Effects of the Optimized Resonator Dimensions on the Performance of the Standing-wave Thermoacoustic Refrigerator

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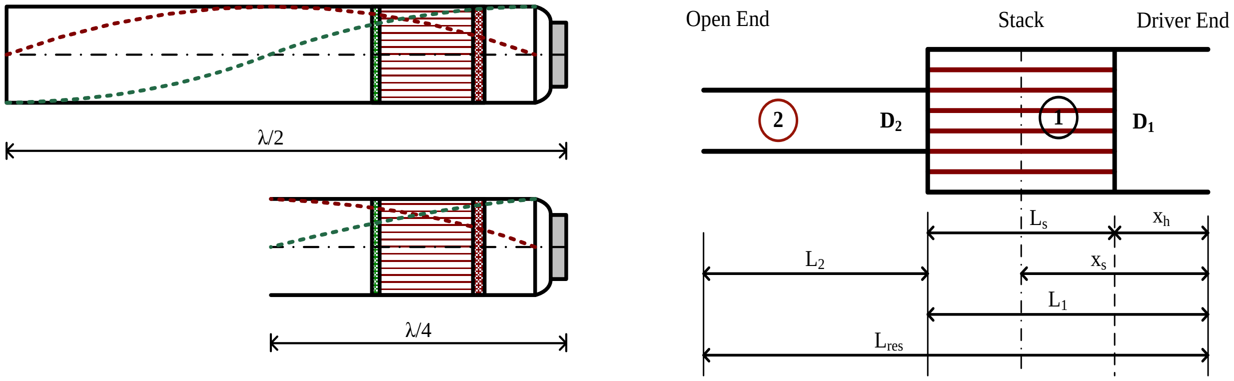
Abstract. Thermoacoustic refrigerator is an alternative cooling system, which is environmentally safe due to the absence of any refrigerants. The resonator tube of the system is of great importance; its design and dimensions influence the design and performance of the entire refrigerator. The central component of the resonator is the stack. So this work describes the design of the stack and the resonator along with the influence of its dimensions on the performance of the standing-wave thermoacoustic refrigerator. The resonator consists of two tubes, one larger than the other, characterized by the diameter ratio of the small over the larger diameter. A Lagrange multiplier method is used as a technique to optimize the coefficient of performance (*COP*) of the system. The computational analyses show that the resonator small diameter tube dissipates minimum acoustic power at a diameter ratio of 0.46, which is about 17 percent (at least) less than the published values. Moreover, the results show that the resonator length increases gradually with the increase of the mean design temperature. The increase of the resonator length leads to increase of the total acoustic power dissipated by the resonator, which reduces the *COP* of the standing-wave thermoacoustic refrigerator.

Introduction

Thermoacoustic refrigerating systems are a conceivable alternative technology to the conventional systems [1]. Sound wave in a gas consists of pressure and displacement oscillations accompanied by temperature oscillations. The interference between these oscillations and the solid boundaries produces thermoacoustic effects. Harnessing these effects is the basis of thermoacoustic devices [2]. Merits of the thermoacoustic refrigerators include the use of non-hazardous inert gases, the relatively simple mechanical design with few or no moving part. Nevertheless, the practical application of this technology is challenged by its low coefficient of performance (*COP*) [3]. Generally, a standing-wave thermoacoustic refrigerator consists of an acoustic driver attached to one end of a resonator tube filled with an inert gas. In addition, it consists of a stack sandwiched between two heat exchangers [4]. The acoustic driver, which involves the only moving part of the refrigerator, sustains the standing waves in the resonator. The stack provides a medium for the heat transfer process as the sound wave oscillates through the resonator. The heat exchangers merely transfer heat between each end of the stack, from the cooling load to the environment.

The resonator stores the acoustic energy and obtains the required phasing between the velocity and the pressure. Its length together with the gas sound speed determines the operating resonance frequency [5]. Thence, the principal role of the resonator is to maintain a desired resonance frequency, meanwhile minimizing the dissipation of the acoustic power [6]. The resonance frequency governs the total resonator length. Typically, the total resonator length corresponds to the half wavelength or the quarter wavelength [7]. The half wavelength resonator is merely a tube closed at both ends while the quarter wavelength resonator is open at one end as shown in Fig. 1(a).

The open end of the quarter wavelength resonator ends with a large buffer volume. Reducing the diameter of the resonator tube at the cold side of the stack optimizes the quarter wavelength resonator [8]. Therefore, the resonator comprises two sections; a large diameter tube (1) and a small diameter tube (2). The large diameter tube contains the stack and the heat exchangers as indicated in Fig. 1(b).



(a) Resonator length. (b) The large and the small diameter tubes

Figure 1: The length and dimensions of the resonator and its parts.

Hardly any research had been devoted to the resonator unit design and its effects on the thermoacoustic refrigerator. Therefore, this paper investigates the influence of the resonator dimensions on the performance of the standing-wave thermoacoustic refrigerator, taking into account the optimized design of the stack unit.

Design of the Refrigerator

Each component of the standing-wave thermoacoustic refrigerator is easy to build, even though its complex design represents a big challenge. Helium at 10 *bar* design pressure and 0.02 drive ratio is selected as a working medium for the existing study. 400 *Hz* resonance frequency and mean design temperature in the range of 200-450 *K* are chosen to compute the performance of the standing-wave thermoacoustic refrigerator.

Stack Design. The stack coefficient of performance, COP_s, is the upper bound of the entire standing-wave thermoacoustic refrigerator *COP* [3]. Therefore, its design determines the dimensions and design of the other components of a thermoacoustic refrigerator. A Mylar parallel plates stack with plate spacing equal to triple the thermal penetration depth [7] is considered herein. The normalized cooling load, Q_{cn}, and the normalized acoustic power, W_n, are used to calculate the stack coefficient of performance according to:

$$COP_s = Q_{cn}/W_n. \tag{1}$$

The optimum stack *COP* and the corresponding stack center position, x_s, stack length, L_s, and stack cross-sectional area, A_s, are computed with Lagrange multiplier method. One can refer to the design and optimization procedure of the stack unit with the Lagrange multiplier method details, which was presented before [9].

Resonator Design. The resonator material should be adequately strong and robust as it usually contains a high pressure working medium. It should have considerable acoustical impedance for the working fluid to see it as a solid boundary. In addition, it should have low thermal conductivity in order to prevent the parasitic heat losses [10]. The selection of the resonator material is also determined by the ease machined, simple construction and low cost. The resonance frequency, f, governs the resonator length, L_r, through the correlation:

$$L_r = a/(nf) = \sqrt{\gamma R T_m}/(nf). \tag{2}$$

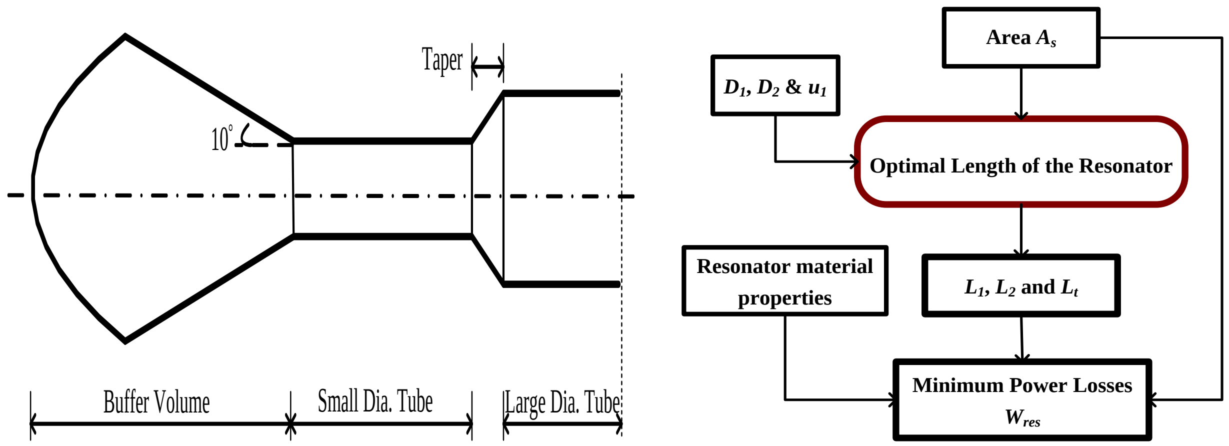
Here, a is the sound velocity, n corresponding to the half wavelength and quarter wavelength are 2.0 and 4.0 respectively, γ is the isobaric to isochoric ratio, R is the gas constant and T_m is the mean design temperature. The resonance frequency, the stack position and the stack length determine the resonator geometry. The resonance frequency in the normal resonance mode corresponds to the non-uniform cross-sectional area of the resonator [6]. The total acoustic power dissipated per unit surface area of the resonator, W_r, is defined as:

$$dW_r/dS = 0.25\rho_m|u_1|^2\delta_v\omega + 0.25|p_1|^2(\gamma\text{-}1)\delta_k\omega/\rho_m a^2(1+\epsilon_s). \tag{3}$$

Here, ρ_m is the mean density, u_1 is the gas particle velocity, δ_v is the viscous penetration depth, ω is the angular frequency, p_1 is the pressure amplitude, δ_k is the thermal and ϵ_s is the plate heat capacity factor. The plate heat capacity factor is so small and usually neglected [4]. The total dissipated acoustic power is proportional to the surface area of the resonator. In order to reduce the acoustic power losses, one should reduce the resonator surface area. The surface area of a quarter wavelength resonator is half the surface area of a half wavelength resonator [5]. Therefore, the quarter wavelength resonator dissipates only half the power dissipated by a half wavelength resonator, thus it is favored. To further reduce the acoustic power losses of the resonator tube, its inner circumference should be reduced. To reduce the inner circumference of the resonator, its cross-sectional area should be smooth and circular. In order to simulate the resonator open end, a cone shaped buffer volume is more favourable owing to the minimal power losses [7]. A smooth gradual connection between the small diameter tube and the full diameter of the buffer volume prevents the boundary layer separation from the resonator wall. A taper with a cone half angle roughly 10-12^{o} yields a convenient pressure recovery coefficient and a rather low loss coefficient [11]. For a non-uniform cross-sectional area the resonance frequency is the normal resonance mode, which dissipates less acoustic power. Thus a non-uniform cross-sectional area reduces the length, the weight and the dissipated acoustic power of the resonator and thence enhances the thermoacoustic refrigerator performance [3]. Matching the acoustic impedances [7] at the boundary between the large and the small diameter tubes yields the resonance condition. The corresponding resonance condition, which governs the resonator lengths and diameters, is given by:

$$\cot(KL_1) = (D_1/D_2)^2\tan[K(L_r - L_1)]. \tag{4}$$

Here, $(L_r - L_1)$ is equivalent to the small diameter tube length. In light of the foregoing mentioned considerations, a Polyvinylchloride (PVC) resonator material with a non-uniform cross-sectional area and 10° half angle cone-shape buffer volume is used for the current study [11]. A schematic of the proposed resonator tube is depicted in Fig. 2(a). The design and optimization procedure for the resonator is shown in Fig. 2(b). First, the dissipated acoustic power within the resonator wall is calculated, using Eq. 4, for different values of the resonator tubes diameter ratio, D_2/D_1. The diameter ratio optimal value, corresponding to the minimum dissipated power, is determined. Then, the optimal diameters ratios are used to determine the corresponding lengths for different parts of the resonator, using Eq. 2. The cross-sectional area of the large diameter tube of the resonator equals the cross-sectional area of the stack. The lengths of the different parts of the resonator along with its cross-sectional area are used to determine the minimum acoustic power losses at the resonator walls, using Eq. 3.



(a) The proposed shape (b) The design and optimization procedure

Figure 2: The proposed resonator tube and the design and optimization procedure

The *COP* of the entire standing-wave thermoacoustic refrigerator, which is a ratio of the cooling load and the total acoustic power, is defined as:

$$COP = Q_c/W_t. \tag{5}$$

The relative *COP* of the thermoacoustic refrigerator, which is a ratio of the coefficient of performance and the Carnot coefficient of performance, COP_C, is defined as:

$$COP_R = COP/COP_C. \tag{6}$$

Discussion

The optimum design of the resonator corresponds to the optimum diameter ratio, (D_2/D_1), of the resonator tubes. The minimum total dissipations of the acoustic power within the resonator walls occur at a diameter ratio equal to 0.46 as shown in Fig. 3. The current ratio is about 17 percent -at least- less than the published values [7,8]. In Fig. 4, the resonator length, in terms of standing-wave thermoacoustic refrigerator optimum performance, shows a steady increase with the increase of the mean design temperature. According to Eq. 2, the resonator length is proportional to the square root of the mean design temperature, which interprets the plot pattern. In addition, the resonator length is inversely proportional to the resonant frequency. Since the resonant frequency is kept at 400 *Hz*, the resonator length should be prolonged so as to ensure this frequency is sufficient to drive a loudspeaker standing-wave thermoacoustic refrigerator. The corresponding total dissipations of the acoustic power in the resonator tube wall show a sharp linear increase with the increase of the mean design temperature as shown in the same figure. The total acoustic power dissipation is proportional to the resonator surface area, the viscous penetration depth and the square of the gas velocity as Eq. 3 stated. A longer resonator means a wider surface area and thus more acoustic power dissipations.

The relative *COP* in Fig. 5 increases as the mean design temperature increases attains its peak at 250 *K* then decreases in a parabolic behavior. The highest relative performance of the refrigerator, which is 47 percent, is almost same as the performance of the conventional refrigerators under the same design and operating conditions. Fig. 6 presents the relation between the relative coefficient of performance, the resonator length and the total acoustic power dissipated within the resonator wall. The relative coefficient of performance intersects with the corresponding total acoustic power dissipated at 250*K*. After that it drops sharply with the increase of the mean design temperature, the resonator length and the total acoustic power dissipations. The plot behaviour shows that it is reasonable to design the standing-wave thermoacoustic refrigerator to operate at 250 *K*. for an optimal refrigeration performance.

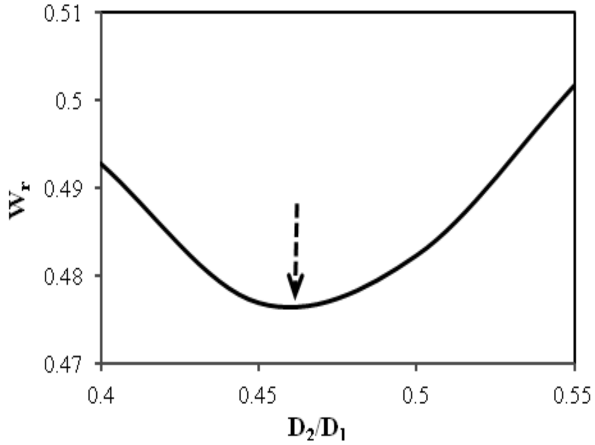


Figure 3: Acoustic power dissipations vs. the diameter ratio of the resonator.

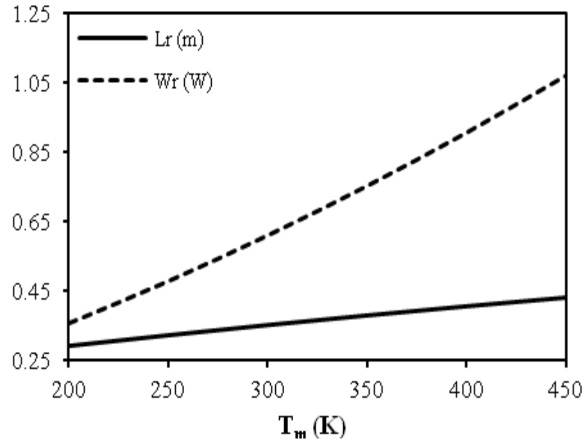


Figure 4: The resonator length and power dissipations vs. the mean design temperature.

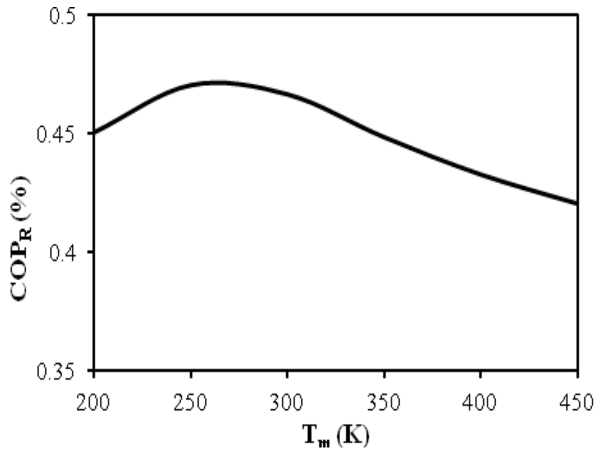


Figure 5: The standing-wave thermoacoustic refrigerator relative coefficient of performance vs. the mean design temperature.

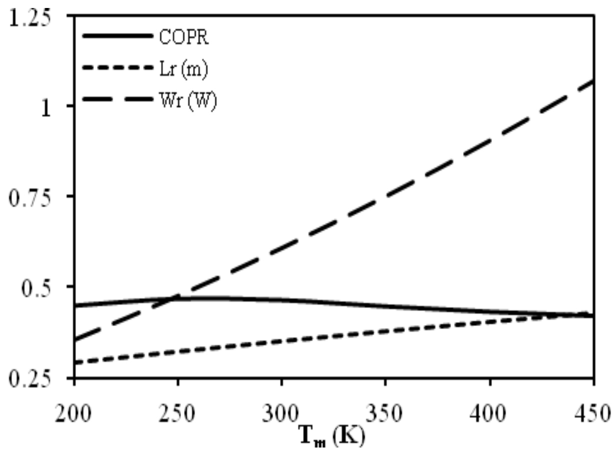


Figure 6: The relation between the relative coefficient of performance, length and the power dissipations.

Conclusion

The influence of the optimized resonator dimensions on the performance of the standing-wave thermoacoustic refrigerator was investigated. The investigation was carried out in terms of the mean design temperature. The design of the resonator depends on the resonator material, the resonance frequency, the resonator geometry and the design parameters of the stack. A higher mean design temperature needs a longer resonator tube to obtain an optimal performance at the same design and operating parameters. However, a longer resonator means a wider surface area and thus more acoustic power dissipations. The relative coefficient of performance attains its peak (47 percent) at 250 *K* then decreases with the increase of the mean design temperature, the resonator length and the total acoustic power dissipations. The dimensions of the resonator play a considerable role in the performance of the entire standing-wave thermoacoustic refrigerator.

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