

# Using of EKF for Speed Sensorless SVM-DTC in Induction Motor at Low Speed

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**Abstract**—Extended Kalman Filter (EKF) is a special type of observer, which filters out measurement and system noises while at the same time estimate the state variables. With the inevitable increased in the temperature due to the continuous operation, the performance of the induction motor drive system, especially at low speed, is more affected by the stator resistance than the rotor resistance. So, at low speed operation, speed estimation using EKF based on rotor flux is more accurate than the stator flux based EKF. In this work, a speed estimation using an EKF based on rotor flux for Space Vector Modulation-Direct Torque Control (SVM-DTC) of a three-phase induction motor at low speed is presented. Simulation results showed excellence performance and the effectiveness of the presented system.

**Keywords**—EKF; low speed; speed estimation; SVM-DTC

## I. INTRODUCTION

Switching Table-Direct Torque Control (ST-DTC) is one of the most popular high performance induction motor drive control techniques. Using this method, the stator flux and torque are estimated and compared with the references. The decoupling of the torque and flux components is accomplished by using two hysteresis controllers [1]. Since ST-DTC uses hysteresis controllers for its implementation, high sampling frequency is mandatory to avoid high torque ripple [2]. In this paper a Direct Torque Control (DTC) of induction motor with Space Vector Modulation (SVM) technique is presented. The switching frequency remained constant by using the space vector modulation based inverter which is used to synthesize the reference voltages, which are generated by the PI controllers. The DTC-SVM of induction motor has gaining popularity in recent years [3]-[7].

In some applications, the speed of the induction motor needs to be controlled and hence need to be available by the control system. On the other hand, in other applications, even though the speed is not the controlled variable, it is still needed by the motor control algorithm in order to estimate the flux and torque. In either case, the speed needs to be measured using mechanical sensors or alternatively estimated using terminal variables. The use of mechanical sensor has some disadvantages, such as increasing cost, size, hardware complexity and decreasing the reliability and robustness. Moreover, in some applications it is almost impossible or inappropriate to install the mechanical sensors. Under these

constraints and disadvantages of mechanical sensors, estimation of rotor speed using terminal variables is the practical solution. In the recent years, the estimation of induction motor parameters has becoming one of the popular topics among researchers. The Extended Kalman Filter (EKF) is one of the most popular methods for estimation of parameters such as speed, torque and resistance in induction motor drive system. The method uses the non-linear model of induction motor and provides optimal filtering to the measurement and system noises [8]-[13].

By using EKF instead of voltage model in estimating the stator flux for the DTC drives, the well-known pure integration problem at low rotor speed [14] is solved. At very low speed operation, the effect in the variation of  $r_s$  (due to the temperature increased) to the accuracy of the speed estimation is more dominant when compared to the variation of  $r_r$  [15]. This is mainly due to the large error in the stator flux estimation that is caused by the EKF, which is based on stator flux. The large error in the stator flux consequently resulted in a large error in the estimated speed. The poor speed estimation problem at low speed can be improved if, instead of stator flux, the EKF for the speed estimation is based on rotor flux. For this reason, the SVM-DTC of induction motor introduced in this paper uses EKF based on rotor flux to estimate the rotor flux and rotor speed. The estimated value of the rotor flux is then used to calculate the stator flux and electromagnetic torque of induction motor.

This paper is organized as follows. In section II, the d-q modeling of the three-phase induction motor in stationary reference frame is presented. The SVM-DTC method is presented in section III. In section IV, the EKF algorithm based on rotor and stator flux for estimation of rotor speed is described. The proposed method for speed sensorless SVM-DTC for induction motor at low speed is also presented in this section. In section V, the effectiveness of the proposed method is verified by simulation using Matlab/Simulink. Finally, conclusion is presented in section VI.

## II. MODEL OF THREE-PHASE MOTOR

The equations of a three-phase motor in the stator reference frame are defined as follows:

$$\mathbf{v}_{sd}^s = r_s \mathbf{i}_{sd}^s + p \boldsymbol{\phi}_{sd}^s \quad (1)$$

$$v_{sq}^s = r_s i_{sq}^s + p \varphi_{sq}^s \quad (2)$$

$$0 = r_r i_{rd}^s + p \varphi_{rd}^s + \omega_r \varphi_{rd}^s \quad (3)$$

$$0 = r_r i_{rq}^s + p \varphi_{rq}^s - \omega_r \varphi_{rd}^s \quad (4)$$

$$\varphi_{sd}^s = L_s i_{sd}^s + L_m i_{rd}^s \quad (5)$$

$$\varphi_{sq}^s = L_s i_{sq}^s + L_m i_{rq}^s \quad (6)$$

$$\varphi_{rd}^s = L_m i_{sd}^s + L_r i_{rd}^s \quad (7)$$

$$\varphi_{rq}^s = L_m i_{sq}^s + L_r i_{rq}^s \quad (8)$$

$$T_e = \frac{3}{2} p_p (\varphi_{rq}^s i_{rd}^s - \varphi_{rd}^s i_{rq}^s) \quad (9)$$

$$p_p (T_e - T_l) = J p \omega_r + F \omega_r \quad (10)$$

Where,  $\varphi_{sd}^s$ ,  $\varphi_{sq}^s$ ,  $\varphi_{rd}^s$ ,  $\varphi_{rq}^s$ ,  $i_{sd}^s$ ,  $i_{sq}^s$ ,  $i_{rd}^s$ ,  $i_{rq}^s$ ,  $v_{sd}^s$  and  $v_{sq}^s$  are the  $dq$  fluxes, currents, and voltages of the stator and rotor in the stationary reference frame (superscript  $s$ ).  $r_s$  and  $r_r$  indicate the stationary and rotor resistances.  $L_s$ ,  $L_r$  and  $L_m$  are the stator, and the rotor self and mutual inductances.  $\omega_r$  is the speed of motor.  $p$  and  $p_p$  are differential operator and pole pairs respectively.  $T_e$ ,  $T_l$ ,  $J$  and  $F$  are electromagnetic torque, load torque, inertia and viscous friction coefficient.

### III. SVM-DTC METHOD FOR INDUCTION MOTOR

The SVM-DTC of induction motor scheme is shown in Fig. 1. In this method, the reference voltage for space vector modulator is generated by two PI controllers: stator flux PI controller and the electromagnetic torque PI controller. Moreover, the stator flux space vector and the electromagnetic torque are calculated as (11)-(13):

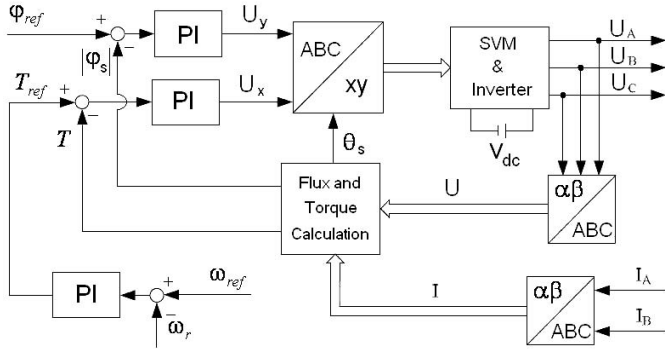


Fig. 1. Scheme of SVM-DTC

$$\varphi_{s\alpha} = \int (v_{s\alpha} - r_s i_{s\alpha}) dt \quad (11)$$

$$\varphi_{s\beta} = \int (v_{s\beta} - r_s i_{s\beta}) dt \quad (12)$$

$$|\varphi_s| = \sqrt{\varphi_{s\alpha}^2 + \varphi_{s\beta}^2} \quad (13)$$

$$\theta_s = \tan^{-1} \left( \frac{\varphi_{s\alpha}}{\varphi_{s\beta}} \right) \quad (14)$$

$$T = \frac{3}{2} p_p (i_{s\beta} \varphi_{s\alpha} - i_{s\alpha} \varphi_{s\beta}) \quad (15)$$

In Fig. 1, the 3-2 phase transformation to the stator flux reference frame are given by (16) and (17)

$$\begin{bmatrix} f_{s\alpha} \\ f_{s\beta} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} f_A \\ f_B \\ f_C \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} f_A \\ f_B \end{bmatrix} = \begin{bmatrix} -\sin \theta_s & \cos \theta_s \\ \frac{1}{2}(-\cos \theta_s + \sqrt{3} \sin \theta_s) & \frac{1}{2}(\sin \theta_s + \sqrt{3} \cos \theta_s) \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} \quad (17)$$

### IV. USING OF EKF IN SVM-DTC METHOD

A dynamic model for a three-phase induction motor can be shown as following equations:

$$\dot{x} = Ax + Bu + w(t) \quad (18)$$

$$y = Cx + v(t) \quad (19)$$

The covariance matrices of noises are defined as follows:

$$Q = \text{cov}(w) = E\{ww^t\} \quad (20)$$

$$R = \text{cov}(v) = E\{vv^t\} \quad (21)$$

The Kalman filter algorithm can be formulated and is given by equations (22)-(28):

*Prediction of State:*

$$x_{n+1|n} = \Phi(n+1, n, x_{n|n-1}, u_n) \quad (22)$$

where:

$$\Phi(n+1, n, x_{n|n-1}, u_n) = A_n(x_{n|n-1})x_{n|n-1} + B_n(x_{n|n-1})u_n \quad (23)$$

*Estimation of Error Covariance Matrix:*

$$P_{n+1|n} = \frac{d\Phi}{dx} \Big|_{x=x_{n|n-1}} P_{n|n-1} \frac{d\Phi^T}{dx} \Big|_{x=x_{n|n-1}} + Q \quad (24)$$

*Computation of Kalman Filter Gain:*

$$K_n = P_{n|n-1} \frac{\partial H^T}{\partial x} \Big|_{x=x_{n|n-1}} \quad (25)$$

$$\left( \frac{\partial H}{\partial x} \Big|_{x=x_{n|n-1}} P_{n|n-1} \frac{\partial H^T}{\partial x} \Big|_{x=x_{n|n-1}} + R \right)^{-1}$$

where:

$$H(x_{n|n-1}, n) = C_n(x_{n|n-1})x_{n|n-1} \quad (26)$$

*State Estimation:*

$$x_{n|n} = x_{n|n-1} + K_n(y_n - H(x_{n|n-1}, n)) \quad (27)$$



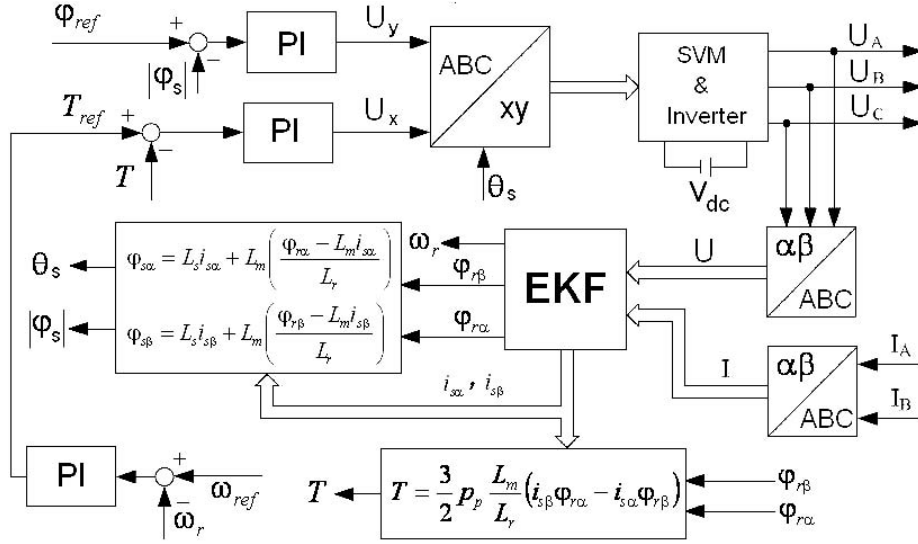


Fig. 3. Proposed scheme of speed sensorless SVM-DTC by using EKF based on rotor flux at low speed

$$A_{2n} = \begin{bmatrix} 1 - \frac{k_3}{k_2} dt & 0 & \frac{L_m r_r}{k_2 L_r} dt & \frac{L_m p_p \omega_r^{(n)}}{k_2 L_r} dt & 0 \\ 0 & 1 - \frac{k_3}{k_2} dt & -\frac{L_m p_p \omega_r^{(n)}}{k_2 L_r} dt & \frac{L_m r_r}{k_2 L_r} dt & 0 \\ \frac{L_m r_r}{L_r} dt & 0 & 1 - \frac{r_r}{L_r} dt & -p_p \omega_r^{(n)} dt & 0 \\ 0 & \frac{L_m r_r}{L_r} dt & p_p \omega_r^{(n)} dt & 1 - \frac{r_r}{L_r} dt & 0 \\ -\frac{3 p_p L_m}{2 J L_r} \omega_r^{(n)} dt & \frac{3 p_p L_m}{2 J L_r} \omega_r^{(n)} dt & 0 & 0 & 1 \end{bmatrix} \quad (39)$$

$$B_{2n} = \begin{bmatrix} \frac{1}{k_2} dt & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{k_2} dt & 0 & 0 & 0 \end{bmatrix}^T \quad (40)$$

$$C_{2n} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (41)$$

where:

$$k_3 = r_s + \frac{r_r L_m^2}{L_r^2} \quad (42)$$

By using of these equations and EKF algorithm, the rotor flux and rotor speed can be estimated.  $\varphi_{s\alpha}$  and  $\varphi_{s\beta}$  are calculated as follows:

$$\varphi_{s\alpha} = L_s i_{s\alpha} + L_m \left( \frac{\varphi_{r\alpha} - L_m i_{s\alpha}}{L_r} \right) \quad (43)$$

$$\varphi_{s\beta} = L_s i_{s\beta} + L_m \left( \frac{\varphi_{r\beta} - L_m i_{s\beta}}{L_r} \right) \quad (44)$$

From equations (43) and (44), torque can be expressed as follows:

$$T = \frac{3}{2} p_p \frac{L_m}{L_r} (i_{s\beta} \varphi_{r\alpha} - i_{s\alpha} \varphi_{r\beta}) \quad (45)$$

Finally, Fig. 3 shows the speed sensorless SVM-DTC induction motor scheme with EKF based on rotor flux.

## V. SIMULATION RESULTS

To study on the effectiveness of proposed system, speed sensorless SVM-DTC with EKF speed estimator based on rotor flux is simulated using MATLAB/Simulink. The system is simulated with a closed-loop speed control system. In the simulation, the stator voltage used by the EKF is obtained from the VSI of the SVM-DTC system. Motor data are as given in appendix.

From  $t = 0$  s to  $t = 2$  s the reference speed is changed from 0 RPM to 150 RPM and from  $t = 2$  s to  $t = 5$  s the reference speed is kept at 150 RPM. Moreover, from  $t = 0$  s to  $t = 3.5$  s the stator resistance is set to  $r_s = 0.014855 \Omega$  (nominal value) and from  $t = 3.5$  s to  $t = 5$  s, it is changed to  $(2 \times 0.014855) \Omega$ . In the simulated system the reference stator flux is set to 1.2 Wb.

Fig. 4 shows the simulation results of the proposed speed sensorless SVM-DTC with EKF speed estimation based on rotor flux. Fig. 4 (a) and (b) show the reference speed and stator resistance respectively. The real and estimated rotor speed is shown in Fig. 4 (c). Fig. 4 (d) shows the rotor speed error. The results indicated that the system is able to operate at low speed satisfactorily. Fig. 4 (e) and 4 (f) show the estimated stator flux magnitude and trajectory respectively. The oscillations of estimated stator flux are about 0.03 Wb around the average amount of 1.2 Wb. The stator flux position and the stator currents are shown in Fig. 4 (g) and Fig. 4 (h) respectively.

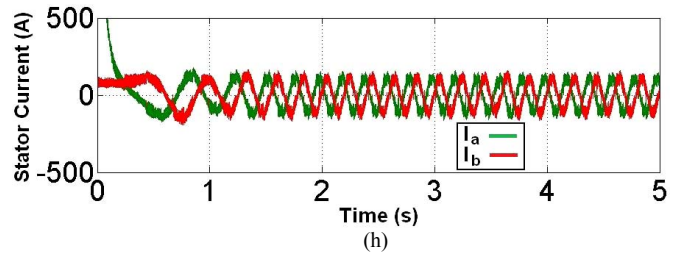
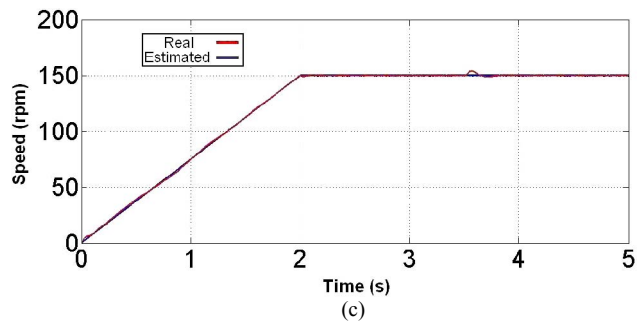
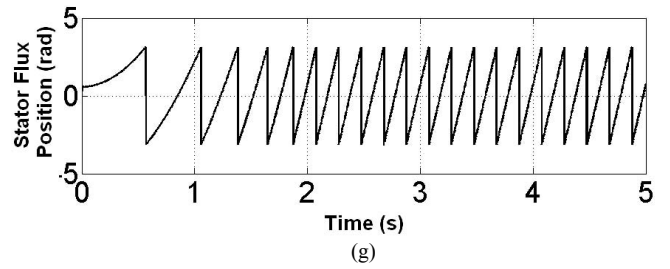
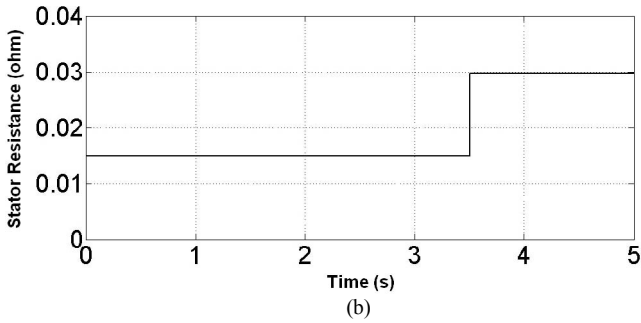
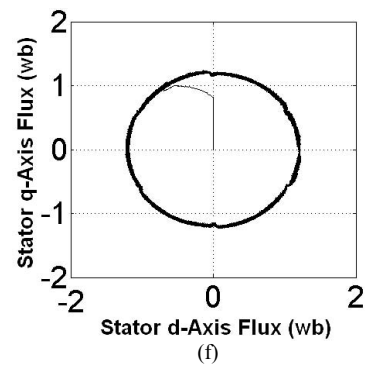
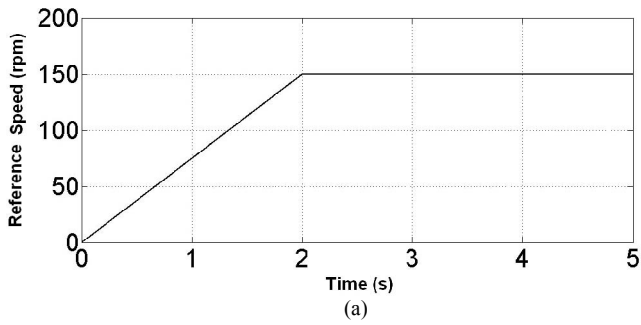
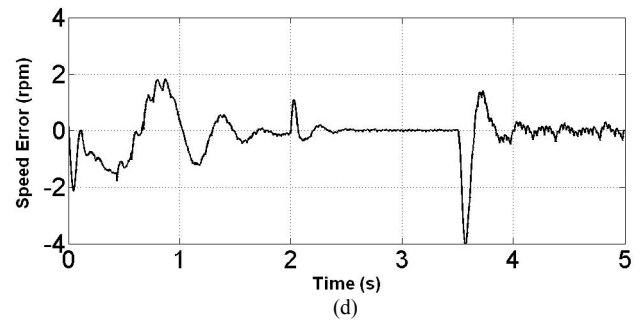


Fig. 4. Simulation results of proposed speed sensorless SVM-DTC in induction motor by using EKF at low speed



For the purpose of comparison, the SVM-DTC with EKF speed estimation based on stator flux is also performed and shown in Fig. 5 (system of Fig. 2). As can be seen that the oscillations in the rotor speed about 1.7 RPM is detected after a step change in the stator resistance is introduced. Under the same condition, oscillation in speed for the rotor flux based EKF (shown in Fig. 4(c)) is insignificant.

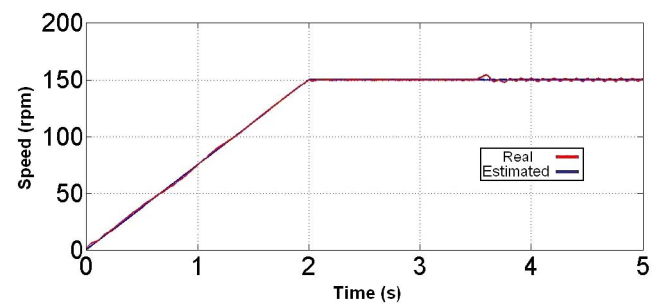
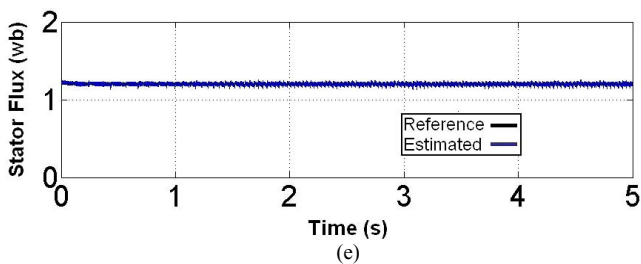


Fig. 5. Simulation result of speed sensorless SVM-DTC in induction motor based on stator flux by using EKF at low speed



## VI. CONCLUSION

This work presents the speed sensorless SVM-DTC of induction motor at low speed operation. The speed estimation is performed using EKF. By considering the fact that at very low speed the variations of stator resistance are more pronounced than the variation in rotor resistance as far as the stator flux estimation is concerned, in this paper a speed sensorless SVM-DTC method by using EKF based on rotor flux is proposed. In this work, by using of estimated rotor flux space vector and stator currents from the EKF, stator flux space vector and electromagnetic torque are calculated. Simulation results showed that the performance of the proposed method is excellence, especially at very low speed operations.

## APPENDIX

Parameters of the simulated induction motor:

Voltage: 460V,  $f = 60\text{Hz}$ , no. of poles=4, Power=149200W,  
 $r_s = 0.014855\Omega$ ,  $r_r = 0.009295\Omega$ ,  $L_{lr} = L_{ls} = 0.0003027\text{H}$ ,  
 $L_m = 0.01046\text{H}$ ,  $J = 3.1\text{kg.m}^2$ ,  $F = 0.08\text{N.m.s}$

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