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Review of Discrete-Continuous Models in Energy and Transportation

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Abstract

Demand modelling of energy and transport incorporate interconnected decision variables, which are either discrete or continuous. After forty years, from McFadden multinomial logit model and incorporating it to Heckman endogenous simultaneous equation, there is an opportunity to determine the parameters of interconnected decisions simultaneously. These models are bounded by the utility theory. Now these models have matured, and their empirical aspects revealed. In this study, the pioneering works on discrete-continuous models that have been developed in the field of energy and transportation have been reviewed with a view of proposing new development to these models. These models theoretically are based on two approaches; McFadden indirect utility function and Gorman polar functional form of utility structures. In both approaches, the models are estimated by maximum likelihood procedures.

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1. Introduction

Most of the decisions in real world are interconnected and require to be taken sequentially or simultaneously. A sustain decision-making system will concern to the other agents demands. Sustainability considerations have forced the modeling system to search for multi-dimensional structures which have the ability to incorporate more than one dependent variable.

On consumer demand behavior, one may encounter a qualitative or discrete variable in addition to traditional continuous analysis of demand; and this is known as a discrete-continuous framework. For instance, a consumer decides on which particular brand of car to buy as well as how long to drive or use it; or a shipper decides on which mode of transport to be used while also concerns about the shipment size. In terms of housing, a consumer decides whether to buy or rent a house coincides with the size of house to live in; or a consumer decides whether to buy an electric or gas heater as well as the amount of electricity usage in a month. These examples require modelers to be concern with more than one subject at a time. These models are first developed in the field of energy as well as transport. In all cases the choice of discrete variable depends partly on the continuous variable and vice-versa. Therefore the two choices should be mutually and consistently modeled. Discrete-continuous model has been applied in energy economics and transportation fields.

In order to model discrete-continuous dependent variable systems two approaches have been proposed in the literature. The first is based on the indirect utility function which is proposed by Dubin and McFadden [1], Hanemann [2], Heckman [3]. The second one is the utility-based (Gorman structured utility function) and multiple discrete-continuous extreme value (MDCEV) model which is proposed by Bhat and Sen [4]. Currently, development in discrete model estimation provided an opportunity to simultaneously estimate a system of equations that include more than one discrete and continuous dependent variables. Identification of new developments in this type of models is the main goal of this study. For this review only pioneering works have been selected and their modeling procedure has been explained. Subsequently future model development works are proposed.

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2. General framework of discrete-continuous models: Indirect utility function and Gorman polar forms

Based on McFadden [5] a consumer indirect utility function gives the consumer maximal utility with respect to product price level and consumer income. It is a consumer preference based on market condition and income level. Utility maximization problem (UMP) based on price and income level is:

$$V(p,I) \equiv max_X \left\{ U(X) \middle| I - \sum_i p_i \cdot x_i \geq 0 \text{ and } X \geq 0 \right\}$$

where; V is observed part of utility function for product x; p is price and I is income.

From the above equation the maximum attainable level of utility as a function of price and income is presented. If there is a unique solution to the consumer's UMP, thereafter Marshallian demand function can simply be substituted into the utility function as:

$$V(p_1, p_2, \dots, p_n, I) \equiv U\big(x_1^*(p_1, p_2, \dots, p_n, I), \dots, x_n^*(p_1, p_2, \dots, p_n, I)\big)$$

where; $x^*=x(p, I)$ and solves $maxU(x)$ subject to budget constraint $(p \cdot x \leq I)$. Here indirect utility value V is determined based on the value of U and any rescaling of U like density function of U is applicable and analogous to V. That is indirect because consumers usually care about what they consume rather than the prices, or "the demand for durables arises from the flow of services provided by durables ownership, the utility associated with a consumer durable is then best characterized as indirect" [1]. The indirect utility function unlike random utility function embodies an optimization process where consumers through a bundle of products try to maximize their utilities. This presumption let the consumers to choose a bundle which is affordable and maximizes their utilities. Definition of indirect utility function and how to optimize the decision will impose some properties which any indirect utility will posses. Some of these properties are:

1. Homogeneous of degree zero in price and income: If prices and income multiplies by the same positive factor, the budget constraint does not change, and therefore the choice and the utility level will not change; $V(t \cdot p, t \cdot I) = V(p, I)$ for $t>0$.
2. Non-increasing in prices and non-decreasing in income.
3. Quasi-convex in prices and income: If we draw the level curves of $v(p,m)$ in the space of (p_1,p_2) all maximum utility curves will be convex towards the origin.
4. Continuous in prices and income.
5. Roy's identity if $V(p, I)$ is differentiable [5]:

$$x_i^*(p_1, \dots, p_n, I) = -\frac{\partial V / \partial p_i}{\partial V / \partial I}, \quad i = 1, 2, \dots, n$$

Roy's identity is the main and useful property of indirect utility function in the case of demand analysis.

The foregoing notations are conditional to the chosen commodity. Here, the discrete choice of which commodity to be selected in a dual-product situation can be represented by a set of binary values $\Delta = (\delta_1, \delta_2, \dots, \delta_n)$ where; $\delta_j=1$ if $x_j>0$ and $\delta_j=0$ if $x_j=0$. The discrete choice can be represented in terms of the conditional indirect utility function as:

$$\delta_j(p, b, y, s, \varepsilon) = \begin{cases} 1 \text{ if } v_j\big(p_j, b_j, y, s, \varepsilon\big) \geq v_i(p_i, b_i, y, s, \varepsilon) & \text{for all } i \\ 0 & \text{otherwise} \end{cases}$$

where; b is the attributes of x's commodity; y is income level; s is decision-maker characteristics like as age, education, …;ε is random component of utility with some joint density function.

Here, for an econometrician or observer, the discrete choice indices δ's are random variables with expected value of $E(\delta) \equiv \pi_j$ given by:

$$\pi_j(p, b, y, s, \varepsilon) = Prob\big\{v_j\big(p_j, b_j, y, s, \varepsilon\big) \geq v_i(p_i, b_i, y, s, \varepsilon), for\ all\ i\big\} = \int_{-\infty}^{+\infty} F_v^j(u, \dots, u)du,$$

where; F_v^j is the derivative of $F_v(.)$ with respect to its jth argument.

The unconditional function and conditional function is related to each other based on the value of δ which is 0 or 1(selected x and its correspondent indirect utility is showed with – mark). Then, this relation will be:

$$x_j(p, b, y, s, \varepsilon) = \delta_j(p, b, y, s, \varepsilon) \cdot \bar{x}_j\big(p_j, b_j, y, s, \varepsilon\big)$$

which $j = 1, \dots, N$

$$v(p, b, y, s, \varepsilon) = max[\bar{v}_1(p_1, b_1, y, s, \varepsilon), \dots, \bar{v}_N(p_N, b_N, y, s, \varepsilon)]$$

Econometricians have used these relations to draw distributions of x_j and v. To do this, they introduced the sets $A_j \equiv [\varepsilon |\ v_j >= v_i]$. From f_ε one can construct $f_{\varepsilon|\varepsilon\ \epsilon\ Aj}$ as a conditional joint density of $\varepsilon_1, \dots, \varepsilon_m$ given that $\varepsilon\ \epsilon\ A_j$ then the probability density of x_j, $f_{\varepsilon|\varepsilon\ \epsilon\ Aj}(x) = prob[x_j=x \mid \varepsilon\ \epsilon\ A_j]$ is obtainable by analogy from $f_{\varepsilon|\varepsilon\ \epsilon\ Aj}$ by a change of variable based on Roy's identification. Thus, the probability density of x_j, $f_{xj}(x)=prob(x_j=x)$ takes the following form:

$$f_{x_j}(x) = f(x) = \begin{cases} 1 - \pi_j, & x = 0 \\ \pi_j f_{x_j|\varepsilon \in A_j}(x), & x > 0 \end{cases}$$

Then, conditional indirect utility function and the joint density of random components are milestones of the random utility discrete-continuous models. If one specifies v_j and f_ε then, densities of f_v and $f_{\varepsilon|\varepsilon\ \epsilon\ Aj}$ can be constructed which are used to form the discrete choice probabilities and the conditional and unconditional densities of x_j's [2].

A close consideration to the unconditional and conditional function for the assumed binary model will let to value of x in the following form based on Roy's identity to be expressed:

$$x = \begin{cases} -\dfrac{\partial \bar{v}_1(p_1, b_1, y, s, \varepsilon)/\partial p_1}{\partial \bar{v}_1(p_1, b_1, y, s, \varepsilon)/\partial p_1} & \text{if } \bar{v}_1(p_1, b_1, y, s, \varepsilon) \geq \bar{v}_2(p_2, b_2, y, s, \varepsilon) \\ -\dfrac{\partial \bar{v}_1(p_1, b_1, y, s, \varepsilon)/\partial p_1}{\partial \bar{v}_1(p_1, b_1, y, s, \varepsilon)/\partial p_1} & \text{otherwise} \end{cases}$$

The above form is a linear single equation switching regression model. This mutually and consistent mix discrete-continuous model is regarded as switching regression model and presented by Amemiya [6], Lee and Trost [7], Heckman [8] where this switching regression model can be written as:

$$x = \begin{cases} W_1\beta_1 + v_1 & \text{if } Z\gamma + \eta \geq 0 \\ W_2\beta_2 + v_2 & \text{otherwise} \end{cases}$$

where; Z represents the exogenous variables; W is exogenous variables and a transformation of Z; β, γ are coefficients to be estimated, β directly related to γ; η is random component; v is a random components derived from η.

Then, random utility discrete-continuous demand model can be estimated by any of the statistical instrument created for switching regression estimation [2]. Based on Heckman [8], if the decisions for discrete and continuous are not independent, then the model will suffer from selection bias. For many consumer choice cases, the error term of the choice equation is correlated with the error term of the continuous equation. Here, a selection correction estimation method is required. The correlation and dependency of error terms in both discrete and continuous equations are the heart of switching regression model.

Whenever the Gorman polar functional form is given by;

$$U_j = U_j(p, y - c_j) = \frac{y - c_j + a(p)(e_j + m_j)}{b(p)}$$

where; *a(.)* and *b(.)* are concave and non-decreasing in p and homogenous in zero degree and m_j is independent of price and income which is alternative-specific. Then, U_j is non-decreasing and convex in prices. Where preferences are associated with alternative-specific m_j as observable part of utility function, e_j is unobservable part and stochastic. Here, the choice probability of jth alternative is the difference between observable and unobservable terms:

$$P_j(B) = P\left[e_j + m_j - \frac{c_j}{a(p)} - \max\left(e_k + m_k - \frac{c_k}{a(p)}\right)\right].$$

With relaxation of extreme value we can get:

$$P_j(B) = \frac{\exp\left(m_j - \frac{c_j}{a(p)}\right)}{\sum_{k \in B} \exp\left(m_k - \frac{c_k}{a(p)}\right)}.$$

Roy's identity in the following continuous part of the model will be attained;

$$x_{rj} = \left(\frac{a(p)b_r(p)}{b(p)} - a_r(p)\right)m_j$$
$$-(y - c_j)\frac{b_r(p)}{b(p)} + \left(\frac{a(p)b_r(p)}{b(p)} - a_r(p)\right)e_j.$$

3. Discrete-continuous model in energy studies

"If, as the theory would suggest, the demand for durables and their use are related decisions by the consumer, specifications which ignore this fact will lead to biased and inconsistent estimates of price and income elasticities" [1]. This notation was an important base to test the exogeneity of durable appliance dummy variables included in demand equation for energy. They consistently estimated appliance choice model (discrete) with demand for electricity (continuous) simultaneously, their model was based on the presented model of Heckman [3], which was the first study of dummy variables in simultaneous equations. Dubin and McFadden [1] developed a discrete-continuous model in which the electricity consumption is determined with Roy's identity (the first model) and the probability of indirect utility function included observed and unobserved components of durable portfolio, unobserved component of decision-maker, income and cost of purchasing durables, and prices of electricity and alternative energy source represented the choice selection of discrete variable. Then, (in the second model) they use of a parametric specification of UEC (Unit Electricity Consumption), which means they define Roy's identity in a way that it is partially differentiable and its solution gives the conditional utility function. And, then, they have defined a discrete choice probability by use of resulted conditional indirect utility function. Here we will discuss their second model.

They assumed the UEC equation is linear in income as bellow:

$$x_1 = \beta_i(y - r_i) + m^i(p_1, p_2) + v_{1i}$$

where; x is unit electricity consumption; β is parameter of income; y is income; r is price of durable appliance or its rental charge annually; p_1, p_2 are price of electricity and other energy source respectively; m^i is linear parameters; ν is stochastic terms with typical distribution based on discrete choice.

A general solution to the indirect utility function which yielding the above linear electricity demand function will be as follow:

$$u = \psi\left\{\left[M^i(p_1, p_2) + (y - r_i) + \left(\frac{v_{1i}}{\beta_i}\right)\right] \cdot e^{-\beta_i p_1}, p_2, v_{2i}\right\}$$

where; ψ is an increasing function in its first argument, then:

$$M^i(p_1, p_2) = \int_{p_1}^{0} m^i(t, p_2) \cdot e^{\beta_i(p_1 - t)} dt$$

and the demand for alternative energy source will satisfy:

$$x_2 = M_2^1(p_1, p_2) - e^{-\beta_i p_1} \cdot \frac{\psi_2}{\psi_1}$$

where; $\psi_2/\psi_1 = \left(\frac{\partial\psi}{\partial p_2} / \frac{\partial\psi}{\partial p_1}\right)$ and $M_2^i = \partial M^i/\partial p_2$.

Linear parameters will be evaluated at the above indirect utility function.

In another recognizable study Vaage [9] modeled a discrete appliance choice as a multinomial logit model with appliance attributes, decision-maker characteristics, income, housing unit characteristics as explanatory variables and then continuous energy use modeled conditional to the appliance choice for heating data of Norway. This study also used of Roy's identity to continuous usage of energy. Davis [10] tested whether efficient durable which cost less to operate will lead to more use of them. He has followed Dubin and McFadden [1] to find the demand for water and energy in household service production. He estimated the model with data from a field trial to test the hypothesis; he found that just some portion of gained efficiency will be used in more usage of durable or less influence on demand for energy. The effect of climate warms on energy usage and energy types have been studied by Mansur et al. [11]. They found that people switched from oil and gas to electricity and in near future the electricity demand will increase highly however the energy expenditure will change from heating houses to cooling them. They also used the Dubin and McFadden [1] methodology in simultaneously determination of energy type and its demand. Same model used in vehicle choice and utilization, in the next section discrete-continuous model in vehicle choice and usage will be discussed in the same form as Dubin and McFadden [1] and other approaches will be discussed thereafter.

4. Discrete-continuous model in transport studies

4.1. Vehicle choice and mileage use

Households based on their socio-economic standing, the built environment, budget, fuel price, and environmental concerns decide which type of vehicle (discrete) to be chosen and how many miles (continuous) to be driven. In any discrete-continuous model the challengeable part of the model is how to link together different dimensions of the single decision about discrete choice of vehicle and continuous equation of usage. Up to now the only way of linking to variables in a discrete-continuous model is the correlation and dependency of unobserved terms in both discrete and continuous equations. In a joint model of vehicle type choice (discrete variable) and utilization or usage (continuous) both equations are linked together with an assumption about the joint distribution of disturbance terms.

Spissu et al. [12] presented a model in which they first presented the vehicle type choice component, and then the vehicle mileage component, and finally they described a joint structure between these two components. Here, their model as one of the recent studies will be reviewed. To construct the vehicle choice part of the mode they supposed the following utility function for the vehicle type choice:

$$u^*_{qi} = \beta_i' x_{qi} + \varepsilon_{qi}$$

where; u^*_{qi} is the latent utility of household q from vehicle type i; β'_i vector of coefficient; x_{iq} is a vector of household attributes; ε_{qi} is error term of unobserved factors associated with vehicle choice.

With the above utility function assumes that a household q will choose vehicle type i when it has the maximum utility among all other alternatives as:

$$u^*_{qi} > \max\ u^*_{qj} \quad \forall\ (j = 1, 2, \dots, I \text{ and } i \neq j).$$

Then Spissu et al. [12] have followed Lee [13] to build a polychotomous discrete choice in a form of a series of binary discrete choice for every vehicle type. To do it they assumed R_{qi} as a dichotomous variable with 1 for chosen vehicle type and 0 for otherwise. With the substitution of right hand side of first equation into second, one can equivalently get bellow forms which is equivalent to multinomial discrete choice model:

$$R_{qi} = 1 \quad \text{if} \quad \beta_i' x_{qi} > v_{qi} \quad \forall i = 1, 2, \dots, I$$

$$v_{qi} = \left(\max\ u^*_{qj}\right) - \varepsilon_{qi} \qquad \forall\ (j = 1, 2, \dots, I \text{ and } i \neq j).$$

Clearly, in the above equations the type of distribution of v_{qi} is depended on the distribution of ε_{qi}. To have a multinomial logit probability of vehicle choice type v_{qi} have to be distributed based on logistic distribution and then ε_{qi} have to be distributed based on type-I extreme value distribution.

In the continuous part of their model Spissu et al. [12] applied a classic log-linear regression model in which logarithms of annual mileage of vehicle type i used as latent dependent variable against observed household attributes and disturbance term like as:

$$m^*_{qi} = \alpha_i' z_{qi} + \eta_{qi}, \qquad m_{qi} = 1\left[R_{qi} = 1\right] m^*_{qi}.$$

At the above equation m^*_{qi} is observed (in the form of m_{qi}) only if household q is observed to hold a vehicle of type i. After the construction of discrete and continuous parts of the model, they have to be linked. Spissu et al. [12] applied copula-based method to capture the extent of the dependency between v_{qi} , η_{qi} terms. Copula method first transform v_{qi} , η_{qi} into a uniform distribution by using of their inverse cumulative distribution functions. Then, copula applied to uniformly distribute inverse cumulative distributions into multivariate joint distributions. Here, if the marginal distribution of v_{qi} , η_{qi} be $F_{vi}(.)$, $F_{\eta i}$ then the joint distribution of v, η will be $F_{vi,\ \eta i}(.,.)$. This joint distribution can be expressed in joint cumulative probability distribution of uniform (0,1) marginal variables U_1, U_2 as follows:

$$\begin{aligned} F_{vi,\eta i}(y_1, y_2) &= P\left(v_{qi} < y_1, \eta_{qi} < y_2\right) \\ &= P[F_{vi}^{-1}(U_1) < y_1, F_{vi}^{-1}(U_2) < y_2] \\ &= P\left[U_1 < F_{vi}(y_1), U_2 < F_{\eta i}(y_2)\right]. \end{aligned}$$

Based on the Sklar's theorem one can generate the above joint distribution with $C_\theta(.,.)$ such that:

$$F_{vi,\eta i}(y_1, y_2) = C_\theta\left(u_1 = F_{vi}(y_1), u_2 = F_{\eta i}(y_2)\right)$$

where; C_θ is a copula function; θ is a dependency parameter.

The copula function has to show the degree of dependency between v_{qi} , η_{qi}. The form of the log-likelihood joint vehicle type choice and mileage model is:

$$L = \prod_{q=1}^{Q}\left[\prod_{i=1}^{I}\left[\frac{1}{\sigma_{\eta i}} \times \frac{\partial C_{\theta i}\left(u^i_{q1}, u^i_{q2}\right)}{\partial u^i_{q2}} f_{\eta i}\left(\frac{m_{qi} - \alpha_i' z_{qi}}{\sigma_{\eta i}}\right)\right]^{R_{qi}}\right].$$

The copula method based on log-likelihood has some advantages to other methods such as; it gives different dependency for every vehicle type based on C_θ copula function, the dependency characterization does not depend on v_{qi} , η_{qi}.

To solve vehicle choice and mileage use problem in transportation, recently MDCEV model proposed by Bhat and Sen [4] and extended by Bhat [14]. This method is based on the new utility structure which introduced by Gorman [15] for the first time and then used by Kim et al. [16]. This neo-structure utility function is based on the translated utility function. With the use of this neo-structure utility function Bhat and Sen [4] propose the same utility structure for vehicle type and mileage use and then by use of Lagrangian form and applying Kuhn-Tucker condition find the optimal solution. For further detail one can see Bhat and Sen [4], Bhat [14], Spissu et al. [12], Castro et al. [17]. Still this model follows the original model just the assumption of the model is improved to make it closer to reality.

4.2. Shipment size and mode choice

The group of models that concern about the weight of shipment size as a dependent or latent variable is categorized as commodity-based models. In studying freight transport José Holguín-Veras [18] developed a model to deal with commercial vehicle type as a discrete and shipment size as a continuous variable. In his conceptual model two main agents participate directly or indirectly to decide about vehicle type and shipment size. Shippers are responsible to define the shipment size, origin/destination, and overall handling requirements. Carriers on the other hand are responsible to determine the mode of transport, vehicle type and size, and routing pattern. He followed Abdelwahab and Sargious [19]. In this approach the decision between shipper-carrier will be done based on joint interactions and to some extent based on a delayed response from previous experiences. The value of joint decision of shipper-carrier has been recognized by Samuelson [20] for the first time. To now two main approaches are identified for discrete-continuous modeling, the first approach discussed already which is based on Roy's Identity which used by Dubin and McFadden [1], Hanemann [2], Hanemann [21], Mannering and Hensher [22] in early studies, the second approach is based on a reduced form structure in which total utility is formulated into joint utility [23,24].

José Holguín-Veras [18] defined the utility of choice of vehicle as:

$$U_i = \beta_i Z_i + \phi y_i + \varepsilon_i$$

where; Z_i=vector of attributes of carrier company and vehicle; β_i=vector of parameters associated with Z_i; y_i=shipment size; Φ=parameter associated with shipment size; ε=disturbance term.

In his model shipment size is a continuous variable and defined as:

$$y_i = \alpha_i X_i + \eta_i$$

where; X_i=vector of shipper attributes; α_i=parameter associated to shipper attributes; η_i=disturbance term.

The continuous choice equation above shows the role of shippers in choice of shipment size and vehicle type. By substituting continuous choice equation into discrete utility equation one can get an equation in terms of shipment size and vehicle type but in reduced form as bellow:

$$U_i = \beta_i Z_i + \phi \alpha_i X_i + \phi \eta_i + \varepsilon_i.$$

Here the major concerned issue is estimation bias which resulted from the disturbance correlation of discrete and continuous equations. To overcome this bias estimation indirect and direct approaches are applied, indirect one is to estimate different models with exogenous variables, the other is to consider interactions between error terms of discrete and continuous choice models or assuming their distribution patterns like various extreme value distribution [2].

Here, José Holguín-Veras [18] in estimating his model used of indirect method. He estimated a shipment size equation in which the shipment size is exogenous and not depended upon vehicle choice or discrete variable, and then this estimated shipment size instead of observed shipment size used as an instrumental variable to be replaced in the actual shipment size equation to estimate discrete variable. He estimated different type of shipment size equation but in most of them shipment size is a function of trip length, a binary variable to represents commodity group, and type of economic activities in destination. The general form of estimated shipment size and correspondent discrete choice is:

$y_i^{estimated} = \alpha_i X_i + \eta_i$ and

$$U_i = \beta_i Z_i + \phi \cdot y_i^{estimatd} + \varepsilon_i.$$

To find the suitability of every shipment size to be transported by a specific vehicle he used of an equation that calculated the difference between average observed shipment size and estimated shipment size in form of absolute value as:

$$XL_i = ABS\left(M_i - y_i^{estimated}\right).$$

In his model vehicle selection process is a function of average unit cost of transport per ton because it estimates the associated resources to vehicle use and XL_i at the above equation. The rationale for including XL_i is that it provides an indication to know the appropriateness of vehicle for transporting the shipment size. He found that to increase the use of certain vehicle class one has to increase the weight-distance tax with rate ϕ based on the type of interested vehicle.

5. New research agenda

The aim of this study was to identify new developments to this type of models and the following suggestions are proposed as new research agenda:

a. Recently, the discrete continuous models have matured in the case of estimation and application. The next phase in developing this model is to include the time equation in order to let the model to be dynamic. This suggestion in transportation means when would the agent start its trip to reduce the congestion and in energy means the exact time of using the electric devices to reduce the energy consumption and relief pressure on the electricity production system.

b. Improvement of first assumption in indirect utility: in the estimation of the model, as the simulated maximum likelihood procedure has been proposed, one can propose the models with random parameters and including heterogeneity of consumers to make the model close to realities.
c. Where pricing in energy and transport is one of the major policy instruments to control amount of usage, it should be included in the model as a separate equation and not as a variable of the continuous model. This innovation directly pushes the discrete-continuous model to be linked to simultaneous equation system.

6. Conclusion

The compilation of developed models in micro-econometrics provided an opportunity to treat system of equations that include both continuous and discrete dependent variables simultaneously. This new way of treating both variables simultaneously is now known as discrete-continuous modeling. The joint decisions of mode choice and shipment size, brand of a car and mileage use, a durable appliance and its energy use are two simultaneous decisions in the field of transport and energy which falls into discrete-continuous modeling framework. Separate estimation of simultaneous decisions will lead to bias estimation and parameters will be over/under-estimated. Indirect utility function and extreme value distribution have facilitated the simultaneous estimation of both discrete and continuous decision. Both approaches can be applied into energy economics to determine joint decision of appliance brand and its energy use and in transport to determine the mode of transport and shipment size or mileage use. This review showed that both approaches have been applied in the field of energy and transport however the extreme value is mostly used to model the transport joint decisions. It is noticeable that estimation of extreme value distribution is more complicated and requires simulation procedures while indirect utility approach is estimable with maximum likelihood procedure which is available in most of commercial software. It is envisaged that these approaches will enable better cost estimates to be obtained thus ensuring the costing related to the economic sustainability pillar to be more accurate.

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