$+b\left(\frac{1+\lambda_1}{\alpha_1^2}\right)H(t)\left[\frac{1}{\pi}\int_0^{\alpha_1}\left(\frac{\alpha_1 x}{\alpha_1-x}\right)e^{-xt}\sin\left(y\sqrt{\alpha_1 x\left(\frac{1+\lambda_1}{\alpha_1-x}\right)}\right)dx\right]$$
$$-b(1+\lambda_1)\left[\left(\frac{y^2\,\mathrm{Pr}_{\mathrm{eff}}}{2}+t\right)\mathrm{erf}\,c\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}_{\mathrm{eff}}}{t}}\right)-y\sqrt{\mathrm{Pr}_{\mathrm{eff}}}\sqrt{\frac{t}{\pi}}e^{-\frac{y^2\mathrm{Pr}_{\mathrm{eff}}}{4t}}\right]$$

$$
\begin{aligned}
&-\frac{b(1+\lambda_1)}{\alpha_1}\frac{e^{-\alpha_1 t}}{2}\left[\begin{aligned}&e^{-y\sqrt{-\alpha_1 \Pr_{\text{eff}}}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr_{\text{eff}}}{t}}-\sqrt{-\alpha_1 t}\right)\\&+e^{y\sqrt{-\alpha_1 \Pr_{\text{eff}}}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr_{\text{eff}}}{t}}+\sqrt{-\alpha_1 t}\right)\end{aligned}\right] \\
&+\frac{b(1+\lambda_1)}{\alpha_1}\left[\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr_{\text{eff}}}{t}}\right)\right],
\end{aligned}
\tag{20}
$$

where $\alpha_1 = \frac{1}{\alpha}$ and $\operatorname{erf}c(.)$ is the complementary error function.

It is important to note that the above solution is valid for $\Pr_{\text{eff}} \neq 1$. The solution for $\Pr_{\text{eff}} = 1$, can be easily obtained by substituting $\Pr_{\text{eff}} = 1$ into equation (13) and follow the similar procedure as discussed above.

3.0 CONCLUSION

In this paper we studied the influence of thermal radiation on free convection flow of Jeffery fluid over an impulsively started infinite vertical plate. Using the constitutive equations of Jeffery fluid, the governing equations are derived. The momentum equation (2.2) for unsteady free convection flow of Jeffery fluid was modeled by Kavita *et al*. [15]. We found that their equation is not correct. In the present paper we have derived the correct form of the momentum equation (5). The closed form solutions for velocity and temperature are obtained. It is found that they satisfy governing equations and imposed conditions.

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