The Effect of Autocorrelation on the Performance of MEWMA Control Chart with Controlled Correlation.

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Abstract. Control charts are made to identify assignable causes of difference that could exist in production processes. When traditional control charts are utilized you have the implied presumption that this observations are independently and identically distributed as time passes. It is usually believed the probability distribution which represents the actual observations includes a known functional form and it is constant as time passes. Nevertheless, in reality, observations produced through continuous in addition to discrete generation procedures in many cases are serially correlated. Autocorrelation not just breaks the actual independence assumption of conventional control charts, but also can impact the efficiency associated with control charts negatively. In this article, we are going to investigate the result associated with autocorrelation around the performance of MEWMA control chart, in which autocorrelated data were utilized to create the MEWMA chart with induced autocorrelation from various levels of correlations (small, moderate as well as large) and different sample sizes. Simulations had been done to create the data set used to construct the MEWMA control chart and the outcomes implies that all of the control charts constructed had their points outside the designed control limits, that confirmed the effect of autocorrelation to the performance of the MEWMA control chart.

Keywords Autocorrelation, MEWMA, ARIMA, Assignable causes, Standard control chart.
1.0 INTRODUCTION

Recently the significance of quality is becoming more and more obvious, and quality control in production has shifted from discovering nonconforming items through assessment in order to finding quality abnormalities along the way through statistical process control (SPC). Where it is used efficiently, SPC performs a vital role in lessening variance within manufactured products as well as enhancing the competition of the manufacturer by increasing item quality yet still time minimizing production costs. Charts such as the Shewhart X and R charts are finding broad use in industry due to their simplicity of use with regard to technicians as well as others with small learning statistics, because the computations and plotting can be achieved manually. Much more lately, nevertheless, technology by means of extremely obtainable and much more effective personal computers are making this easy to speed up SPC and offer technicians and managers the flexibility to spend a shorter period around the mechanics of charting and much more time in identifying methods for enhancing the process. Furthermore, the new technology provides managers a choice of using modern-day SPC models that precisely reveal the procedure being supervised through relaxing many of the assumptions stated in traditional SPC, which are generally disregarded in reality. For example, traditional SPC charts, such as Shewhart, exponentially weighted moving average (EWMA) and the cumulative sum (CUSUM) charts, have already been designed within the assumptions that this system observation tends to be independent and identically distributed (i.i.d) normal with regard to a central mean

\[ x_t = \mu + a_t \]  

(1.1)

Where:
- \( x_t \) = the observation at time \( t \),
- \( \mu \) = the fixed process mean,
- \( \{ a_t \} \) = a sequence of independently distributed errors with mean zero and variance \( \sigma_a^2 \).

However, often within industrial practice, with discrete in addition to continuous production procedures, findings usually are not independent, but they are really serially correlated, for example, in machining and forging procedures [1]. Under such circumstance, traditional SPC charts might be improper for checking process quality, and therefore modern-day correlated models is highly recommended. This sort of correlated models might not just be well suited for checking process quality, but probably enable
the possibility of predicting long term quality as the presence of correlation results in forecasting.

Despite the seeming advantages of the [2] approach, little or no work has been done to investigate its performance through traditional SPC procedures which do not explicitly account for correlated observations.

Under the conditions of autocorrelation, traditional SPC techniques are not enough as pointed out by so many authors. [3] discussed the effects of autocorrelation has on the Shewhart chart which is to increase the number of false alarms and investigations for assignable causes. [4], also discussed the effects of autocorrelation on the performance of the CUSUM chart. [5] discussed the impact on EWMA charts and pointed out that the average and median run lengths of these charts are sensitive to presence of autocorrelation.

To accommodate autocorrelated data, some additional SPC methodologies have been developed in recent years.

i. Process residual charts using models and plotting the model residuals using the traditional SPC charts.

ii. Adjusting the traditional SPC chart control limits.

iii. One-step-ahead EWMA.

Just to mention but few methods of accommodating the autocorrelation of which in this research we shall consider the process residuals chart using models and plotting the model residuals.

The process makes it necessary that the procedure database design while using time series model for example, the autoregressive integrated moving average (ARIMA) model. Using the assumption of the accurate model, the process residualstend to be statistically independent and will be charted, while using conventional multivariate SPC charts. In case a change in the mean takes place, the determined model has stopped being appropriate and also the model error is going to be changed into the other residuals.

A control chart applied to these residuals will ultimately detect the shift in the mean [6]. The traditional multivariate are: Hotelling's, MEWMA, and MCUSUM.

According to [6], they suggested that the presence of autocorrelation has a profound effect on control charts developed using the assumptions of independent observations. The necessary impact is to increase the frequency with which false action signals are generated.

[7] used observations generated by a multivariate AR(1) process to investigate the effects of autocorrelation on the performance of MCUSUM control chart, a procedure based on Healy's method that will improved the ARL values. Based on their study, they found that if the residuals from a time series model are used instead of the original data then the ARL properties of MCUSUM control chart can be improved considerably.
1.2 Organization of the Article

In this article, is on processes in which we determine the effect of autocorrelation on the performance of Multivariate Exponentially Weighted Moving Average (MEWMA) control chart when induced autocorrelation at different levels of correlation (low, moderate and high) are used to construct the control chart. In section 2, we stated the problem statement. Section 3, the simulated data were generated and used to construct the MEWMA chart and the results were obtained. Finally in section 4, we summarize and conclude on the findings.

2.0 PROBLEM STATEMENT

The problem statement is stated below as:

The impact of autocorrelation on the performance of Multivariate Exponentially Weighted moving Average (MEWMA) control chart when controlled at different levels of correlation.

The objective of this paper is to investigate the effect of autocorrelation to the performance of MEWMA control chart. The effect of the autocorrelation on the traditional control charts, the Shewhart, the CUSUM and EWMA control charts have been studied by many researchers, for example:[8]studied the impact of autocorrelation on the retrospective X chart, where they concluded that positive autocorrelation results in an increased number of false alarms and negative autocorrelation leads to an increase in time required to detect process shift. [9]considered process observations following a first order autoregressive vector(VAR(1)) stationary process and they proposed a method for monitoring the mean of the autocorrelated multivariate process where the method was characterized by the diagnostic ability to identify the out of control. Based on the outcome of their studies, and to the knowledge of the authors so far there is no-one who did research on the impact of the autocorrelation with controlled correlation at different levels on the performance of MEWMA control chart.

3.0 SIMULATIONS AND RESULTS

In this study, we consider a process that follows an ARMA (1, 1) model. The ARMA (1, 1) model was chosen because it is stationary, as many SPC systems are in practice, and because it contains both an autoregressive and a moving-
average component; hence the effect of each type of parameter could be examined. We assume for the study that only one observation is available (or practical) at each time period.

3.1. The ARMA (1, 1) Process

The ARMA (1,1) model is described as follows:[10]:

\[ x_t = (1 - \phi_1) \mu + \phi_1 x_{t-1} + a_t - \theta_1 a_{t-1} \]  

(3.1)

Where:
\( x_t \) = the observation at time t,
\( a_t \) = the random noise term at time t,
\( \phi_1 \) = the autoregressive parameter,
\( \theta_1 \) = moving average parameter,
\( \mu \) = mean of the process

The parameters \( \phi_1 \) and \( \theta_1 \) must be fit to the data using standard techniques introduced by [10]. If \( \phi_1 = 0 \) the process is said to be purely moving average (commonly written as MA (1), and if \( \theta_1 = 0 \) then the process is purely autoregressive or AR (1). If both \( \phi_1 \) and \( \theta_1 \) are 0 then the process degenerates to the random process given in (1.1).

3.2. The Control Charts

This section is aimed at constructing multivariate exponentially weighted moving average control chart with induced autocorrelation at different levels of correlation for some specified samples sizes of 1000, 500 and 100 each for the correlation values of 0.9, 0.7 and 0.2 respectively.

3.2.1 MEWMA (Multivariate Exponentially Weighted Moving Average) Control Chart

MEWMA is the multivariate extension of the EWMA chart proposed by Roberts (1959). It was introduced by Lowry et al. (1992) and is more sensible in detecting non-random changes in the process and based on the principle of the weighted average of the previously observed vectors.
The MEWMA chart has the statistic:

\[ T^2 = Z_i^T \Sigma_z^{-1} Z_i > h \quad (3.2) \]

where

\[ Z_i = \lambda X_i + (1 - \lambda) X_{i-1} \]

being \( Z_0 = 0 \), \( \lambda \) is diagonal \((p \times p)\) matrix of the smoothing constant with \( 0 < \lambda < 1 \), although in practice there is no reason to employ different values of \( \lambda \) in the same problem. In particular case, when rational subgroups are obtained, i.e., \( n > 1 \), just replace \( X_i \) with \( \overline{X}_i \).

Lowry et al. (1992) provided two alternatives to compute the \( \Sigma_z \), the exact covariance matrix:

\[ \Sigma_z = \frac{\lambda [1-(1-\lambda)^{zi}]}{2-\lambda} (\Sigma) \quad (3.3) \]

And asymptotic covariance matrix

\[ \Sigma_z = \frac{\lambda}{2-\lambda} (\Sigma) \quad (3.4) \]

The exact covariance matrix is better in performance than the asymptotic according to [11]. Also, they point out that the ARL performance of the chart depends only on no centrality parameter \( \theta \):

\[ \theta = [((\mu_1 - \mu_0) \Sigma (\mu_1 - \mu_0)]^{1/2} \quad (3.5) \]

### 3.2.2 MEWMA Control Chart Construction

![MEWMA Control Chart](image)
In Figures 1, 2 and 3 depicts the MEWMA control chart with induced autocorrelation at different level of controlled correlation of 0.9, 0.7 and 0.2 respectively, with the same number of observation simulated 1000 times and 2 variables as quantity characteristics. Almost all the points of the charts are outside the designed control limits. The autocorrelation adversely affected the performance of the MEWMA control as shown above.
In Figures 4, 5 and 6 shows the MEWMA control chart with induced 
autocorrelation at different levels of controlled correlation (0.9, 0.7 and 0.2) 
respectively with same number of observations simulated 500, it is clearly 
evident that the process is out of control with only very few points fall within the 
designed control limits.

![Figure 5: MEWMA Control Chart with Autocorrelation, n=500, p=2, r=0.7](image)

![Figure 6: MEWMA Control Chart with Autocorrelation, n=500, p=2, r=0.2](image)

![Figure 7: MEWMA Control Chart with Autocorrelation, n=100, p=2, r=0.9](image)
Figures 7, 8 and 9 shows the MEWMA control charts based on 100 simulated data set with autocorrelation with controlled correlation of r=0.9, 0.7 and 0.2 respectively. Where the charts exhibits out control process hence the autocorrelated had adversely affected the performance of the MEWMA control chart.

3.3 Dealing with Autocorrelation

Because of the autocorrelation exists, an effective plan of action should be established, the most efficient strategy depends upon the specific application. In many cases, the effect of autocorrelation is to increase the variability of the process, the need to first attempt to remove the source of the autocorrelation if possible. If the source of autocorrelation cannot be removed directly, then it may be possible to model the autocorrelation and use a feedback control scheme to reduce variability about a specified target value. \[12\] Fitting a time series model to the process data so that forecasts of each observation can be made using the previous observation and then applies to the residuals of the traditional control charts. See \[13\], [6] and \[14\] to mention but few.
The typical time series model usually employed is the autoregressive integrated moving average (ARIMA) model

\[ \Phi_p(B)\nabla^d X_t = \Theta_q(B)\epsilon_t \]

Where \( \Phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p) \) is an autoregressive polynomial of order \( p \),
\( \Theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) \) is a moving average polynomial of order \( q \), \( \nabla \) is a backward difference operator, \( B \) is the backshift operator, and \( \epsilon_t \) is a sequence of normally and independently distributed random shock with mean zero and constant variance \( \sigma^2 \). If \( \hat{X}_t \) is the predicted value obtained from an appropriately-identified and fitted ARIMA model then the residuals \( e_t = X_t - \hat{X}_t \) will behave like independent and identically distributed random variables [10]. Therefore, control charts may be applied to the residuals. If a shift in the mean occurs, the identified model will no longer be correct and the model misspecification will be transferred to the residuals. Hence the shift will be detected on a control chart that applied the residuals.

![Time Series Plot of c](image-url)

Figure 10: Time series plot for \( r=0.9 \), \( n=1000 \)
Figure 11: Time series plot for $r=0.7$ and $n=1000$

Figure 12: Time series plot for $r=0.2$, $n=1000$
Time Series plot for $r=0.9$, $n=500$

Figure 13: Time series plot for $r=0.9$, $n=500$

Time Series plot for $r=0.7$, $n=500$

Figure 14: Time series plot for $r=0.7$, $n=500$
Figure 15: Time series plot for $r=0.2$, $n=500$

Figure 16: Time series plot for $r=0.9$, $n=100$
Time Series plot for r=0.7, n=100

Figure 17: Time series plot for r=0.7, n=100
Figures 10-18 shows the time series plot of the simulated data set with autocorrelation at the 3 levels of correlation controlled (0.9, 0.7, and 0.2), for various sample sizes of 1000, 500 and 100. All the plots are patterned-like and are positively linear. It is quite possible then that the points outside the control limits on the original individual charts were outside the limits because of due to the occurrence of special causes. Many researchers investigates the properties of the proposed charts for autocorrelated processes, determine their relative performance, or compare the performance to that of the traditional charts. Even though the use of traditional control charts for the autocorrelated processes seems to be difficult. [15]

3.4 Fitting the ARIMA model

In this section the simulated data with autocorrelation were fitted to the ARIMA model in order to find the best model that could fit the data. With the plots in Figures 10-18 they are structured and on trying to fit the ARIMA model, which could not fit any of the family as the data set cannot be estimated to enable the plotting of the residuals to the individual control chart. This could be attributable to the special causes, which is the induced autocorrelation.
4.0 SUMMARY AND CONCLUSION

The effect of autocorrelation on the performance of MEWMA control chart with controlled correlation has been investigated at different level of correlations (low, moderate and high) with different sample sizes were simulated and found that it has an adverse effect on the performance of MEWMA as we can see that from the output of control charts in Figures 1-9 shows all are out of control. Many suggestions were made on the methods to deal with autocorrelation in the literatures for example: [2], [14] but for this research we decided to use one of the methods, fit an appropriate time series model to the observations and then apply the individual control charts to the residuals from this model, which is usually the (ARIMA) autoregressive integrated moving average model.

Based on our finding that the effect of autocorrelation with controlled correlation to the performance of MEWMA control chart is certain, which is also same as other control charts constructed from continuous and discrete processes and the residual charts can be able to detect the shift as soon as it occurs or at later time [16], but unlike in this case where the residual charts are not possible.

This conclude that the simulated data with autocorrelation with controlled level of correlation have an adverse effect on the performance of the MEWMA control chart, where the suggested methods of dealing with general autocorrelation failed to correct the measures by using the fitting the ARIMA model method to the data.

Further research is suggested by the authors to look into the impact of the induced autocorrelation on the performance of other multivariate traditional control charts such as the Hotelling’s, MCUSUM etc. when the correlation is controlled.
REFERENCES


