Modeling and Speed Estimation of A Faulty 3-Phase Induction Motor by Using Extended Kalman Filter

M. Jannati, S. H. Asgari, A. Monadi, N. R. N. Idris and M. J. A. Aziz
UTM-PROTON Future Drive Laboratory
Universiti Teknologi Malaysia
Skudai, Johor 81310, Malaysia
jannatim94@yahoo.com

D. Dehghani and A. A. M. Faudzi
Centre for Artificial Intelligence & Robotics (CAIRO)
Universiti Teknologi Malaysia
Skudai, Johor 81310, Malaysia
athif@fke.utm.my

***Abstract*—This paper presents a modeling technique for a faulty 3-phase Induction Motor (3-phase Induction Motor IM under open-phase fault). The developed model has the same structure of equations as the balanced 3-phase model. It is shown that the model can be utilized to estimate the speed of a faulty IM based on an Extended Kalman Filter (EKF) estimation technique, which was developed for a balanced 3-phase IM. Simulation and experimental results are presented to show the validity of the proposed techniques.**

***Keywords*—EKF; estimation of rotor speed; modeling of faulty 3-phase IM**

I. INTRODUCTION

The AC motors are widely used in industry. In these drives, AC motors like Induction Motors (IMs) and permanent magnet synchronous motors are widely used. Typical applications of AC machines are Heating, Ventilation and Air Conditioning (HVAC), fans, mixers, etc. Open-circuit is one of most popular faults in the stator windings of IMs [1-3]. This failure is caused by blown fuse, the opening of a coil at a junction box, mechanical shakings of the machine and etc. The fault detection in this research is supposed to be implemented using the on-line calculations that need the motor current and voltage as presented in [1]. The method enables the detection immediately depending on the sampling of the system by reconstructing the current space phasor based on the IM equations and comparing it with its actual measured value of current. Modeling of a faulty or an unbalanced 3-phase IMs is extremely important in some critical applications, such as military, space explorations, to ensure fault-tolerant operations. In order for the same control and speed observer algorithms as used in the balanced 3-phase IM can be directly applied for the faulty motor, the model of the faulty IM should have the same structure of equations as the balanced 3-phase IM model [4]. Rotor speed estimation instead of rotor speed measurement by using sensor will be reduced the complexity, size and cost of the system. Some methods have been proposed by researchers for the speed estimation for unbalanced two-phase IMs or single-phase IMs [5-9]. The majority of these methods use stator voltages and currents to perform the estimation. Obviously, this will result in significant errors in the estimated speed whenever fault occur if the same balanced model is used by the speed estimator. Specifically, in this paper, an Extended Kalman Filter (EKF) based speed estimation technique developed for a balanced 3-phase system will be used for the unbalanced system using the proposed model. Overall the work proposed contributes to a wide area of research on sensorless control scheme of IMs in the fault conditions such as [4, 10-12]. This work is organized as follows: in section II, d-q-0 model of 3-phase IM is presented. d-q-0 model of 3-phase IM when one of its stator phases is open-circuit is proposed in section III. In section IV, equations of EKF for speed estimation in faulty IM are shown. Simulation and experimental results that validate the proposed methods are presented in section V. Finally, section VI concludes the paper.

II. D-Q-0 MODEL OF 3-PHASE IM

For obtaining of d-q-0 model of 3-phase IM, the following two assumptions are made:

- Sinusoidal distribution of motor windings
- The effects of magnetic saturation, eddy currents and iron losses are neglected

The distribution of rotor and stator windings is shown in Fig. 1:

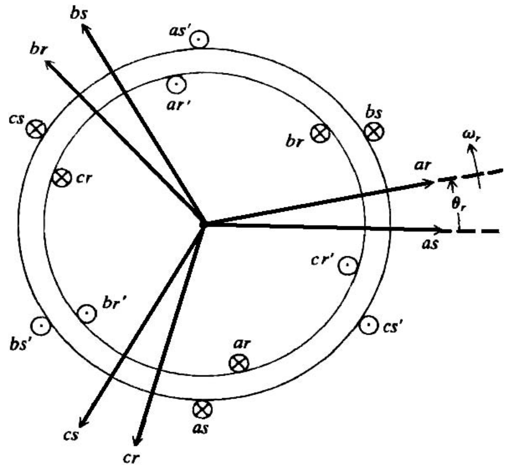

Fig. 1. Distribution of stator and rotor windings

The equations of stator and rotor for 3-phase IM can be written as follows [9]:

Stator and rotor equations:

$$\begin{aligned}[v_{abcs}] &= [R_s][i_{abcs}] + \frac{d}{dt}[\lambda_{abcs}] \\ [\lambda_{abcs}] &= [L_{ss}][i_{abcs}] + [L_{sr}][i_{abcr}]\end{aligned} \tag{1}$$

$$\begin{aligned}[v_{abcr}] &= [R_r][i_{abcr}] + \frac{d}{dt}[\lambda_{abcr}] \\ [\lambda_{abcr}] &= [L_{rr}][i_{abcr}] + [L_{rs}][i_{abcs}]\end{aligned} \tag{2}$$

Stator and rotor winding axis and moreover d and q axis (Park conversions) are shown as Fig. 2:

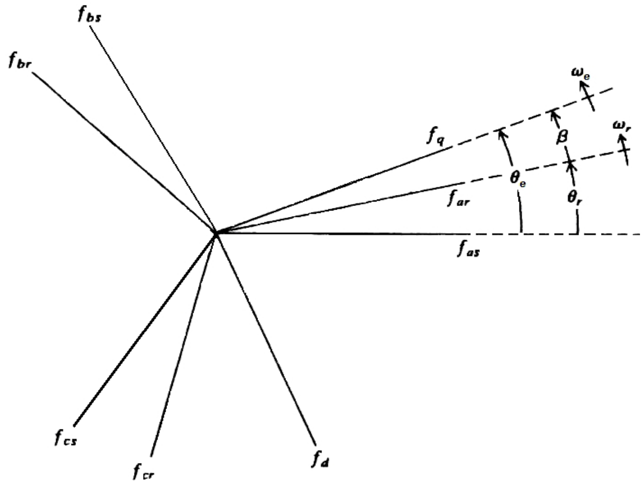

Fig. 2. Stator, rotor, d and q axis

By considering Fig. 2, the Park conversions for stator variables are obtained as follows:

$$\begin{aligned}f_d &= f_{as}\cos(-\theta_e + \frac{\pi}{2}) + f_{bs}\cos(-\theta_e + \frac{7\pi}{6}) + f_{cs}\cos(\theta_e + \frac{\pi}{6}) \\ f_q &= f_{as}\cos(\theta_e) + f_{bs}\cos(-\theta_e + \frac{2\pi}{3}) + f_{cs}\cos(\theta_e + \frac{2\pi}{3}) \\ f_o &= f_{as} + f_{bs} + f_{cs}\end{aligned} \tag{3}$$

As a result, the basic $\bar{d}$, $\bar{q}$ and $\bar{0}$ vectors can be written as following equations:

$$\begin{aligned}\bar{d} &= \begin{bmatrix}\sin\theta_e & \sin(\theta_e - \frac{2\pi}{3}) & \sin(\theta_e + \frac{2\pi}{3})\end{bmatrix} \\ \bar{q} &= \begin{bmatrix}\cos\theta_e & \cos(\theta_e - \frac{2\pi}{3}) & \cos(\theta_e + \frac{2\pi}{3})\end{bmatrix} \\ \bar{0} &= \begin{bmatrix}1 & 1 & 1\end{bmatrix}\end{aligned} \tag{4}$$

The Park conversion matrix for stator variables, after normalization is obtained as follows:

$$[T_s] = \sqrt{\frac{2}{3}}\begin{bmatrix}\sin\theta_e & \sin(\theta_e - \frac{2\pi}{3}) & \sin(\theta_e + \frac{2\pi}{3}) \\ \cos\theta_e & \cos(\theta_e - \frac{2\pi}{3}) & \cos(\theta_e + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \tag{5}$$

Similar to equation (6) the normalized Park conversion matrix for rotor variables is obtained as follows:

$$[T_r] = \sqrt{\frac{2}{3}}\begin{bmatrix}\sin\beta & \sin(\beta - \frac{2\pi}{3}) & \sin(\beta + \frac{2\pi}{3}) \\ \cos\beta & \cos(\beta - \frac{2\pi}{3}) & \cos(\beta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \tag{6}$$

Using the Park transformation matrices for stator and rotor variables the equations of 3-phase IM in d-q-0 frame can be obtained as:

Stator equations:

$$\begin{aligned}[T_s][v_{abcs}] &= [T_s][R_s][T_s]^{-1}[T_s][i_{abcs}] \\ &+ [T_s]\frac{d}{dt}\left([T_s]^{-1}[T_s][\lambda_{abcs}]\right)\end{aligned} \tag{7}$$

$$\begin{aligned}[T_s][\lambda_{abcs}] &= [T_s][L_{ss}][T_s]^{-1}[T_s][i_{abcs}] \\ &+ [T_s][L_{sr}][T_r]^{-1}[T_r][i_{abcr}]\end{aligned} \tag{8}$$

Therefore, we have:

$$[v_{dqos}] = [T_s][R_s][T_s]^{-1}[i_{dqos}] + [T_s]\frac{d}{dt}\left([T_s]^{-1}[\lambda_{dqos}]\right) \tag{9}$$

$$[\lambda_{dqos}] = [T_s][L_{ss}][T_s]^{-1}[i_{dqos}] + [T_s][L_{sr}][T_r]^{-1}[i_{dqor}] \tag{10}$$

Rotor equations:

$$\begin{aligned}[T_r][v_{abcr}] &= [T_r][R_r][T_r]^{-1}[T_r][i_{abcr}] \\ &+ [T_r]\frac{d}{dt}\left([T_r]^{-1}[T_r][\lambda_{abcr}]\right)\end{aligned} \tag{11}$$

$$\begin{aligned}[T_r][\lambda_{abcr}] &= [T_r][L_{rr}][T_r]^{-1}[T_r][i_{abcr}] \\ &+ [T_r][L_{rs}][T_s]^{-1}[T_s][i_{abcs}]\end{aligned} \tag{12}$$

So, we have:

$$[v_{dqor}] = [T_r][R_r][T_r]^{-1}[i_{dqor}] + [T_r]\frac{d}{dt}\left([T_r]^{-1}[\lambda_{dqor}]\right) \tag{13}$$

$$[\lambda_{dqor}] = [T_r][L_{rr}][T_r]^{-1}[i_{dqor}] + [T_r][L_{rs}][T_s]^{-1}[i_{dqos}] \tag{14}$$

where:

$$\begin{aligned}&[v_{dqos}] = [v_{ds}\ \ v_{qs}\ \ v_{os}]^T, [i_{dqos}] = [i_{ds}\ \ i_{qs}\ \ i_{os}]^T \\ &[\lambda_{dqos}] = [\lambda_{ds}\ \ \lambda_{qs}\ \ \lambda_{os}]^T, [v_{dqor}] = [v_{dr}\ \ v_{qr}\ \ v_{or}]^T \\ &[i_{dqor}] = [i_{dr}\ \ i_{qr}\ \ i_{or}]^T, [\lambda_{dqor}] = [\lambda_{dr}\ \ \lambda_{qr}\ \ \lambda_{or}]^T\end{aligned} \tag{15}$$

After simplifying these equations, the equations of 3-phase IM in the arbitrary reference frame (superscript "*e*") are obtained as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \\ v_{os}^e \\ v_{dr}^e \\ v_{qr}^e \\ v_{or}^e \end{bmatrix} = \begin{bmatrix} r_s + L_s p & -\omega_e L_s & 0 & Mp & -M\omega_e & 0 \\ \omega_e L_s & r_s + L_s p & 0 & M\omega_e & Mp & 0 \\ 0 & 0 & r_s + L_{ls} p & 0 & 0 & 0 \\ Mp & -(\omega_e - \omega_r)M & 0 & r_r + L_r p & -(\omega_e - \omega_r)M & 0 \\ (\omega_e - \omega_r)M & Mp & 0 & (\omega_e - \omega_r)M & r_r + L_r p & 0 \\ 0 & 0 & 0 & 0 & 0 & r_r + L_{lr} p \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \\ i_{os}^e \\ i_{dr}^e \\ i_{qr}^e \\ i_{or}^e \end{bmatrix} \quad (16)$$

where:

$$p = \frac{d}{dt}\ ,\ L_s = L_{ls} + \frac{3}{2}L_{ms}\ ,\ L_r = L_{lr} + \frac{3}{2}L_{mr}$$
$$M = \frac{3}{2}L_{ms}\ ,\ \omega_r = \frac{d\theta_r}{dt}\ ,\ \omega_e = \frac{d\theta_e}{dt} \quad (17)$$

For obtaining equations of motor in the stationary reference frame, in the equation (16), ω_e is set to 0. On the other hand, for obtaining the equations of motor in the rotor reference frame, in the equation (16), then $\omega_e=\omega_r$. The equation of the electromagnetic torque can be obtained by considering the electrical power input of the motor as follows:

$$\begin{aligned} P_e &= v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs} = [i_{abcs}]^T[v_{abcs}] \\ &= v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs} = [i_{abcs}]^T[v_{abcs}] \\ &= [i_{abcs}]^T[R_s][i_{abcs}] + [i_{abcs}]^T[L_{ss}]\{p[i_{abcs}]\} \\ &+ [i_{abcs}]^T\{p[L_{sr}]\}[i_{abcr}] + [i_{abcs}]^T[L_{sr}]\{p[i_{abcr}]\} \end{aligned} \quad (18)$$

Based on equation (18), it can be shown that the electromagnetic torque equation is given by:

$$T_e = \frac{p_m}{\omega_r} = \frac{1}{\omega_r}[i_{abcs}]^T\{p[L_{sr}]\}[i_{abcr}] \quad (19)$$

In the d-q-0 frame the equation can be written as:

$$T_e = \frac{1}{\omega_r}[i_{dqos}]^T\{[T_s]^{-1}\}^T\{p[L_{sr}]\}[T_r]^{-1}[i_{dqor}] \quad (20)$$

After simplifying, we have:

$$T_e = Mi_{qs}^e i_{dr}^e - Mi_{ds}^e i_{qr}^e \quad (21)$$

In generally, the electromagnetic torque equation with "pole" number of poles is given by:

$$T_e = \frac{Pole}{2}M\left(i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e\right) \quad (22)$$

III. D-Q-0 Model of 3-Phase IM When One of Its Stator Phases Is Open-Circuit

Suppose that a phase cut out fault happened in the phase "c" of a 3-phase IM. The equations of stator and rotor for unbalanced 3-phase IM are written as follows:

Stator equations:

$$\begin{aligned} [v_{abs}] &= [R_s^{fault}][i_{abs}] + \frac{d}{dt}[\lambda_{abs}] \\ [\lambda_{abs}] &= [L_{ss}^{fault}][i_{abs}] + [L_{sr}^{fault}][i_{abcr}] \end{aligned} \quad (23)$$

Rotor equations:

$$\begin{aligned} [v_{abcr}] &= [R_r][i_{abcr}] + \frac{d}{dt}[\lambda_{abcr}^{fault}] \\ [\lambda_{abcr}^{fault}] &= [L_{rr}][i_{abcr}] + [L_{rs}^{fault}][i_{abs}] \end{aligned} \quad (24)$$

where:

$$[v_{abs}] = [v_{as} \quad v_{bs}]^T, [i_{abs}] = [i_{as} \quad i_{bs}]^T$$

$$[\lambda_{abs}] = [\lambda_{as} \quad \lambda_{bs}]^T, [R_s^{fault}] = r_s \cdot I_{2\times 2} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}$$

$$[L_{ss}^{fault}] = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix}, [\lambda_{abcr}^{fault}] = [\lambda_{ar} \quad \lambda_{br} \quad \lambda_{cr}]^T$$

$$[L_{sr}^{fault}] = L_{ms}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \quad (25)$$

$$[L_{rs}^{fault}] = [L_{sr}^{fault}]^T$$

As can be seen [R_s^{fault}], [L_{ss}^{fault}], [L_{rs}^{fault}] and [L_{sr}^{fault}] are the same as the equations for a balanced machine. The difference is in the missing row and column of phase "c". The stator and rotor flux axis can be shown as Fig. 3:

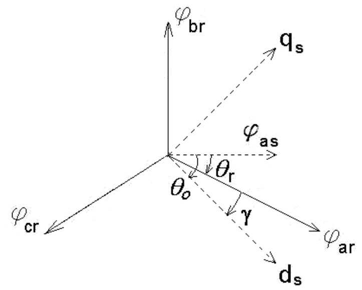

Fig. 3. Stator and rotor flux axis

By considering Fig. 3, stator and rotor d-q components can be written as follows:

$$\varphi_{ds} = \begin{bmatrix} \cos(\theta_o) & \cos(\theta_o + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} \varphi_{as} \\ \varphi_{bs} \end{bmatrix}$$

$$\varphi_{qs} = \begin{bmatrix} \sin(\theta_o) & \sin(\theta_o + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} \varphi_{as} \\ \varphi_{bs} \end{bmatrix}$$

$$\varphi_{dr} = \begin{bmatrix} \cos(\gamma) & \cos(\gamma + 2\pi/3) & \cos(\gamma + 4\pi/3) \end{bmatrix} \begin{bmatrix} \varphi_{ar} \\ \varphi_{br} \\ \varphi_{cr} \end{bmatrix} \quad (26)$$

$$\varphi_{qr} = \begin{bmatrix} \sin(\gamma) & \sin(\gamma + 2\pi/3) & \sin(\gamma + 4\pi/3) \end{bmatrix} \begin{bmatrix} \varphi_{ar} \\ \varphi_{br} \\ \varphi_{cr} \end{bmatrix}$$

where:

$$\gamma = \theta_o - \theta_r \tag{27}$$

Therefore, the basic $\bar{d}$ and $\bar{q}$ vectors can be written as follows:

$$\begin{aligned}
\bar{d}_s &= \begin{bmatrix} \cos(\theta_o) & \cos(\theta_o + \frac{2\pi}{3}) \end{bmatrix} \\
\bar{q}_s &= \begin{bmatrix} \sin(\theta_o) & \sin(\theta_o + \frac{2\pi}{3}) \end{bmatrix} \\
\bar{d}_r &= \begin{bmatrix} \cos(\gamma) & \cos(\gamma + \frac{2\pi}{3}) & \cos(\gamma + \frac{4\pi}{3}) \end{bmatrix} \\
\bar{q}_r &= \begin{bmatrix} \sin(\gamma) & \sin(\gamma + \frac{2\pi}{3}) & \sin(\gamma + \frac{4\pi}{3}) \end{bmatrix}
\end{aligned} \tag{28}$$

From (29) θ_o can be obtained as:

$$d^T.q = q^T.d = 0 \quad \Rightarrow \quad \theta_o = \frac{\pi}{6} \tag{29}$$

The normalized Park transformation matrix for stator variables is obtained as:

$$\left[T_s^{fault}\right] = \sqrt{2}\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{30}$$

From (28) the Park transformation matrix for rotor variables after normalization is obtained as follows:

$$[T_r] = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(\gamma) & \cos(\gamma + \frac{2\pi}{3}) & \cos(\gamma + \frac{4\pi}{3}) \\ \sin(\gamma) & \sin(\gamma + \frac{2\pi}{3}) & \sin(\gamma + \frac{4\pi}{3}) \end{bmatrix} \tag{31}$$

With the transformation matrices of the stator and rotor variables for the unbalanced machine, the equations of unbalanced IM in d-q frame are obtained as follows:

Stator equations:

$$\begin{aligned}
&\left[T_s^{fault}\right][v_{abs}] = \left[T_s^{fault}\right]\left[R_s^{fault}\right]\left[T_s^{fault}\right]^{-1}\left[T_s^{fault}\right][i_{abs}] \\
&+ \left[T_s^{fault}\right]\frac{d}{dt}\left(\left[T_s^{fault}\right]^{-1}\left[T_s^{fault}\right][\lambda_{abs}]\right) \\
&\left[T_s^{fault}\right]\left[L_{ss}^{fault}\right]\left[T_s^{fault}\right]^{-1}\left[T_s^{fault}\right][i_{abs}] \\
&+ \left[T_s^{fault}\right]\left[L_{sr}^{fault}\right][T_r]^{-1}[T_r][i_{abcr}]
\end{aligned} \tag{32}$$

In which we obtain:

$$\begin{aligned}
&[v_{dqs}] = \left[T_s^{fault}\right]\left[R_s^{fault}\right]\left[T_s^{fault}\right]^{-1}[i_{dqs}] \\
&+ \left[T_s^{fault}\right]\frac{d}{dt}\left(\left[T_s^{fault}\right]^{-1}[\lambda_{dqs}]\right) \\
&[\lambda_{dqs}] = \left[T_s^{fault}\right]\left[L_{ss}^{fault}\right]\left[T_s^{fault}\right]^{-1}[i_{dqs}] \\
&+ \left[T_s^{fault}\right]\left[L_{sr}^{fault}\right][T_r]^{-1}[i_{dqr}]
\end{aligned} \tag{33}$$

Rotor equations:

$$\begin{aligned}
&[T_r][v_{abcr}] = [T_r][R_r][T_r]^{-1}[T_r][i_{abcr}] \\
&+ [T_r]\frac{d}{dt}\left([T_r]^{-1}[T_r]\left[\lambda_{abcr}^{fault}\right]\right) \\
&[T_r][L_{rr}][T_r]^{-1}[T_r][i_{abcr}] \\
&+ [T_r]\left[L_{rs}^{fault}\right]\left[T_s^{fault}\right]^{-1}\left[T_s^{fault}\right][i_{abs}]
\end{aligned} \tag{34}$$

Which gives:

$$\begin{aligned}
&[v_{dqr}] = [T_r][R_r][T_r]^{-1}[i_{dqr}] + [T_r]\frac{d}{dt}\left([T_r]^{-1}[\lambda_{dqr}]\right) \\
&[\lambda_{dqr}] = [T_r][L_{rr}][T_r]^{-1}[i_{dqr}] + [T_r]\left[L_{rs}^{fault}\right]\left[T_s^{fault}\right]^{-1}[i_{dqs}]
\end{aligned} \tag{35}$$

where:

$$\begin{aligned}
&[v_{dqs}] = [v_{ds} \quad v_{qs}]^T, [i_{dqs}] = [i_{ds} \quad i_{qs}]^T \\
&[\lambda_{dqs}] = [\lambda_{ds} \quad \lambda_{qs}]^T, [v_{dqr}] = [v_{dr} \quad v_{qr}]^T \\
&[i_{dqr}] = [i_{dr} \quad i_{qr}]^T, [\lambda_{dqr}] = [\lambda_{dr} \quad \lambda_{qr}]^T
\end{aligned} \tag{36}$$

After simplifying of these equations, it can be shown that the equations of the unbalanced IM in the arbitrary reference frame (superscript "*e*") are obtained as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_{ds}p & -\omega_e L_{qs} & M_d p & -\omega_e M_q \\ \omega_e L_{qs} & r_s + L_{qs}p & \omega_e M_d & M_q p \\ M_d p & -(\omega_e - \omega_r)M_q & r_r + L_r p & -(\omega_e - \omega_r)L_r \\ (\omega_e - \omega_r)M_d & M_q p & (\omega_e - \omega_r)L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \\ i_{dr}^e \\ i_{qr}^e \end{bmatrix} \tag{37}$$

where:

$$\begin{aligned}
&L_{ds} = L_{ls} + \frac{3}{2}L_{ms}, L_{qs} = L_{lr} + \frac{1}{2}L_{ms} \\
&M_d = \frac{3}{2}L_{ms}, M_q = \frac{\sqrt{3}}{2}L_{ms}
\end{aligned} \tag{38}$$

Electromagnetic torque equation:

$$T_e = \frac{1}{\omega_r}[i_{dqs}]^T\left\{\left[T_s^{fault}\right]^{-1}\right\}^T\left\{p\left[L_{sr}^{fault}\right]\right\}[T_r]^{-1}[i_{dqr}] \tag{39}$$

After simplifying, the electromagnetic torque equation for unbalanced IM is given as (40):

$$T_e = \frac{pole}{2}\left(M_q i_{qs}^e i_{dr}^e - M_d i_{ds}^e i_{qr}^e\right) \tag{40}$$

In summary, the comparison between model of balanced and unbalanced IM equations are summarized in TABLE I.

IV. EQUATIONS OF EKF FOR ROTOR SPEED ESTIMATION IN FAULTY 3-PHASE IM

The state space model of unbalanced 3-phase IM can be expressed as:

$$\dot{x} = Ax + Bu + w(t) \qquad y = Cx + v(t) \tag{41}$$

TABLE I. COMPARISON BETWEEN MODEL OF BALANCED AND UNBALANCED IM EQUATIONS

Balanced IM	Unbalanced IM
mutual inductance (q-axis) as follow: $M = \frac{3}{2} L_{ms}$	mutual inductance (q-axis) as follow: $M_q = \frac{\sqrt{3}}{2} L_{ms}$
stator self inductance (q-axis) as follows: $L_s = L_{ls} + \frac{3}{2} L_{ms}$	stator self inductance (q-axis) as follows: $L_{qs} = L_{ls} + \frac{1}{2} L_{ms}$

The covariance matrices of noises are defined as:

$$Q = \mathrm{cov}(w) = E\{ww^t\} \qquad R = \mathrm{cov}(v) = E\{vv^t\} \tag{42}$$

In (44), A_n, B_n and C_n are input and output matrices of system. Moreover, $w(t)$ is the system noise and $v(t)$ is the measurement noise. For estimation of rotor speed in faulty motor, the state (x_n), input (u_n) and output matrix (y_n) is given by following equations:

$$\begin{aligned} x_n &= \begin{bmatrix} i_{ds}^{(n)} & i_{qs}^{(n)} & \lambda_{dr}^{(n)} & \lambda_{qr}^{(n)} & \omega_r^{(n)} \end{bmatrix}^T \\ u_n &= \begin{bmatrix} v_{ds}^{(n)} & v_{qs}^{(n)} \end{bmatrix}^T, y_n = \begin{bmatrix} i_{ds}^{(n)} & i_{qs}^{(n)} \end{bmatrix}^T \end{aligned} \tag{43}$$

From equation (43), the matrixes A_n, B_n and C_n are obtained as given by equations (44) and (45). By considering of these equations and EKF algorithm (equations (46-52), the rotor speed in unbalanced IM can be estimated.

$$A_n =$$
$$\begin{bmatrix} 1-\frac{1}{k_1}\left(r_s+\frac{M_d^2 r_r}{L_r^2}\right)dt & 0 & \frac{M_d r_r}{k_1 L_r^2}dt & \frac{M_d r_r}{k_1 L_r}dt & 0 \\ 0 & 1-\frac{1}{k_2}\left(r_s+\frac{M_q^2 r_r}{L_r^2}\right)dt & \frac{-M_q r_r}{k_2 L_r}dt & \frac{M_q r_r}{k_2 L_r^2}dt & 0 \\ \frac{M_d r_r}{L_r}dt & 0 & 1-\frac{r_r}{L_r}dt & -r_r dt & 0 \\ 0 & \frac{M_q r_r}{L_r}dt & r_r dt & 1-\frac{r_r}{L_r}dt & 0 \\ -\frac{1.5 p_p^2 M_d \lambda_{qr}^s}{J L_r}dt & \frac{1.5 p_p^2 M_q \lambda_{dr}^s}{J L_r}dt & 0 & 0 & 1 \end{bmatrix}$$

$$B_n = \begin{bmatrix} \frac{1}{k_1}dt & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{k_2}dt & 0 & 0 & 0 \end{bmatrix}^T, C_n = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

where:

$$k_1 = L_{ds} - \frac{M_d^2}{L_r} \quad , \quad k_2 = L_{qs} - \frac{M_q^2}{L_r} \tag{45}$$

EKF algorithm:

Prediction of State:

$$x_{n+1\ n} = \Phi(n+1, n, x_{n\ n-1}, u_n) \tag{46}$$

where:

$$\Phi(n+1, n, x_{n\ n-1}, u_n) = A_n(x_{n\ n})x_{n\ n} + B_n(x_{n\ n})u_n \tag{47}$$

Estimation of Error Covariance Matrix:

$$P_{n+1\ n} = \frac{d\Phi}{dx}\Big|_{x=x_{n\ n}} P_{n\ n} \frac{d\Phi^T}{dx}\Big|_{x=x_{n\ n}} + Q \tag{48}$$

Computation of Kalman Filter Gain:

$$K_n = P_{n\ n-1} \frac{\partial H^T}{\partial x}\Big|_{x=x_{n\ n-1}} \left(\frac{\partial H}{\partial x}\Big|_{x=x_{n\ n-1}} P_{n\ n-1} \frac{\partial H^T}{\partial x}\Big|_{x=x_{n\ n-1}} + R \right)^{-1} \tag{49}$$

where:

$$H(x_{n\ n-1}, n) = C_n(x_{n\ n-1})x_{n\ n-1} \tag{50}$$

State Estimation:

$$x_{n\ n} = x_{n\ n-1} + K_n(y_n - H(x_{n\ n-1}, n)) \tag{51}$$

Update of the Error Covariance Matrix:

$$P_{n\ n} = P_{n\ n-1} - K_n \frac{\partial H}{\partial x}\Big|_{x=x_{n\ n-1}} P_{n\ n-1} \tag{52}$$

In equations (46-52), H is the matrix of output prediction, P_n error covariance matrix and Φ is the matrix of state prediction.

V. SIMULATION AND EXPREMENTAL RESULTS

For validation purposes, the simulation and experimental using a 2-Poles, 50-Hz, 3000 RPM, 3-phase IM are conducted. The motor is fed by a 3-phase sinusoidal voltage source. The ratings and parameters of the simulated and experimented motor are as given in Appendix.

Fig. 4 (right) shows the dynamic behavior of the 3-phase IM under open-phase fault (a phase cut-off is introduced during start-up (t = 0s)). Moreover Fig. 4 (left) shows the dynamic behavior of the balanced 3-phase IM. In this Fig. a load torque steps from 0N.m to 10N.m is introduced at t = 0.5s. As expected, the results showed the similarity of the proposed modeling technique for faulty IM with balanced 3-phase IM. Fig. 5 demonstrates the experimental results of the rotor speed estimation in a faulty 3-phase IM (in this Fig. a phase cut-off is introduced at t = 0.06s). The result clearly showed the validity of the proposed modeling technique and its usage in speed estimation based on EKF technique.

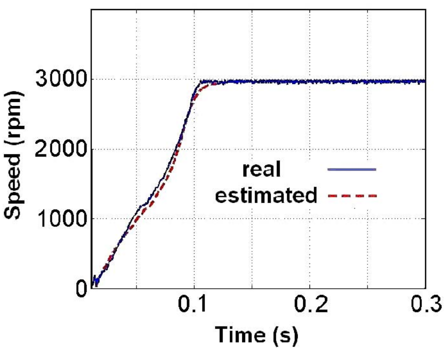

Fig. 4. Experimental result of the rotor speed estimation in a faulty 3-phase IM using EKF

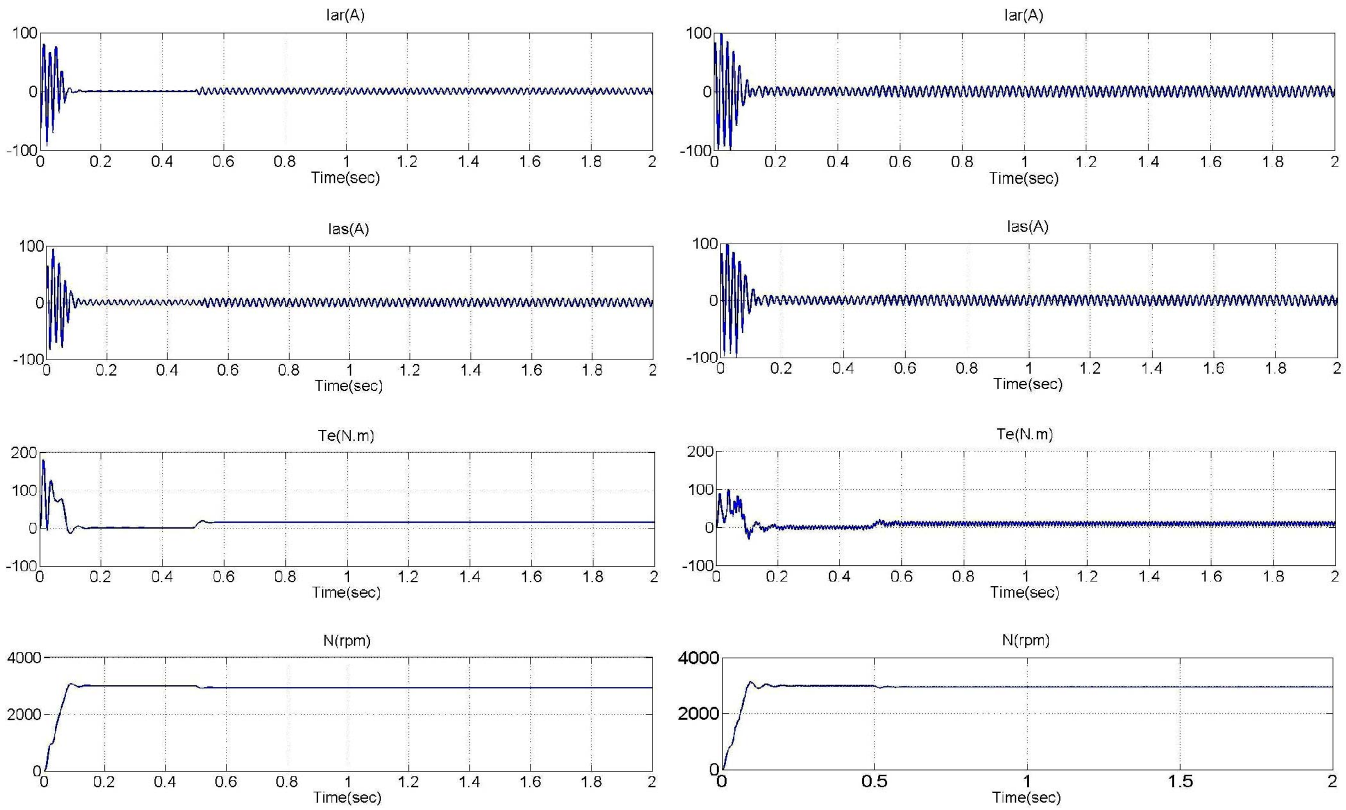

Fig. 5. Simulation results of the faulty 3-phase IM (Rotor a-axis current, Stator a-axis current, Electromagnetic torque, Speed)

VI. Conclusion

A method for modeling a phase cut out fault of a 3-phase IM is presented. In this method, faulty motor is modeled by an equivalent unbalanced two-phase motor, which has the same structure of equations as the balanced 3-phase IM. Using the developed model, a method of speed estimation based on EKF is presented. Simulation and experimental results are presented to verify the validity of the developed modeling technique.

Appendix

Parameters of 3-phase IM:

$v = 400V$, $f = 50HZ$, $Pole = 2$, $r_s = 1.405\Omega$, $r_r = 1.395\Omega$

$L_{ls} = L_{lr} = 0.005839H$, $L_{ms} = .01722H$, $J = 0.0131 kg.m^2$

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