

Single-objective optimization of a thermoacoustic refrigerator

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Abstract: Optimization of energy-related systems with by-products that involve environmental degradation has never been so crucial today with depleting resources and global concerns over negative impacts on our environment. This paper reports the results of an optimization scheme on the coefficient of performance (COP) of a standing wave thermoacoustic refrigerator based on genetic algorithm. The environmentally friendly refrigerator operates without any CFCs, which has been associated with the depletion of ozone, a substance that prevents uv light from reaching the earth's atmosphere. A single-objective optimization to maximize the COP of a thermoacoustic refrigerator has been completed. The variables investigated include the length of the stack, L_{sn} , center position of the stack, x_{sn} , blockage ratio, B and drive ratio, DR . The results show that a COP of up to 1.64 is achievable which provides promise for future improvements in the present systems.

Introduction

In thermoacoustic devices, an acoustic wave interacts with a porous solid material (stack) either (i) to produce acoustic power, induced by a temperature gradient imposed on the solid, or (ii) to obtain a temperature gradient along the solid, induced by an imposed acoustic wave. The thermal interaction of acoustic waves with the stack creates an additional phase shift between the oscillating particle velocity and temperature in the standing acoustic wave, when the stack is placed in a quarter wavelength resonator driven by a loudspeaker.

Thermoacoustic refrigerator is an alternative environmentally friendly system with inert gasses as the working fluid based on the acoustic-solid interactions. Although several prototypes and working systems have been out, the coefficient of performance (COP) of these thermoacoustic refrigeration systems is relatively low compared to its conventional counter parts [1].

Optimization of a thermoacoustic refrigerator is important in the design to make it competitive for consideration by the general public. The optimization of the thermoacoustic refrigerator can be divided into two main categories; geometrical and operating parameters. The former was done by Wetzel and Herman [2], Tijani et al. [3], Babaei and Kamran [4], Zink et al. [5]. Their parameters include the stack length and position, the plate spacing and the resonator length. Minner et al. [6], Emmanuel and Azrai [7], Tasnim et al. [8] were involved in the operating parameters optimization which include the frequency, working fluid, temperature, and pressure in the resonator.

Although thermoacoustic refrigeration is an attractive perspective, its efficiency, particularly for the standing wave type, is still low compared to conventional refrigeration. Thus, tools for optimizing the design variables to improve the performance of the thermoacoustic refrigerator must

be explored [9]. The preceding brief review indicates the limited comprehensive optimization schemes that have been applied to date on thermoacoustic refrigeration systems. They are limited to the local optimum/minimum. Furthermore, the use of DeltaE in past work to design thermoacoustic devices needs tremendous amount of efforts [7]. Genetic algorithm (GA) is a relatively recent optimization scheme with a strong ability in global search for the optimized solution(s) [10]. This paper reports the results of a GA optimization scheme applied to a thermoacoustic refrigerator. The system investigated follows that of Tijani et al [11] because their system was built and tested for its COP.

Theory of a thermoacoustic refrigerator

The thermoacoustic refrigerator being considered in this optimization is made up of a resonator tube, a stack, an acoustic driver attached to the end of the resonator tube to generate acoustic standing wave inside the tube [10], and the working fluid, as shown in Fig. 1. The heart of the thermoacoustic system is the stack because this is where the desired thermoacoustic effects occur. In a thermoacoustic refrigerator, the heat removed from the cold heat exchanger (HE) is Q_c and the work used to accomplish this effect is W , the acoustic power by an acoustic driver used to sustain the standing wave against the thermal and viscous dissipations. These two terms[11],

$$Q_{cn} = \frac{\delta_{kn} DR^2 \sin 2x_n}{8\gamma(1+\sigma)A} x \left(\frac{\Delta T_{mn} \tan x_n}{(\gamma-1)BL_{sn}} \frac{1+\sqrt{\sigma}+\sigma}{1+\sqrt{\sigma}} - (1 + \sqrt{\sigma} - \sqrt{\sigma}\delta_{kn}) \right) \quad (1)$$

and

$$W_n = \frac{\delta_{kn} L_{sn} DR^2}{4\gamma} (\gamma-1) B \cos^2 x_n x \left(\frac{\Delta T_{mn} \tan x_n}{BL_{sn}(\gamma-1)(1+\sqrt{\sigma})A} - 1 \right) - \frac{\delta_{kn} L_{sn} DR^2 \sqrt{\sigma} \sin^2 x_n}{4\gamma EA} \quad (2)$$

where A is defined as

$$A = 1 - \sqrt{\sigma}\delta_{kn} + \frac{1}{2}\sigma\delta_{kn}^2 \quad (3)$$

determine the COP of the stack where [2],

$$\text{COP} = \frac{Q_{cn}}{W_n} \quad (4)$$

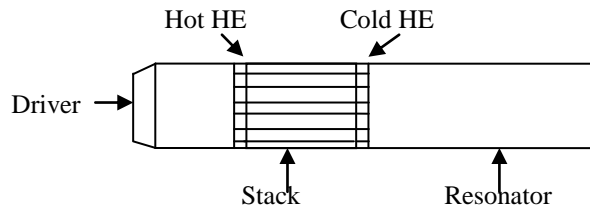


Figure 1 Schematic diagram of a simple standing wave thermoacoustic refrigerator

The terms appearing in Eqs. 1 and 2 are normalized parameters described in Table 1 with the related parameters values included in Table 2. Maximization of Eq.1 and minimization of Eq.2 should maximize the COP. As explained earlier, past optimization results are dependent on the variation of the selected parameter in Eqs.1 and 2 at discrete values while other parameters are held constant. If the stack center position, x_{sn} , are to be optimized, the other parameters take on predetermined values while x_{sn} itself are changed and the outcome of the COP analyzed.

Table 1 Dimensionless parameters in the thermoacoustic refrigerator system

Operation parameters Drive ratio: $D = p_o/p_m$ Normalized cooling power: $Q_{cn} = Q_c/p_m aA$ Normalized acoustic power: $Wn = W/p_m aA$ Normalized temperature difference: $\Delta T_{mn} = \Delta T_m/T_m$
Gas parameters Prandtl number: σ Normalized thermal penetration depth: $\delta_{kn} = \delta_k/y_o$
Stack geometry parameters Normalized stack length: $L_{sn} = kL_s$ Normalized stack position: $x_n = kx$ Blockage ratio or porosity: $B = y_o/(y_o + l)$

Table 2 Operating parameters and properties

Operation parameters	Gas parameters
$p_m = 10$ bar	$a = 935$ m/s
$T_m = 250$ K	$\sigma = 0.68$
$\Delta T_{mn} = 0.3$	$\gamma = 1.67$
$DR = 0.02$	$B = 0.75$
$f = 400$ Hz	$k = 2.68$ m ⁻¹
	$\delta_{kn} = 0.66$

Generally, a parameter to be optimized had to be varied singly while others are unchanged. In the current study, four parameters appearing in Eqs.1 and 2 are simultaneously optimized while trying to optimize the COP using Eq.4. These parameters are the stack length, L_{sn} , the stack center position, x_{sn} , the blockage ratio, B , and the drive ratio, DR . These have been identified as the parameters that can be controlled by the designer to achieve the desired COP as high as possible. The blockage ratio, B , is defined by

$$B = \frac{y_o}{y_o + l} \quad (5)$$

where y_o is the half spacing of the stack while the drive ratio, DR , is defined by

$$DR = \frac{p_o}{p_m} \quad (6)$$

where the p_o is the dynamic pressure and p_m is mean pressure

Single-objective Genetic Algorithm (GA)

In this study, the GA optimization scheme is applied to a system designed by Tijani et al [11]. The single objective function chosen is the COP to optimize the performance of the thermoacoustic refrigerator. GA optimization scheme is applied in the MATLAB toolbox. By optimizing the geometry and operating parameters in one work, the performance can be enhanced. Fundamental ideas of genetic are borrowed and used artificially to construct search algorithms that are robust and require minimal problem information [12]. The main idea of the genetic algorithm is to minimize/maximize $f(X)$ subject to equality constrains $g_j (x_1, x_2, x_3, \dots, x_n) = 0$ and inequality constraints $h_j (x_1, x_2, x_3, \dots, x_n) \geq 0$. In this study, the objective is to optimize the COP of the thermoacoustic refrigerator. Then the complete mathematical formulation of the optimization problem may be written in the following form:

$$\text{maximize } f(X) = COP(X) \quad (7)$$

subject to the imposed constrains for (X) given by

$$\begin{aligned} 0 &\leq L_{sn} \leq 1 \\ 0.06 &\leq x_{sn} \leq 0.42 \\ 0.67 &\leq B \leq 0.8 \\ 0.015 &\leq DR \leq 0.03 \end{aligned}$$

Parameter L_{sn} , x_{sn} , B , and DR are the decision variables allowed to vary in the bound. The decision variables are represented in binary strings in order to find the optimum solution which satisfy the constraints and maximise $f(X)$. Fig. 2 shows the flowchart of the process involved in a GA application. It begins with a search among the random population of solution sets of solutions. Once a string is created by genetic operators, it is necessary to evaluate the solution, particularly in the context of the underlying objective and constraint. The genetic operators used in the thermoacoustic refrigerator optimization are listed in Table 3.

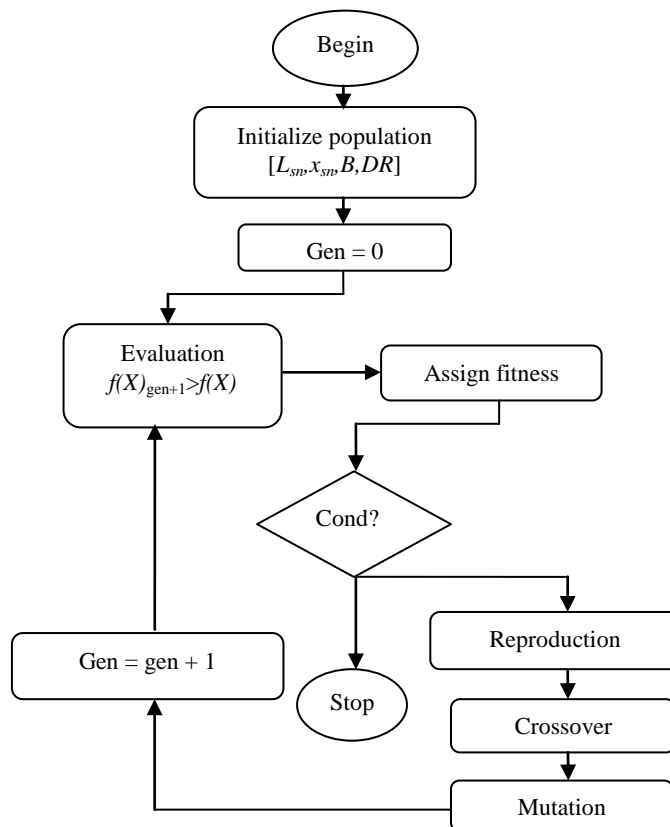


Figure 2 Flowchart of the GA optimization for a thermoacoustic refrigerator

Table 3: Genetic Algorithm operators

Genetic operator	Value
Population size	100
Fitness scaling	Rank
Selection function	Stochastic uniform
Crossover function	Arithmetic
Crossover fraction	0.8

Results and Discussions

The working gas of the thermoacoustic refrigerator investigated here is pure helium and helium-xenon. Fig. 3 shows the fitness value which is the COP and the generations for the single objective optimization.

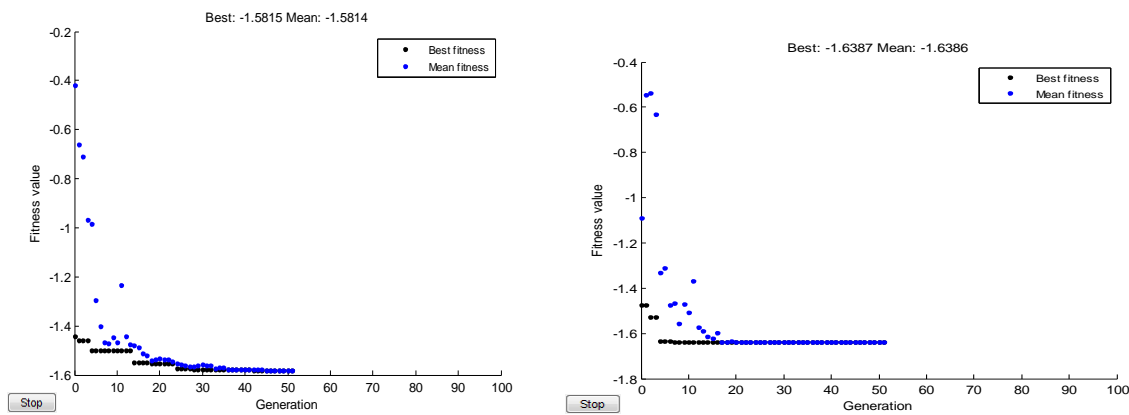


Figure 3 The fitness function for every generation a) Helium b) He-xe

The highest COP accomplished is by using helium, 1.58. This highest COP achieved is from using acoustic work, W , of 4.33 Watt and providing a cooling power, Q_c , of 6.84 Watt. The combination of the optimized variables are $x_s = 6.7$ cm, $L_s = 6.7$ cm, $B = 0.8$, $D = 0.026$. For a mixture of helium and xenon (He-xe), the single objective genetic algorithm optimization with COP as the objective function results in the best COP of 1.64. To achieve this highest COP, the acoustic power needed is 1.18 Watt which provides a cooling power, $Q_c = 1.93$ Watt. This comes from the combination of optimized variables; $x_s = 3.57$ cm, $L_s = 3$ cm, $B = 0.8$, $D = 0.026$. Compared with Tijani et al [3], the COP has been enhanced from 1.3 to 1.64, an increase of 26%.

The maximum cooling power extracted by using helium gas is 6.84 W and by using He-xe, 1.93 W. This phenomenon was also observed by Tasnim et al [8] and Tijani [10] in their works. The results were obtained because the cooling power, Q_c is inversely proportional to the product $p_m a$, where a is the adiabatic speed of sound. The product of $p_m a$ increases when the gas mixture is used thereby reducing the cooling power of the thermoacoustic device. Although helium provides a higher cooling power compared to He-xe, the COP of the thermoacoustic refrigerator is much higher if He-xe is the working fluid. The COP of He-xe is higher because when the Prandtl number is low, the viscous losses are kept at a minimum.

By changing the working fluid from pure to a binary gas mixture, both of which is still from the same group, the transport coefficient which is the Prandtl number changed. The Prandtl number depends on the dynamic viscosity, μ , thermal conductivity, K , isobaric specific heat, c_p , and density, ρ . The viscosity gives the negative effect on the performance of the thermoacoustic refrigerator, reduction of the viscous effects means an increase in the efficiency. The transport coefficient such as the diffusivity, viscosity and thermal conductivity affects the transport of mass, momentum, and

energy by means of molecular motion and molecular collision. The low Prandtl number increases the transport coefficient of the gas and improves the performance of the thermoacoustic refrigerator. Unfortunately, by using He-xe as the working fluid, the cooling power that can be extracted from the system is lower because the addition of xenon increases the density of the whole working fluid which subsequently decreases the cooling power.

Conclusion

Optimization of four relevant parameters in the design of the stack in a thermoacoustic refrigerator has been done using single-objective genetic algorithm (GA). The procedure produced the optimum coefficient of performance (COP) with combination of four optimized parameters. The improved COP of the stack achievable are 1.58 and 1.64 for the working fluid of helium and helium-xenon respectively, compared to that of Tijani et al [3] which is 1.3. Results of the optimization of the stack unit, reduces the losses of the resonator since this provides a more compact stack but still produces a better performance. It shows that GA used in this study has the potential of providing options in the design of the optimized operations of a thermoacoustic refrigerator, particularly in the stack component which is central to the whole system.

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