The Effect of Autocorrelation on the Performance of MEWMA Control Chart with ControlledCorrelation.

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Abstract. Control charts are made to identify assignable causes of difference that could exist in production processes. When traditional control charts are utilized you have the implied presumption that this observations are independently and identically distributed as time passes. It is usually believed the probability distribution which represents the actual observations includes a known functional form and it is constant as time passes. Nevertheless, in reality, observations produced through continuous in addition to discrete generation procedures in many cases are serially correlated. Autocorrelation not just breaks the actual independence assumption of conventional control charts, but also can impact the efficiency associated with control charts negatively. In this article, we are going to investigate the result associated with autocorrelation around the performance of MEWMA control chart, in which autocorrelated data were utilized to create the MEWMA chart with induced autocorrelation from various levels of correlations (small, moderate as well as large) and different sample sizes. Simulations had been done to create the data set used to construct the MEWMA control chart and the outcomes implies that all of the control charts constructed had their points outside the designed control limits, that confirmed the effect of autocorrelation to the performance of the MEWMA control chart.

*Keywords*Autocorrelation, MEWMA, ARIMA, Assignable causes, Standard control chart.

1.0 INTRODUCTION

Recentlythe significance ofqualityis becomingmore and moreobvious, andquality controlin productionhasshiftedfromdiscovering nonconforming itemsthroughassessmentin order tofindingqualityabnormalitiesalong the waythroughstatisticalprocesscontrol (SPC). Whereit isusedefficiently, SPC performsa vital rolein lesseningvariancewithinmanufacturedproductsas well as inenhancing thecompetitionof themanufacturerbyincreasingitemqualityyet stilltimeminimizingproductioncosts. Chartssuch as the Shewhart XandRchartsare findingbroaduse withinindustrydue to theirsimplicity of usewith regard totechniciansas well as otherswithsmalllearningstatistics, because thecomputationsandplottingcan be achievedmanually. Much morelately, nevertheless, technologyby means ofextremelyobtainableand much moreeffectivepersonal computersare makingthis easyto speedup SPC and offertechniciansandmanagersthe flexiblenessto spenda shorter periodaround themechanicsofchartingand much moretime inidentifyingmethods forenhancing theprocess. Furthermore, the newtechnologyprovidesmanagersa choice ofusing modern-day SPC models thatpreciselyrevealthe procedurebeingsupervisedthroughrelaxingmany of theassumptionsstated intraditional SPC, which are generallydisregardedin reality. For example, traditional SPC charts, such as Shewhart, exponentiallyweightedmovingaverage (EWMA) and thecumulativesum (CUSUM) charts, have already beendesignedwithin theassumptionthat thissystemobservationstend to beindependentandidenticallydistributed (i.i.d) normalwith regards to acentralmean

$$x_t = \mu + a_t \tag{1.1}$$

Where:
x_t= the observation at time t,
μ = the fixed process mean,
$\{a_t\}$ = a sequence of independently distributed errors with mean zero and variance σ_a^2.
However, oftenwithinindustrialpractice, withdiscretein addition tocontinuousproductionprocedures, findingsusually are notindependent, but they arereally serially correlated, for examplein machining andforgingprocedures [1]. Undersuchcircumstancestraditional SPC chartsmight beimproperforcheckingprocessquality, and therefore modern-day correlatedmodelsis highlyrecommendedThis sort ofcorrelatedmodelsmight notjust bewell suited forcheckingprocessquality, butprobablyenable

thepossibility ofpredictinglong termqualityas thepresence ofcorrelationresults inforecasting.

Despite the seeming advantages of the [2] approach, little or no work has been done to investigate its performance through traditional SPC procedures which do not explicitly account for correlated observations.

Under the conditions of autocorrelation, traditional SPC techniques are not enough as pointed out by so many authors. [3] discussed the effects of autocorrelation has on the Shewhart chart which is to increase the number of false alarms andinvestigations for assignable causes. [4], also discussed the effects of autocorrelation on the performance of the CUSUM chart. [5] discussed the impact on EWMA charts and pointed out that the average and median run lengths of these charts are sensitive to presence of autocorrelation.

To accommodate autocorrelated data, some additional SPC methodologies have been developed in recent years.

i. Process residual charts using models and plotting the model residuals using the traditional SPC charts.

ii. Adjusting the traditional SPC chart control limits.

iii. One-step-ahead EWMA.

Just to mention but few methods of accommodating the autocorrelation of which in this research we shall consider the process residuals chart using models and plotting the model residuals.

The processmakes itnecessary thatthe proceduredatabedesignwhile usingtimeseriesmodelfor example, the autoregressive integratedmovingaverage (ARIMA) model. Using theassumptionof theaccuratemodel, the processresidualstend to bestatisticallyindependentand will becontrol charted,while usingconventional multivariate SPC charts. In case achange inthemeantakes place, thedeterminedmodelhas stopped beingappropriateand also themodelerroris going to bechanged intotheresiduals.

A control chart applied to these residuals will ultimately detect the shift in the mean [6]. The traditional multivariate are: Hotelling's, MEWMA, and MCUSUM.

According to [6], they suggested that the presence of autocorrelation has a profound effect on control charts developed using the assumptions of independent observations. The necessary impact is to increase the frequency with which false action signals are generated.

[7] used observations generated by a multivariate AR(1) process to investigate the effects of autocorrelation on the performance of MCUSUM control chart, a procedure based on *Healy's* method that will improved the ARL values. Based on their study, they found that if the residuals from a time series model are used instead of the original data then the ARL properties of MCUSUM control chart can be improved considerably.

1.2 Organization of the Article

In this article, is on processes in which we determine the effect of autocorrelation on the performance of Multivariate Exponentially Weighted Moving Average (MEWMA) control chart when induced autocorrelation at different levels of correlation (low, moderate and high) are used to construct the control chart. In section 2, we stated the problem statement. Section 3, the simulated data were generated and used to construct the MEWMA chart and the results were obtained. Finally in section 4, we summarize and conclude on the findings.

2.0 PROBLEM STATEMENT

The problem statement is stated below as:

> *The impact of autocorrelation on the performance of Multivariate Exponentially Weighted moving Average (MEWMA) control chart when controlled at different levels of correlation.*

The objective of this paper is to investigate the effect of autocorrelation to the performance of MEWMA control chart. The effect of the autocorrelation on the traditional control charts, the Shewhart, the CUSUM and EWMA control charts have been studied by many researchers, for example:[8]studied the impact of autocorrelation on the retrospective X chart, where they concluded that positive autocorrelation results in an increased number of false alarms and negative autocorrelation leads to an increase in time required to detect process shift. [9] considered process observations following a first order autoregressive vector(VAR(1)) stationary process and they proposed a method for monitoring the mean of the autocorrelated multivariate process where the method was characterized by the diagnostic ability to identify the out of control.Based on the outcome of their studies, and to the knowledge of the authors so far there is no-one who did research on the impact of the autocorrelation with controlled correlation at different levels on the performance of MEWMA control chart.

3.0 SIMULATIONS AND RESULTS

In this study, we consider a process that follows an ARMA (1, 1) model. The ARMA (1, 1) model was chosen because it is stationary, as many SPC systems are in practice, and because it contains both an autoregressive and a moving-average component; hence the effect of each type of parameter could be examined. We assume for the study that only one observation is available (or practical) at each time period.

3.1. The ARMA (1, 1) Process

The ARMA(1,1) model is described as follows:[10]:

$$x_t = (1 - \phi_1)\mu + \phi_1 x_{t-1} + a_t - \theta_1 a_{t-1} \qquad (3.1)$$

Where:
x_t = the observation at time t,
a_t= the random noise term at time t,
ϕ_1 = the autoregressive parameter,
θ_1= moving average parameter,
μ= mean of the process
The parameters ϕ_1 and θ_1 must be fit to the data using standard techniques introduced by [10]. If ϕ_1= 0 the process is said to be purely moving average (commonly written as MA (1), and if θ_1 = 0 then the process is purely autoregressive or AR (1). If both ϕ_1 and θ_1 are 0 then the process degenerates to the random process given in (1.1).

3.2 TheControlcharts

This section is aimed at constructing multivariate exponentially weighted moving average control chart with induced autocorrelation at different levels of correlation for some specified samples sizes of 1000, 500 and 100 each for the correlation values of 0.9, 0.7 and 0.2 respectively.

3.2.1 MEWMA (Multivariate Exponentially Weighted Moving Average) Control Chart

MEWMA is the multivariate extension of the EWMA chart proposed by Roberts (1959). it was introduced by Lowry et al. (1992) and is more sensible in detecting non-random changes in the process and based on the principle of the weighted average of the previously observed vectors.

The MEWMA chart has the statistic:

$$T^2 = Z_i' \Sigma_{z_i}^{-1} Z_i > h \qquad (3.2)$$

where

$$Z_i = \lambda X_i + (1-\lambda) X_{i-1}$$

being $Z_0 = 0$, λ is diagonal *(p x p)* matrix of the smoothing constant with $0<\lambda_i<1$, although in practice there is no reason to employ different values of λ in the same problem. In particular case, when rational subgroups are obtained, i.e. , n>1, just replace X_i with $\overline{X}_i$.

Lowry et al. (1992) provided two alternatives to compute the Σ_{z_i}, the exact covariance matrix:

$$\Sigma_{Z_i} = \frac{\lambda\,[1-(1-\lambda)^{2i}]}{2-\lambda}(\Sigma) \qquad (3.3)$$

And asymptotic covariance matrix

$$\Sigma_{Z_i} = \frac{\lambda}{2-\lambda}(\Sigma) \qquad (3.4)$$

The exact covariance matrix is better in performance than the asymptotic according to [11].Also, they point out that the ARL performance of the chart depends only on no centrality parameter θ:

$$\theta = [(\mu_1 - \mu_0)' \Sigma (\mu_1 - \mu_0)]^{1/2}, \qquad (3.5)$$

3.2.2 MEWMA Control Chart Construction

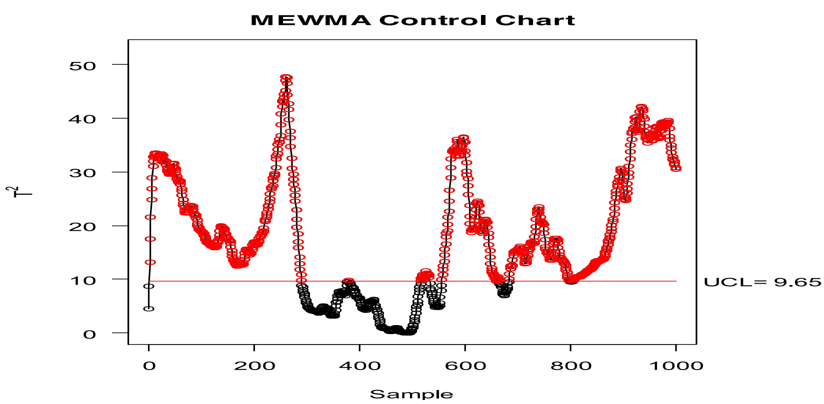

Figure1: MEWMA Control Chart with Autocorrelation, n=1000, p=2, r=0.9

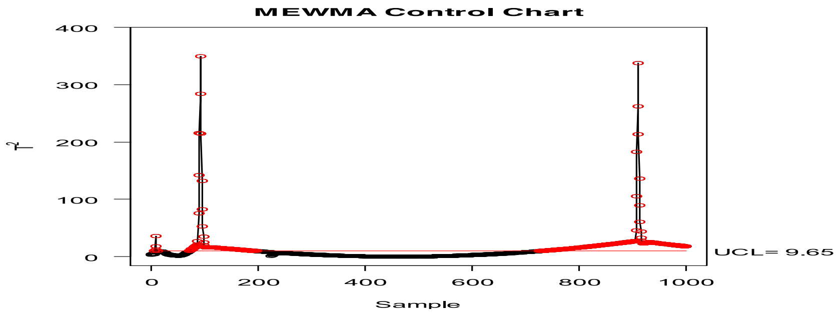

Figure 2 : MEWMA Control Chart with Autocorrelation, n=1000, p=2, r=0.7

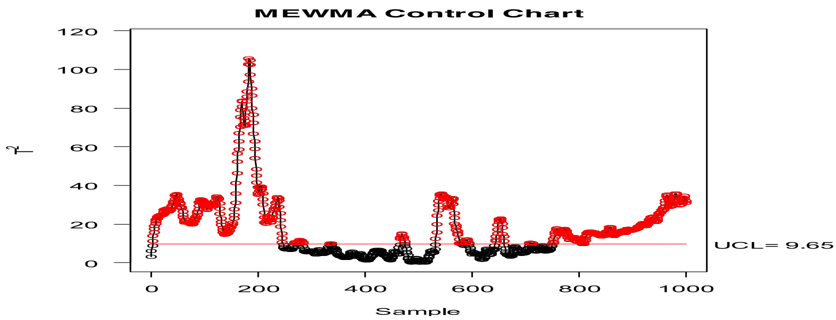

Figure 3 MEWMA Control Chart with Autocorrelation, n=1000, p=2, r=0.2

In Figures 1,2 and 3 depicts the MEWMA control chart with induced autocorrelation at different level of controlled correlation of 0.9, 0.7 and 0.2 respectively, with the same number of observation simulated 1000 times and 2 variables as quantity characteristics. Almost all the points of the charts are outside the designed control limits. The autocorrelation adversely affected the performance of the MEWMA control as shown above.

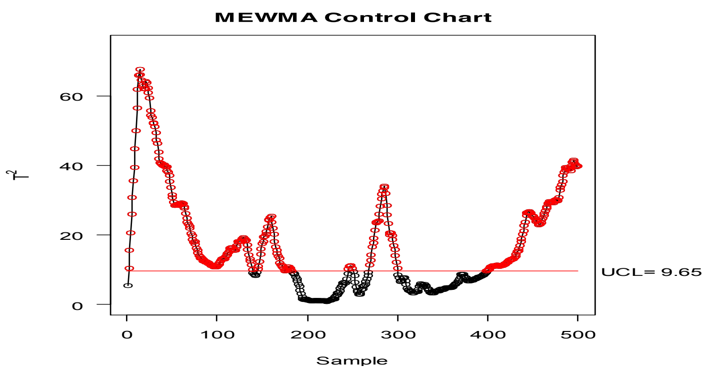

Figure 4: MEWMA Control Chart with Autocorrelation, n=500, p=2, r=0.9

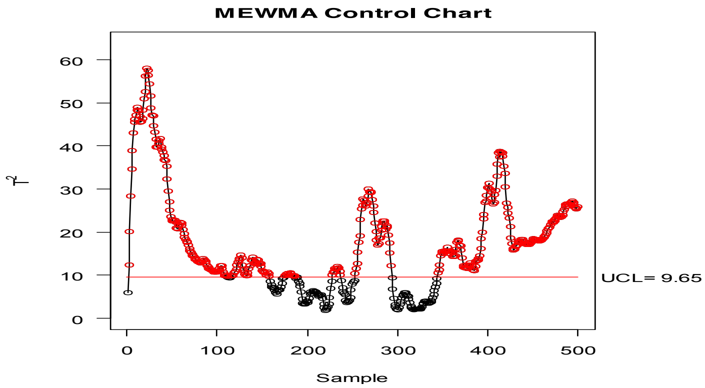

Figure 5: MEWMA Control Chart with Autocorrelation, n=500, p=2, r=0.7

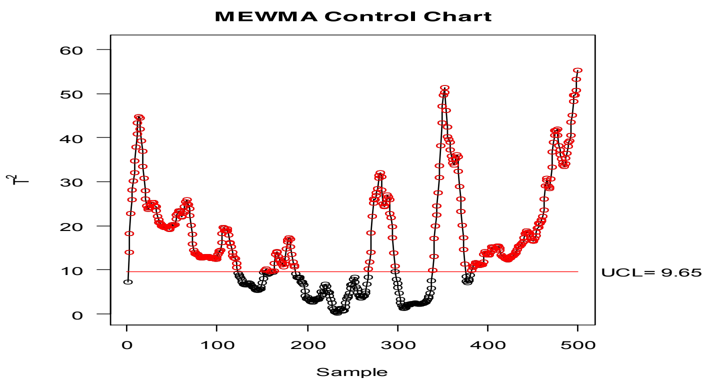

Figure 6: MEWMA Control Chart with Autocorrelation, n=500, p=2, r=0.2

In Figures 4, 5 and 6 shows the MEWMA control chart with induced autocorrelation at different levels of controlled correlation (0.9, 0.7 and 0.2) respectively with same number of observations simulated 500, it is clearly evident that the process is out of control with only very few points fall within the designed control limits.

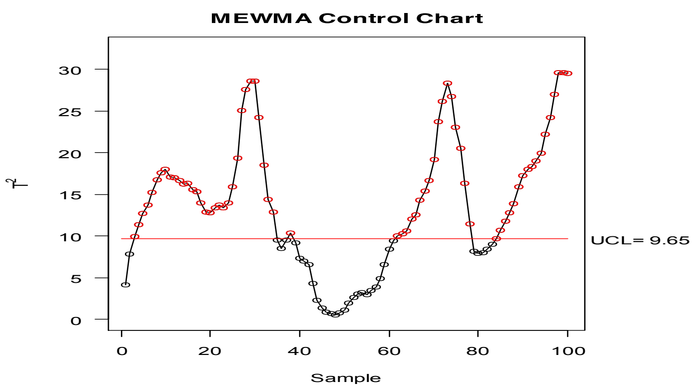

Figure 7: MEWMA Control Chart with Autocorrelation, n=100, p=2, r=0.9

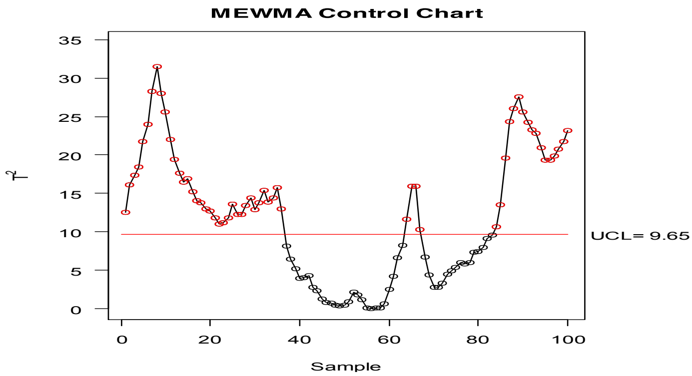

Figure 81: MEWMA Control Chart with Autocorrelation, n=100, p=2, r=0.7

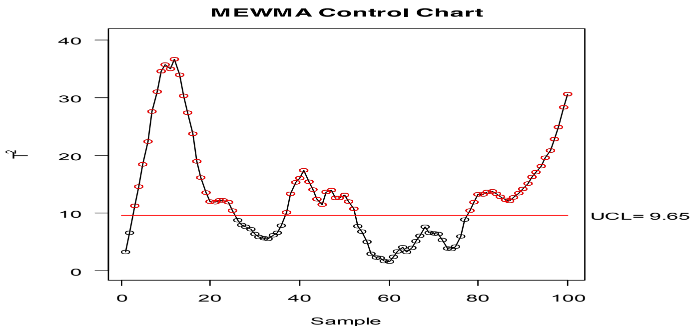

Figure 9: MEWMA Control Chart with Autocorrelation, n=100, p=2, r=0.2

Figures 7, 8 and 9 shows the MEWMA control charts based on 100 simulated data set with autocorrelation with controlled correlation of r=0.9,0.7 and 0.2 respectively. Where the charts exhibits out control process. hence the autocorrelated had adversely affected the performance of the MEWMA control chart.

3.3 Dealing with Autocorrelation

.Because of the autocorrelation exists, an effectiveplan of actionshould beestablished, the most efficientstrategydepends uponthe specificapplication
In many cases, the effect of autocorrelation is to increase the variability of the process, the need to first attempt to remove the source of the autocorrelation if possible. If the source of autocorrelation cannot be removed directly, then it may be possible to model the autocorrelation and use a feedback control scheme to reduce variability about a specified target value. [12]
Fitting a time series model to the process data so that forecasts of each observations can be made using the previous observation and then applies to the residuals of the traditional control charts. See [13],[6] and [14] to mention but few.

The typical time series model usually employed is the autoregressive integrated moving average (ARIMA) model

$$\Phi_p(B)\nabla^d X_t = \Theta_q(B)\epsilon_t$$

Where $\Phi_p(B)$ =(1-$\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$) is an autoregressive polynomial of order p,

$\Theta_q(B)$ = (1-$\theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$) is a moving average polynomial of order q, ∇ is a backward difference operator, B is the backshift operator, and ϵ_t is a sequence of normally and independently distributed random shock with mean zero and constant variance σ_p^2. If $\hat{X}_t$ is the predicted value obtained from an appropriately-identified and fitted ARIMA model then the residuals $e_t = X_t - \hat{X}_t$ will behave like independent and identically distributed random variables [10]. Therefore, control charts may be applied to the residuals. If a shift in the mean occurs, the identified model will no longer be correct and the model misspecification will be transferred to the residuals. Hence the shift will be detected on a control chart that applied the residuals.

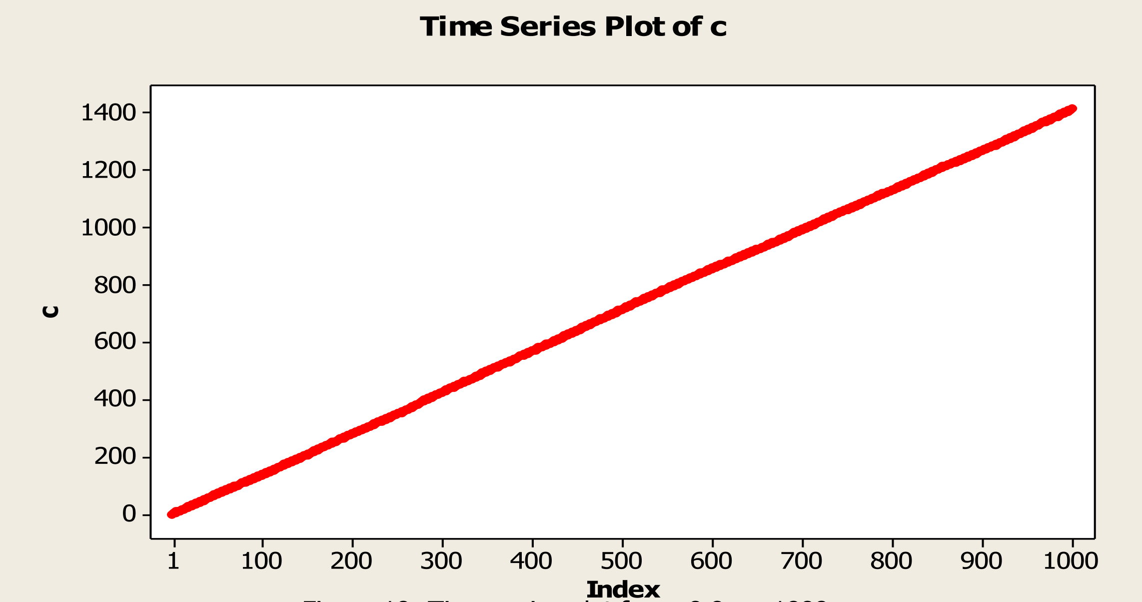

Figure 10: Time series plot for r=0.9, n=1000

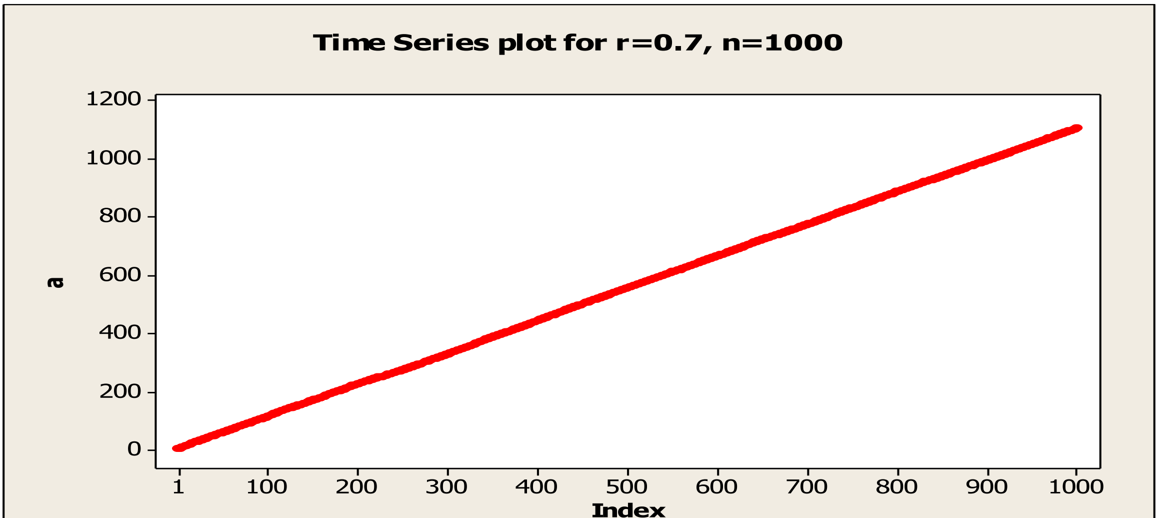

Figure 11: Time series plot for r=0.7 and n=1000

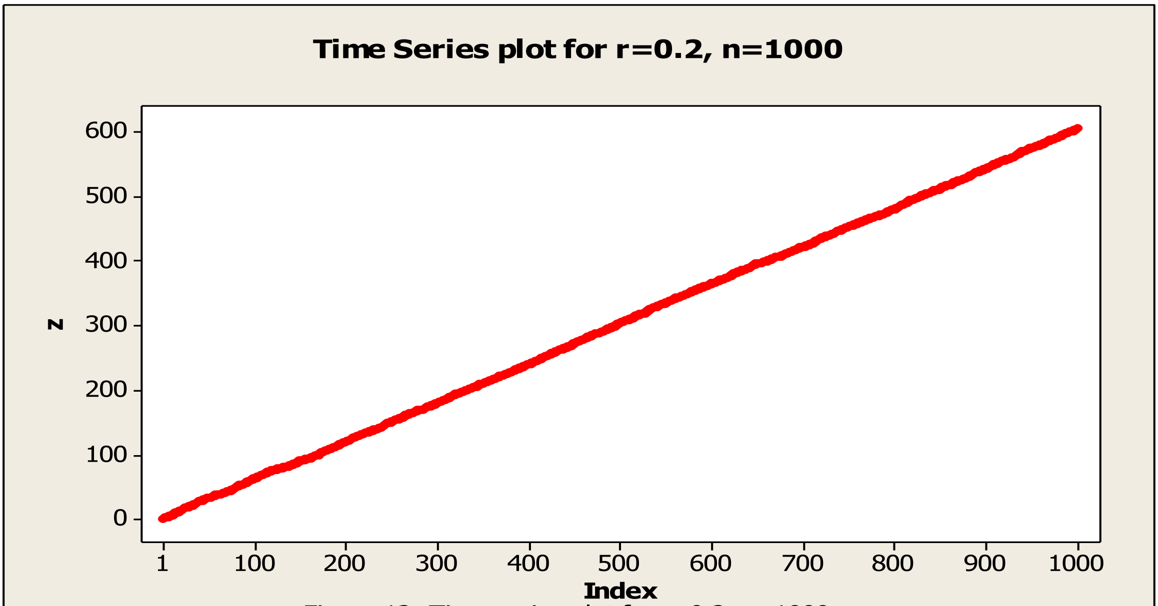

Figure 12: Time series plot for r=0.2, n=1000

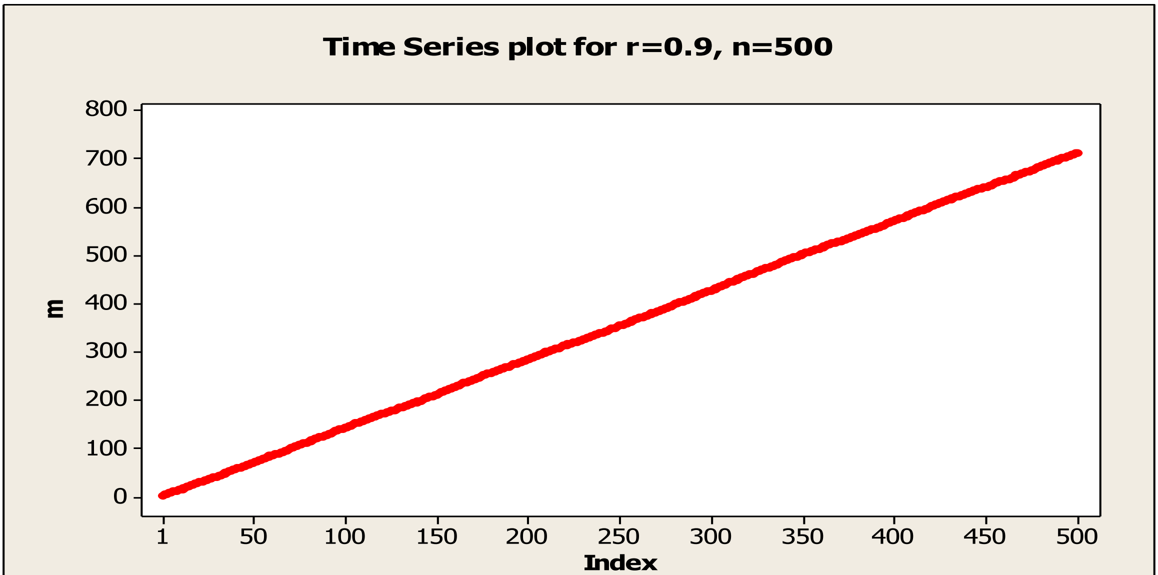

Figure 13: Time series plot for r=0.9, n=500

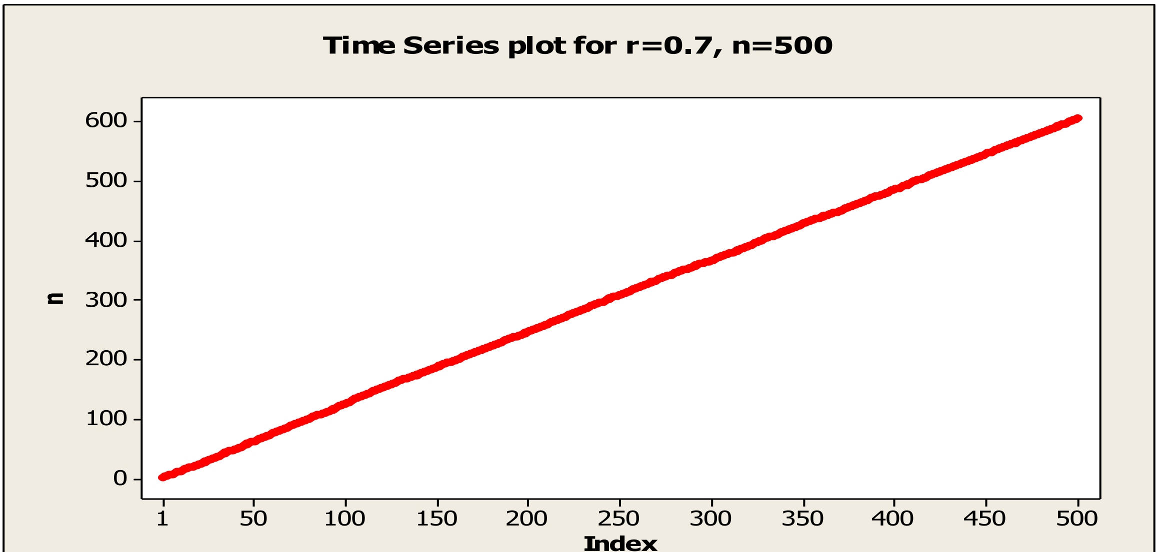

Figure 14: Time series plot for r=0.7, n=500

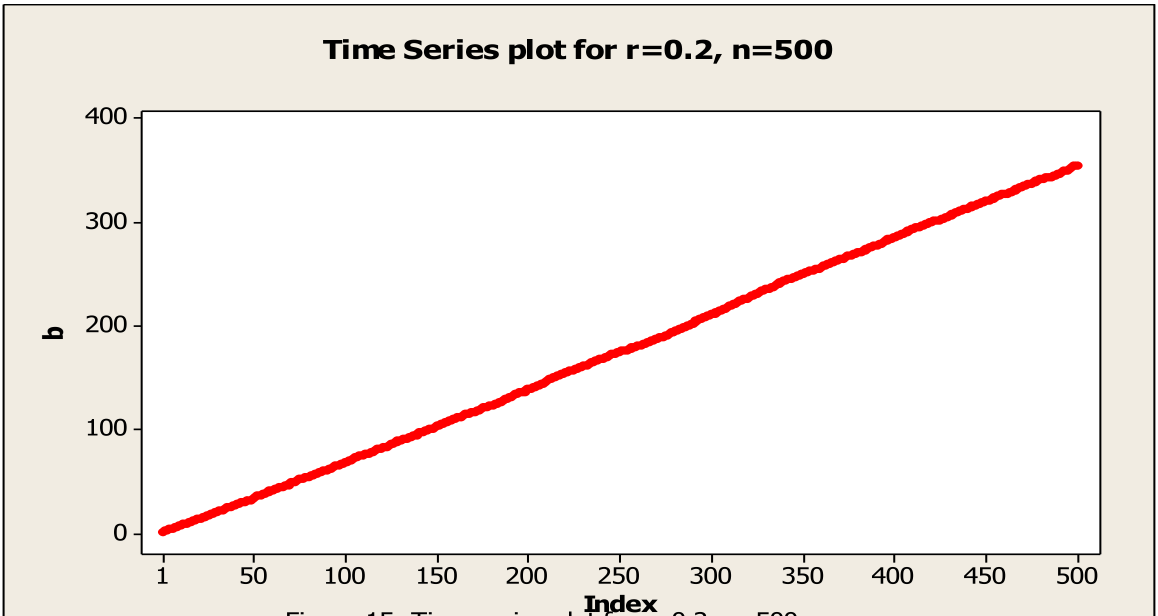

Figure 15: Time series plot for r=0.2, n=500

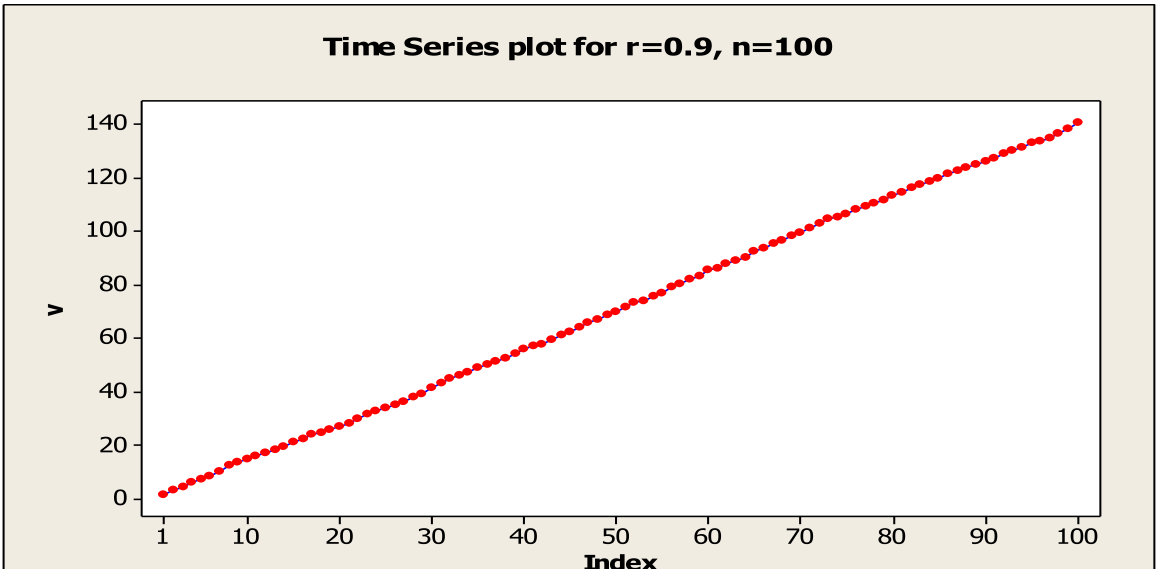

Figure 16: Time series plot for r=0.9, n=100

Figure 17: Time series plot for r=0.7, n=100

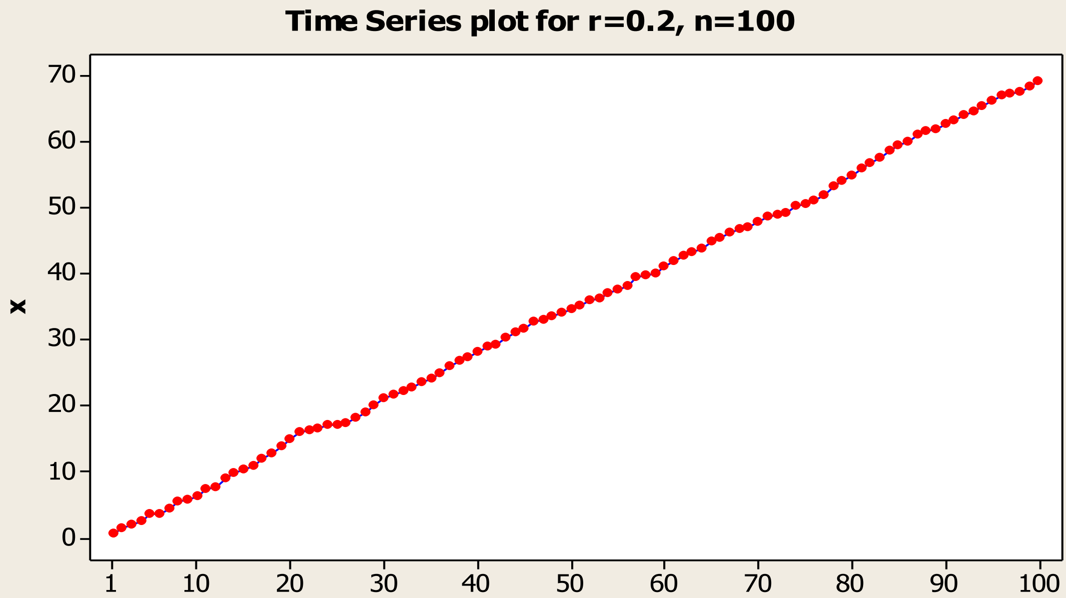

Figure 18: Time series plot for r=0.2, n=100

Figures 10-18 shows the time series plot of the simulated data set with autocorrelation at the 3 levels of correlation controlled(0.9, 0.7, and 0.2), for various sample sizes of 1000, 500 and 100. All the plots are patterned-like and are positively linear. It is quite possible then that the points outside the control limits on the original individual charts were outside the limits because of due to the occurrence of special causes. Many researchers investigates the properties of the proposed charts for autocorrelated processes, determine their relative performance, or compare the performance to that of the traditional charts. Even though the use of traditional control charts for the autocorrelated processes seems to be difficult. [15]

3.4 Fitting the ARIMA model

In this section the simulated data with autocorrelation were fitted to the ARIMA model in order to find the best model that could fit the data. With the plots in Figures 10-18 they are structured and on trying to fit the ARIMA model, which could not fit any of the family as the data set cannot be estimated to enable the plotting of the residuals to the individual control chart. This could be attributable to the special causes, which is the induced autocorrelation.

4.0 SUMMARY AND CONCLUSION

The effect of autocorrelation on the performance of MEWMA control chart with controlled correlation has been investigated at different level of correlations (low, moderate and high) with different sample sizes were simulated and found that it has an adverse effect on the performance of MEWMA as we can see that from the output of control charts in Figures 1-9 shows all are out of control. Many suggestions were made on the methods to deal with autocorrelation in the literatures for example: [2], [14] but for this research we decided to use one of the methods, fit an appropriate time series model to the observations and then apply the individual control charts to the residuals from this model, which is usually the (ARIMA) autoregressive integrated moving average model.

Based on our finding that the effect of autocorrelation with controlled correlation to the performance of MEWMA control chart is certain, which is also same as other control charts constructed from continuous and discrete processes and the residual charts can be able to detect the shift as soon as it occurs or at later time [16], but unlike in this case where the residual charts are not possible.

This conclude that the simulated data with autocorrelation with controlled level of correlation have an adverse effect on the performance of the MEWMA control chart, where the suggested methods of dealing with general autocorrelation failed to correct the measures by using the fitting the ARIMA model method to the data.

Further research is suggested by the authors to look into the impact of the induced autocorrelation on the performance of other multivariate traditional control charts such as the Hotelling's, MCUSUM etc. when the correlation is controlled.

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