

A New Method for RFOC of Single-Phase Induction Motor Based on Rotational Transformations

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Abstract—This paper presents a new technique for Rotor Field-Oriented Control (RFOC) of a single-phase induction motor (IM) with two asymmetrical main and auxiliary windings. Using this technique, transformation matrices are introduced and applied to the equations of single-phase IM. It is shown that by applying these transformation matrices to the equations of single-phase IM, asymmetrical equations of RFOC for single-phase IM are transformed into symmetrical equations. These rotational transformations are derived based on a simplification to the steady state equivalent circuit of a single-phase IM. Finally, simulation results are presented to demonstrate the excellence performance of the presented method.

Keywords—Rotor-Field Oriented Control; single-phase IM; transformation matrices

I. INTRODUCTION

Single-phase Induction Motors (IMs) are widely employed in the low power domestic and industrial applications where a three-phase AC supply is not available; they are used in mixers, dryers, washers, vacuum cleaners, compressors and many other applications. In these constant speed applications, the single-phase IM is fed by a single-phase AC supply. Single-phase IMs have two main and auxiliary winding and its operation usually needs capacitors. Single-phase IMs can be classified as split-phase motors, shaded-pole motors, capacitor-start motors, capacitor-run motors and two-capacitor motors [1]. In general, single-phase IM can be considered as an asymmetric two-phase IM. Recently, the use of single-phase IMs for variable speed applications is gaining popularity. In particular, vector control techniques applied to single-phase IMs are proven to be very effective in improving the performance and efficiency of conventional constant speed single-phase induction motor drives [2-21]. In [10, 18, 19, 20] Stator FOC (SFOC) technique are presented, whereas in [12, 14, 15, 21] Rotor FOC (RFOC) method for single-phase IMs or unbalanced two-phase IMs are studied. In this study, single-phase IM without startup and running capacitors controlled by RFOC is investigated. Unlike three-phase IM, different power converter topologies have been adopted for supplying single-phase IMs. Some examples are one-leg inverter [3], two-leg inverter [10, 12], three-leg inverter [4, 7] and four-leg inverter in [6] have been employed for supplying single-phase IM. Fig.

1 shows a two-leg inverter connected to a single-phase IM which is used in this work as it has been studied in [10, 12].

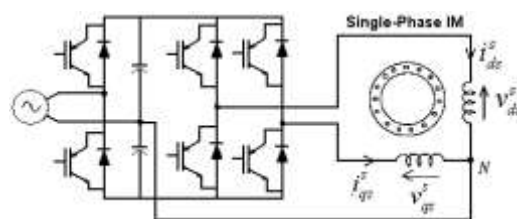


Fig. 1. Single-phase IM drive system

Single-phase IM can be modeled as two balanced forward and backward circuits based on the double rotating magnetic field theory [1]. Based on this concept, the transformation matrices are developed in this paper that transform unbalanced structure of equations into a balanced equations. Next, a general rotational transformation matrix for single-phase vector control of IM is developed. This rotational transformation matrix can also be applied to an unbalanced or faulty 3-phase IM (phase cut-off faulty). The presented method is analyzed and verified by using computer simulations for single-phase IM at zero, low and high-speed operations. The rest of this paper is organized as follows. In section II, the d-q model of single-phase IM is presented. Proposed method to control of single-phase IM is discussed in section III. The RFOC equations of single-phase IM are presented in section IV. The simulation results are presented in section V. Finally, conclusion is presented in section VI.

II. THE SINGLE-PHASE IM MODEL

The equations of single-phase IM with unequal main and auxiliary windings in a stationary reference frame (superscript "s") can be expressed as follows:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_{ds} + L_{ds} \frac{d}{dt} & 0 & M_d \frac{d}{dt} & 0 \\ 0 & r_{qs} + L_{qs} \frac{d}{dt} & 0 & M_q \frac{d}{dt} \\ M_d \frac{d}{dt} & \omega_s M_q & r_r + L_r \frac{d}{dt} & \omega_s L_r \\ -\omega_s M_d & M_q \frac{d}{dt} & -\omega_s L_r & r_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 & M_d & 0 \\ 0 & L_{qs} & 0 & M_q \\ M_d & 0 & L_r & 0 \\ 0 & M_q & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (1)$$

$$\tau_e = \frac{\text{Pole}}{2} (M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s)$$

$$\frac{\text{Pole}}{2} (\tau_e - \tau_l) = J \frac{d\omega_r}{dt} + F\omega_r$$

where, $v_{ds}^s, v_{qs}^s, i_{ds}^s, i_{qs}^s, i_{dr}^s, i_{qr}^s, \lambda_{ds}^s, \lambda_{qs}^s, \lambda_{dr}^s$ and λ_{qr}^s are the d-q axes voltages, currents, and fluxes of the stator and rotor in the stator reference frame. r_{ds}, r_{qs} and r_r denote the stator and rotor resistances. L_{ds}, L_{qs}, L_r, M_d and M_q denote the stator and the rotor self and mutual inductances. ω_r is the machine speed. τ_e, τ_l, J and F are electromagnetic torque, load torque, inertia and viscous friction coefficient, respectively.

In rotor flux oriented control method, the equations of machine are transformed into the rotor flux oriented reference frame by employing a rotational transformation matrix given in (2).

$$\begin{bmatrix} T_s^e \end{bmatrix} = \begin{bmatrix} \cos \theta_e & \sin \theta_e \\ -\sin \theta_e & \cos \theta_e \end{bmatrix} \quad (2)$$

where θ_e is the angle between the stationary reference frame and the rotor flux oriented reference frame. If this equation is applied to the equations of single-phase IM, the equations of the forward and backward components are generated; this is because of the unbalance windings in a single-phase IM equations. It is possible to control a single-phase IM by controlling these forward and backward components however, the control system will be very complex. In this paper, new transformation matrices are introduced. By employing these transformation matrices, the equations of single-phase IM are transformed into equations that have the same structure as balanced equations.

III. PROPOSED METHOD FOR VECTOR CONTROL OF SINGLE-PHASE IM

The main idea of using transformation matrices is obtained from the steady state equivalent circuit of a single-phase IM. Fig. 2 shows the equivalent circuit of single-phase IM. In this figure, V_m, V_a are the main and auxiliary voltages, I_m and I_a are the main and auxiliary currents, " a " is the turn ratio ($\alpha = N_a/N_m$) and " j " is the square root of "-1". E_{bm}, E_{ba}, E_{fm} and E_{fa} are the backward and forward voltage of magnetizing branch of the main and auxiliary windings. R_f, R_b, X_f and X_b are the forward and backward stator resistance and inductance in main winding. R_{lm}, R_{la}, X_{lm} and X_{la} are the leakage resistance and inductance of the main and auxiliary winding. According to Fig. 2, the motor main and auxiliary voltage is obtained as:

$$V_m = Z_{lm} I_m + E_{fm} - \frac{j}{\alpha} E_{fa} + E_{bm} + \frac{j}{\alpha} E_{ba} \quad (3)$$

$$V_a = Z_{la} I_a + E_{fa} + j\alpha E_{fm} + E_{ba} - j\alpha E_{bm}$$

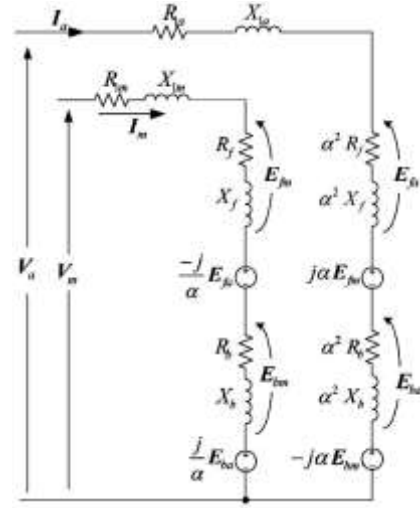


Fig. 2. Steady state equivalent circuit of the single-phase IM

where:

$$E_{fm} = Z_f I_m, E_{bm} = Z_b I_m, E_{fa} = \alpha^2 Z_f I_a$$

$$E_{ba} = \alpha^2 Z_b I_a, Z_f = R_f + jX_f, Z_b = R_b + jX_b \quad (4)$$

$$Z_{lm} = R_{lm} + jX_{lm}, Z_{la} = R_{la} + jX_{la}$$

By applying of following change of variables,

$$I_m = \frac{1}{2}(I_1 + I_2), I_a = \frac{j}{2\alpha}(I_1 - I_2) \quad (5)$$

ratio of windings currents is obtained as follows:

$$\frac{I_m}{I_a} = \frac{Z_{la} + \alpha^2(Z_f + Z_b) + j\alpha(Z_f - Z_b)}{Z_{lm} + Z_f + Z_b - j\alpha(Z_f - Z_b)} \quad (6)$$

$$\frac{I_1}{I_2} = \frac{\alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1)}{-\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1)}$$

By using of following change of variables,

$$V_1 = Z_3 V_m + jZ_4 V_a \quad (7)$$

Fig. 2 can be simplified as Fig. 3.

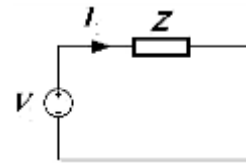


Fig. 3. Simplified equivalent circuit of single-phase IM

In Fig .3:

$$Z = (Z_3 + Z_4) \frac{Z_1(Z_{lm} + 2Z_f) + Z_2(Z_{lm} + 2Z_b)}{2Z_1} \quad (8)$$

$$Z_1 = \alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1)$$

$$Z_2 = -\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1)$$

It can be seen that by using the change of variables, the equivalent circuit of single-phase IM changed into a balanced circuit. By choosing suitable values of Z_3 and Z_4 , Fig. 2, can be simplified as a balanced circuit. With changing of Z_3 and Z_4 , only the impedance of the Fig. 3 circuit is changed. Equations (5) and (7) can be written as following equations:

$$\begin{cases} jV_1 = -Z_4V_a + jZ_3V_m \\ V_1 = jZ_4V_a + Z_3V_m \end{cases}, \begin{cases} jI_1 = \frac{N_a}{N_m}I_a + jI_m \\ I_1 = -j\frac{N_a}{N_m}I_a + I_m \end{cases} \quad (9)$$

With following substitutions:

$$j \rightarrow \sin \theta_e, 1 \rightarrow \cos \theta_e$$

$$\frac{N_m}{N_a} \rightarrow \frac{N_q}{N_d} = \frac{M_q}{M_d}, jV_1 \rightarrow v_{ds}^e$$

$$(10)$$

$$V_1 \rightarrow v_{qs}^e, V_a \rightarrow v_{ds}^s, V_m \rightarrow v_{qs}^s$$

$$jI_1 \rightarrow i_{ds}^e, I_1 \rightarrow i_{qs}^e, I_a \rightarrow i_{ds}^s, I_m \rightarrow i_{qs}^s$$

The proposed rotational transformations for stator voltage and current variables are obtained as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \end{bmatrix} = \begin{bmatrix} T_{vs}^e \\ T_{vs}^e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} -Z_4 \cos \theta_e & Z_3 \sin \theta_e \\ Z_4 \sin \theta_e & Z_3 \cos \theta_e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix}$$

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} = \begin{bmatrix} \frac{M_d}{M_q} \cos \theta_e & \sin \theta_e \\ -\frac{M_d}{M_q} \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (11)$$

Equations (11) transform unbalanced voltages and currents into balanced voltages and currents. It is expected by using of these transformation matrices (equation (11)) equations of unbalanced single-phase IM become similar to the equations of balanced motor. By applying (11) to the equations of single-phase IM, we have:

Rotor voltage equations:

$$\begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} M_d \frac{d}{dt} & \omega_r M_q \\ -\omega_r M_d & M_q \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix}^{-1} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

$$+ \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} r_r + L_r \frac{d}{dt} & \omega_r L_r \\ -\omega_r L_r & r_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^{-1} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (12)$$

After simplifying the equations of rotor voltages can be written as:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} M_q \frac{d}{dt} & (\omega_r - \omega_e) M_q \\ -(\omega_r - \omega_e) M_q & M_q \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix}$$

$$+ \begin{bmatrix} r_r + L_r \frac{d}{dt} & (\omega_r - \omega_e) L_r \\ -(\omega_r - \omega_e) L_r & r_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{dr}^e \\ i_{qr}^e \end{bmatrix} \quad (13)$$

Electromagnetic torque equation:

$$\tau_e = \frac{Pole}{2} (M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s)$$

$$= \frac{Pole}{2} \begin{bmatrix} i_{dr}^s & i_{qr}^s \end{bmatrix} \begin{bmatrix} 0 & M_q \\ -M_d & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

$$= \left(\frac{Pole}{2} \begin{bmatrix} i_{dr}^s & i_{qr}^s \end{bmatrix} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \right)^T \left(\begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^{-1} \right)^T$$

$$\begin{bmatrix} 0 & M_q \\ -M_d & 0 \end{bmatrix} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix}^{-1} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

$$(14)$$

After simplifying the equation of electromagnetic torque can be written as follows:

$$\tau_e = \frac{Pole}{2} M_q (i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e) \quad (15)$$

It can be shown that by using the proposed rotational transformations, equations of rotor voltages and electromagnetic torque are transformed into balanced equations. Rotational transformation for stator voltage variables as mentioned before can be considered as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \end{bmatrix} = \begin{bmatrix} T_{vs}^e \\ T_{vs}^e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} -Z_4 \cos \theta_e & Z_3 \sin \theta_e \\ Z_4 \sin \theta_e & Z_3 \cos \theta_e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix}$$

$$= \begin{bmatrix} a_v & b_v \\ c_v & d_v \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} \quad (16)$$

By applying of this matrix to the stator voltage equations of single-phase IM we have:

Stator voltage equations:

$$\begin{bmatrix} T_{vs}^e \\ T_{vs}^e \end{bmatrix} \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} T_{vs}^e \\ T_{vs}^e \end{bmatrix} \begin{bmatrix} r_{ds} + L_{ds} \frac{d}{dt} & 0 \\ 0 & r_{qs} + L_{qs} \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix}^{-1} \begin{bmatrix} T_{is}^e \\ T_{is}^e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

$$+ \begin{bmatrix} T_{vs}^e \\ T_{vs}^e \end{bmatrix} \begin{bmatrix} M_d \frac{d}{dt} & 0 \\ 0 & M_q \frac{d}{dt} \end{bmatrix} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix}^{-1} \begin{bmatrix} T_s^e \\ T_s^e \end{bmatrix} \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (17)$$

One of the problems in equation (17) is the presence of di_{qs}^s/dt and di_{qr}^s/dt in the equation for v_{ds}^e and as well as the presence of di_{ds}^s/dt and di_{dr}^s/dt in the equation for v_{qs}^e . One way of solving this is to use PI controller, which cause the controlling system to be very complex. By considering the

coefficient of di_{qs}^s/dt and coefficient of di_{ds}^s/dt equal to zero, the equations of stator voltage are obtained as follows:

$$\begin{aligned} & \left(a_v \cos \theta_e \left(\frac{L_{ds}M_q}{M_d} - \frac{M_dM_q}{L_r} \right) + b_v \sin \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) \right) \frac{di_{ds}^e}{dt} \\ & + \left(a_v \sin \theta_e \left(-\frac{L_{ds}M_q}{M_d} + \frac{M_dM_q}{L_r} \right) + b_v \cos \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) \right) \frac{di_{qs}^e}{dt} = 0 \\ & \left(c_v \cos \theta_e \left(\frac{L_{ds}M_q}{M_d} - \frac{M_dM_q}{L_r} \right) + d_v \sin \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) \right) \frac{di_{ds}^e}{dt} \\ & + \left(c_v \sin \theta_e \left(-\frac{L_{ds}M_q}{M_d} + \frac{M_dM_q}{L_r} \right) + d_v \cos \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) \right) \frac{di_{qs}^e}{dt} = 0 \end{aligned} \quad (18)$$

From equation (18), we have:

$$\begin{aligned} a_v \sin \theta_e \left(-\frac{L_{ds}M_q}{M_d} + \frac{M_dM_q}{L_r} \right) + b_v \cos \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) &= 0 \\ c_v \cos \theta_e \left(\frac{L_{ds}M_q}{M_d} - \frac{M_dM_q}{L_r} \right) + d_v \sin \theta_e \left(L_{qs} - \frac{M_q^2}{L_r} \right) &= 0 \end{aligned} \quad (19)$$

Therefore, the expressions for a_v , b_v , c_v and d_v can be obtained as follows:

$$a_v = -b_v Z_5 \cot \theta_e, \quad c_v = d_v Z_5 \tan \theta_e \quad (20)$$

where:

$$Z_5 = \left(\frac{L_{qs} - \frac{M_q^2}{L_r}}{-\frac{L_{ds}M_q}{M_d} + \frac{M_dM_q}{L_r}} \right) \quad (21)$$

With supposition a_v and c_v as follows:

$$a_v = -\cos \theta_e, \quad c_v = \sin \theta_e \quad (22)$$

b_v and d_v is obtained as:

$$\begin{aligned} b_v &= \left(\frac{-L_{ds}L_rM_q + M_d^2M_q}{L_{qs}L_rM_d - M_dM_q^2} \right) \sin \theta_e \\ d_v &= \left(\frac{-L_{ds}L_rM_q + M_d^2M_q}{L_{qs}L_rM_d - M_dM_q^2} \right) \cos \theta_e \end{aligned} \quad (23)$$

Subsequently, the proposed rotational transformation for stator voltage variables is obtained as follows:

$$\begin{bmatrix} T_{vs}^e \end{bmatrix} = \begin{bmatrix} -\cos \theta_e & \left(\frac{-L_{ds}L_rM_q + M_d^2M_q}{L_{qs}L_rM_d - M_dM_q^2} \right) \sin \theta_e \\ \sin \theta_e & \left(\frac{-L_{ds}L_rM_q + M_d^2M_q}{L_{qs}L_rM_d - M_dM_q^2} \right) \cos \theta_e \end{bmatrix} \quad (24)$$

By considering of equation (11), the value of Z_3 and Z_4 can be written as follows:

$$Z_3 = \left(\frac{-L_{ds}L_rM_q + M_d^2M_q}{L_{qs}L_rM_d - M_dM_q^2} \right), \quad Z_4 = 1 \quad (25)$$

IV. EQUATIONS OF RFOC FOR SINGLE-PHASE IM

In RFOC method, the rotor flux vector is aligned with d-axis as:

$$\lambda_{dr}^e = |\lambda_r^e|, \quad \lambda_{qr}^e = 0 \quad (26)$$

Based on this condition and equations (13) and (15), the equations of RFOC can be shown as follows:

$$|\lambda_r^e| = \frac{M_q i_{ds}^e}{1 + T_r \frac{d}{dt}}, \quad T_r (\omega_e - \omega_r) |\lambda_r^e| - M_q i_{qs}^e = 0 \quad (27)$$

$$\tau_e = \frac{Pole}{2} \left(\frac{M_q}{L_r} \right) (|\lambda_r^e| i_{qs}^e)$$

In equation (27), T_r is rotor time constant. By using equations (11), (17), (25) and (26), and considering of $L_{ds}/L_{qs} = (M_d/M_q)^2$ [10, 12, 18] stator voltage equation can be simplified as (28)-(31).

$$v_{ds}^{e*} = v_{ds}^{d*} + v_{ds}^{ref*} \quad (28)$$

$$v_{qs}^{e*} = v_{qs}^{d*} + v_{qs}^{ref*}$$

where:

$$\begin{aligned} v_{ds}^{ref*} &= \left(\frac{r_{ds}M_q^2 + r_{qs}M_d^2}{2M_d^2} \right) i_{ds}^e + \left(L_{qs} - \frac{M_q^2}{L_r} \right) \frac{di_{ds}^e}{dt} \\ v_{qs}^{ref*} &= \left(\frac{r_{ds}M_q^2 + r_{qs}M_d^2}{2M_d^2} \right) i_{qs}^e + \left(L_{qs} - \frac{M_q^2}{L_r} \right) \frac{di_{qs}^e}{dt} \end{aligned} \quad (29)$$

$$\begin{aligned} v_{ds}^{d*} &= -\omega_e i_{qs}^e \left(L_{qs} - \frac{M_q^2}{L_r} \right) + \left(\frac{M_q}{L_r} \right) \left(\frac{M_q i_{ds}^e - |\lambda_r^e|}{T_r} \right) \\ v_{ds}^{d*} &= -\omega_e i_{qs}^e \left(L_{qs} - \frac{M_q^2}{L_r} \right) + \left(\frac{M_q}{L_r} \right) \left(\frac{M_q i_{ds}^e - |\lambda_r^e|}{T_r} \right) \\ &+ \left(\frac{r_{ds}M_q^2 - r_{qs}M_d^2}{2M_d^2} \right) \left(\cos 2\theta_e i_{ds}^e - \sin 2\theta_e i_{qs}^e \right) \end{aligned} \quad (30)$$

$$v_{qs}^{d*} = \omega_e i_{ds}^e \left(L_{qs} - \frac{M_q^2}{L_r} \right) + \omega_e M_q \frac{|\lambda_r^e|}{L_r} + \left(\frac{r_{ds}M_q^2 - r_{qs}M_d^2}{2M_d^2} \right) \left(-\sin 2\theta_e i_{ds}^e - \cos 2\theta_e i_{qs}^e \right) \quad (31)$$

$$v_{ds}^{d*} = \omega_e i_{qs}^e \left(L_{qs} - \frac{M_q^2}{L_r} \right) + \omega_e M_q \frac{|\lambda_r^e|}{L_r} + \left(\frac{r_{ds}M_q^2 - r_{qs}M_d^2}{2M_d^2} \right) \left(-\sin 2\theta_e i_{ds}^e - \cos 2\theta_e i_{qs}^e \right)$$

$$v_{qs}^{d*} = \omega_e i_{ds}^e \left(L_{qs} - \frac{M_q^2}{L_r} \right) + \omega_e M_q \frac{|\lambda_r^e|}{L_r} + \left(\frac{r_{ds}M_q^2 - r_{qs}M_d^2}{2M_d^2} \right) \left(-\sin 2\theta_e i_{ds}^e - \cos 2\theta_e i_{qs}^e \right)$$

In (28), v_{ds}^{ref*} and v_{qs}^{ref*} is generated by PI controller and v_{ds}^{d*} and v_{qs}^{d*} is generated by the Decoupling Circuit (see Fig. 4). Based on equations (27)-(31), Fig. 4 shows the proposed RFOC of a single-phase IM.

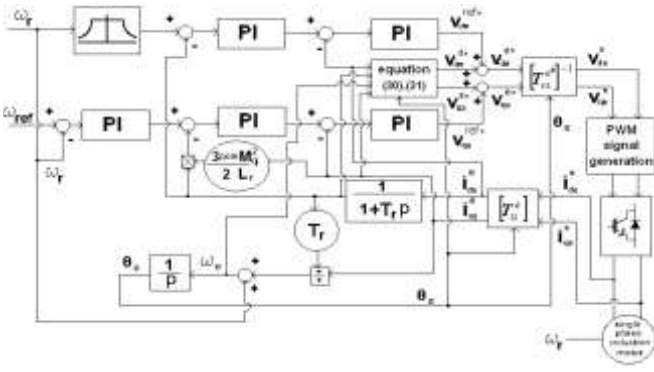


Fig. 4. Block diagram of proposed RFOC for single-phase IM

By considering of $L_{ds} / L_{qs} = (M_d / M_q)^2$, equation (24) can be simplified as (32).

$$\begin{bmatrix} T_{vs}^* \\ T_{vs}^* \end{bmatrix} = \begin{bmatrix} -\cos \theta_e & -\frac{M_d}{M_q} \sin \theta_e \\ \sin \theta_e & -\frac{M_d}{M_q} \cos \theta_e \end{bmatrix} \quad (32)$$

Equation (18) is the general form of the transformation matrix for the stator voltage variables. Comparing this with (32), it can be deduced that $Z_3 = -M_d / M_q$ and $Z_4 = 1$. In [10, 12, 18-21] the values of Z_3 and Z_4 are 1 and $-M_q / M_d$ respectively. In fact, by choosing any value of the Z_3 and Z_4 new transformation matrices can be obtained. A comparison between the steady state speed response of the RFOC with and without considering $(M_d / M_q)^2 = L_{ds} / L_{qs}$ is shown in Fig. 5. Small magnitude of oscillations at the nominal reference speed can be seen in the speed response when the assumption $(M_d / M_q)^2 = L_{ds} / L_{qs}$ is used.

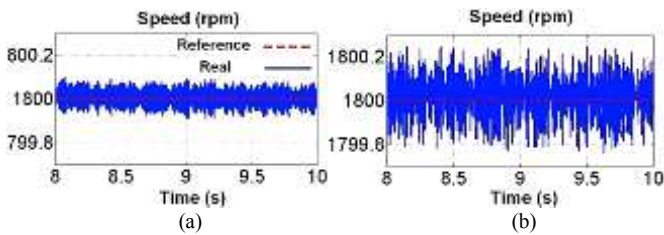


Fig. 5. Simulation results of comparison between speed response in single-phase IM (a) not assuming $(M_d / M_q)^2 = L_{ds} / L_{qs}$, (b) assuming $(M_d / M_q)^2 = L_{ds} / L_{qs}$

V. SIMULATION RESULTS

In this section, the simulation results obtained from the RFOC of single-phase IM shown in Fig. 4 are presented. A Pulse Width Modulation (PWM) two-leg Voltage Source Inverter (VSI) feeds the single-phase IM, as it was shown in Fig. 1. The parameters of simulated single-phase IM are as: Voltage: 110V, $f=60\text{Hz}$, No. of poles=4, $r_{ds}=7.14\Omega$, $r_r=4.12\Omega$, $r_{qs}=2.02\Omega$, $L_r=0.1826\text{H}$, $L_{ds}=0.1885\text{H}$, $L_{qs}=0.1844\text{H}$, $M_q=0.1772\text{H}$, $J=0.0146\text{kg.m}^2$. Fig. 6 (a) shows simulation results of the reference and actual rotor speed using the proposed controller, when the speed reference changes from zero to the nominal value of 1800 rpm. In this figure a load torque of 0.9 Nm is applied at $t = 8\text{ s}$. Fig. 6 (b) shows the electromagnetic torque

and in Fig. 6 (c) the stator main and auxiliary currents are shown. It is evident from Fig. 6 that the single-phase IM can follow the reference speed even after applying load torque without any overshoot and steady-state error.

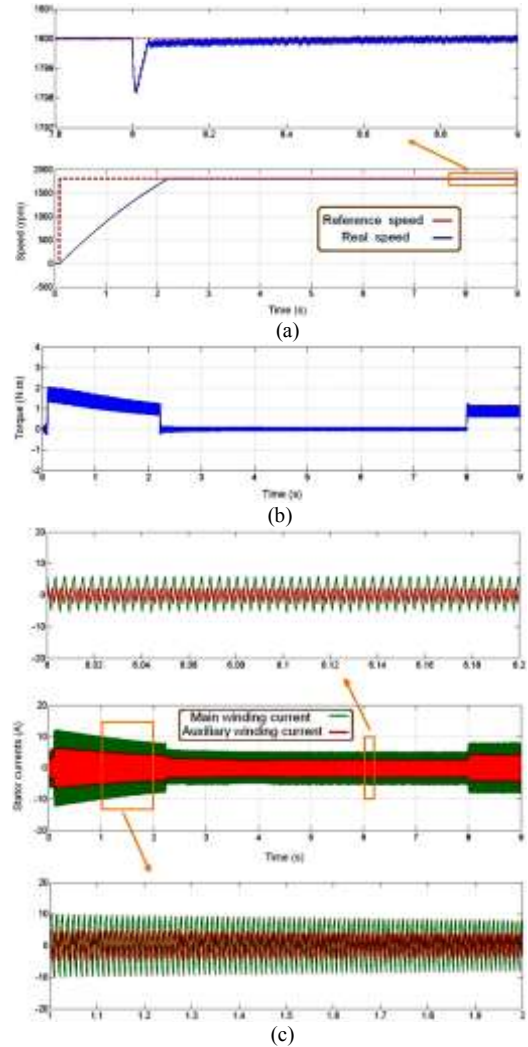
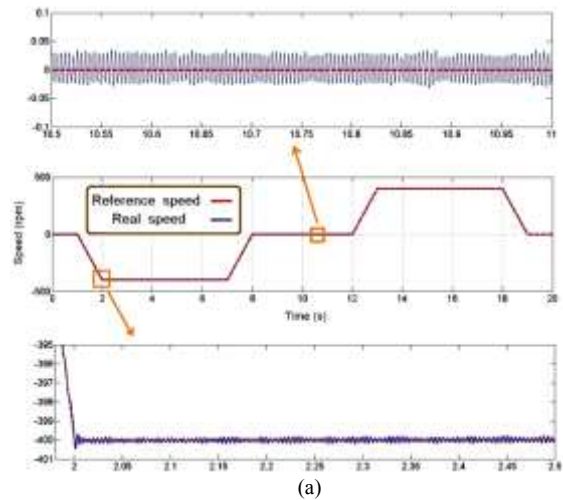


Fig. 6. Simulation results of RFOC for single-phase IM at nominal speed; (a) Speed, (b) Torque, (c) Stator currents



(a)

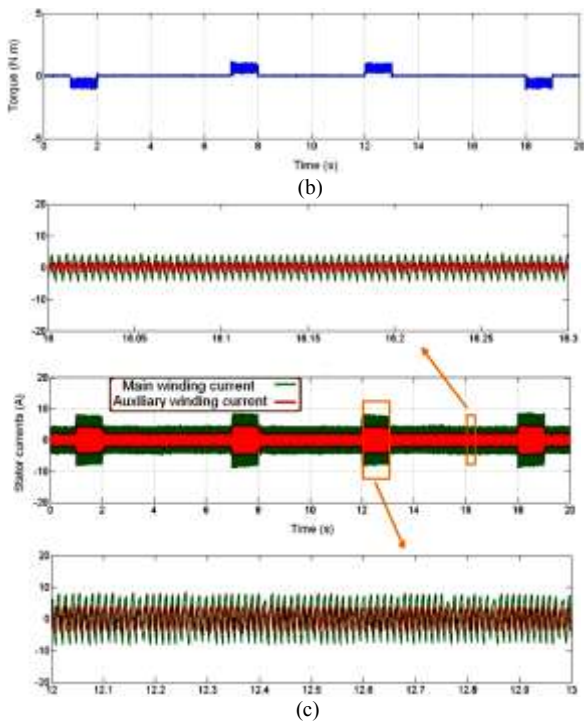


Fig. 7. Simulation results of RFOC for single-phase IM for a trapezoidal and zero reference speed; (a) Speed, (b) Torque, (c) Stator currents

Fig. 7 displays simulation results of vector control for a trapezoidal and zero reference speed. Fig. 8 shows the dynamic behavior of the presented method in the difference values of reference speed. Figs. 6–8 show that the system can operate stably at high, zero and low speed. It can be seen from the simulation results that the dynamic performance of the system is highly satisfactory.

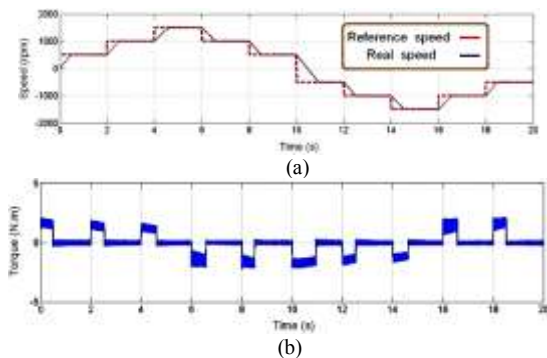


Fig. 8. Simulation results of RFOC for single-phase IM for different values of speed; (a) Speed, (b) Torque

VI. CONCLUSION

This paper presents a new method for the control of a single-phase IM, based on RFOC. The theory and analysis for the vector control technique based on using unbalanced rotational transformations is presented. This method can also be used for vector control of the unbalanced IMs or vector control of the IMs under one or two open-phase fault. The simulation results are presented to show the capability of the presented method in the speed control of single-phase IM at different values of speed.

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