# A PARTIAL POWER GRADIENT AT COUPLING REGION FOR COUPLED WAVEGUIDE FIBER 

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#### Abstract

Coupled fibers are successfully fabricated by injecting hydrogen flow at 1 bar and heating torch flame in the range of 800-1350C. During the fusion process some optical parameters are not clear due to vary. For empirical and theoretical calculation, coupling coefficient and refractive index have been estimated from experimental result of coupling ratio distribution from $1 \%$ until $75 \%$. The change in structural and geometrical of fibers affects normalized frequency even for single mode fiber. Coupling ratio as the function of coupling coefficient and separation of fiber axis also changes the normalized frequency at coupling region. The normalized frequency is derived from the radius, the wavelength and the refractive index parameters. Parametric variations are performed on the left and right hand side of the coupling region. At the center of the coupling region, coupling length splits the power to another fiber where the normalized frequency is assumed to be constant. A partial power is modeled and derived using normalized frequency ( $V$ ), normalized lateral phase constant ( $u$ ), and normalized lateral attenuation constant, $(w)$ through the second form of modified Bessel function of the $l$ order, which obeys the normal mode, $L P_{01}$ and normalized propagation constant (b). Total power is maintained constant in order to comply with the energy conservation law. The partial power gradient affected by $V$, $u$ and $w$ are integrated along $z$ direction coupling region. The model is solved over the pulling length in the range of $7500-9500 \mu \mathrm{~m}$ for 1-D where the radial and angle directions were ignored for a scalar magnitude. The core radius of fiber significantly affects normalized frequency and power partially at coupling region rather than wavelength and refractive index of core and cladding. This model can be compared to application of power transmission and reflection of coupled fibers in industrial application.


Keywords: single mode fiber, coupling ratio, coupling coefficient, normalized frequency, power

## 1. Introduction

Although the coupling ratio research has shown good progress in the experimental and theoretical calculation; coupled waveguide fibers still have power reflection and power losses due to effects of fabrication. Coupling fiber fabrications do not only consider a source and waveguide but also involve some parametric function that emerges along the process when information transfer to fibers occurs ${ }^{1}$. This resulted in a complicated problem, particularly at the junction as the electric field and power are affected by the waveguide, the structure and the geometry of the fiber itself. Nowadays, developments of
those are investigated to obtain good resolutions in transmitting power after passing the junction. Coupled fiber using some junctions at near field distances has more effect of reflection to communication. The loss of transmission power is significant especially in delivering the power ratio. One of the main phenomena occurring to the optical couplers, as coupling of mode in space ${ }^{2}$ which contributes to power propagation along the coupled fiber is coupling coefficient. Coupling coefficient can be expressed as an effective power range that transmits from one fiber to another ${ }^{3}$. The separation between the two fibers is significant in coupling coefficient as it determines the effective power transmission from one fiber.

A fusion process changed the structures and geometries of coupled fibers at the coupling region. These changes are complicated as the refractive indices and fiber geometries are made uncertain due to the coupling ratio effect ${ }^{4}$. However, they tend to decrease along the fibers from one edge to the center of the coupling region and again increase to the other. It also occurs to the wave and power propagation partially at the propagation direction but total power obeys law of energy conservation. Power transmission depends on a distribution of coupling ratio having a fractional power which mainly occurs at the coupling region. The coupling region itself has three regions based on the core and cladding geometry which is situated at the left, center and right. At the center of the coupling region, is where the main coupling occurs as the power propagation splits from one core to another through the cladding.

The waveguide carrying electric field is a single mode fiber (SMF) which is coupled by two fibers with the same geometry 1X2 splitting one source to become two transmission lines as Y junction. The structure is assumed to be homogeneous, isotropic materials and with very small gradient of refractive index along the propagation. The fibers are approximately heated with a slightly unstable torch within a temperature range of $800-1350 \mathrm{C}$. A laser diode source $\lambda=1310 \mathrm{~nm}$ is used to guide a complete power transfer in a distance of $z$. The coupling ratio set cannot determine that the cladding diameter is constant even though the $L P_{01}$ diameter position has been achieved. It is of course, the decrease of the refractive index at the junction fibers is due to effects of fiber structure and geometry by pulling them at a coupling region, while the 2 cores distance is closer than the radius of those two claddings ${ }^{5}$. The SMF- $28 \mathrm{e}^{\circledR}$ core after fusion is reduced from $80.5 \%$ to $94 \%{ }^{6}$. A half distance of pulling length of fiber coupler increases significantly over the coupling ratio. The coupling length increases over coupling ratio due to the longer time taken at the coupling region by a few ms to reach a complete coupling power.

In obtaining a good coupling ratio, the experimental result should meet the power transmits at the coupling region with a larger coupling length than the coupling. In the range of $0.6-0.9 / \mathrm{mm}^{6}$ the coupling coefficient exists along it as the coupling ratio increases. It is assumed that the power transmission and coupling occurs during the fusion and fluctuating some parameters such as twisting fibers, fibers heating, and refractive index changes ${ }^{6,7}$ which cannot be controlled easily in measurement. It was experimentally
measured to be in the range of $0.9-0.6 / \mathrm{mm}$. This corresponds to the determination of refractive index by the empirical equation of the core and cladding which is $n_{l}=1.4640$ 1.4623 and $n_{2}=1.4577-1.4556$ respectively for coupling ratio of $1-75 \%{ }^{6}$. The separation of fibers between the two cores was obtained at a mean value of $10-10.86 \mu \mathrm{~m}{ }^{6}$. It may be expressed that the empirical calculation to reach the coupling ratio can be detected and imposed by power.

The power propagates along SMF-28e ${ }^{\circledR}$. It guides only one mode which can be seen from the normalized frequency. Normalized frequency depends on the core radius, wavelength, and the core and cladding refractive index. To investigate the coupling region in the range of coupling ratio, the power is simply derived and modeled. The power change and its dependence on normalized frequency parameters were studied. This paper describes power gradient as computed from coupling coefficient range and coupling region data which is experimentally obtained from the coupling ratio distribution.

## 2. Partial Power Gradient

SMF has dominant mode, $L P_{0 I}$ with $V=2.405$. When coupled fibers are being fused and pulled, the value changes depending on the wavelength source and material of fibers. At coupling region the changes of some optical parameters are due to the structural and geometrical properties of the fibers. Fiber sizes are decreased and increased at the left and right coupling region. At the center of the coupling region they are assumed to be constant. Consider the pulling length of fibers is as follows,

$$
P_{L}=P L_{1}+P L_{2}+P L_{3}
$$

where $P L_{1}=P L_{3}$ and $P L_{2}=C_{L}$ ( $C_{L}$ is coupling length).
Power propagation $(P)$ along coupling region can be reflected and transmitted as a normalized frequency; where total power input and output must however be conservative. Total scalar power can be defined as follows ${ }^{8}$ :

$$
\begin{equation*}
P=C \pi a^{2}\left(V^{2} / u^{2}\right)\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right] \tag{2.1}
\end{equation*}
$$

where $C$ is constant, $a$ is core radius, $u$ is normalized lateral phase constant, $w$ is the normalized lateral attenuation constant, $K$ is the second kind of modified Bessel function of order $l$. For a $k$ range species of coupling region, total power can be written as a sum of partial power,

$$
P=\sum_{k=(+, 0,-)}^{3} P_{k}
$$

The value of $\nabla \sum_{k=(+, 0,-)}^{3} P_{k}$ is a gradient function of $\nabla P=\nabla P(a, V, u, w)$, resulting in a set of equation in $z$ direction,

$$
\begin{align*}
& \nabla \sum_{k=(+, 0,-)}^{3} P_{k}=\nabla\left\{C \pi a^{2}\left(V^{2} / u^{2}\right)\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \\
& =\left\{\begin{array}{c}
\left\{C \pi a^{2}\left(V^{2} / u^{2}\right) \frac{\partial}{\partial z}\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \\
+\left\{C \pi a^{2}\left(V^{2}\right) \quad\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \frac{\partial}{\partial z}\left(1 / u^{2}\right) \\
+\left\{C \pi a^{2}\left(1 / u^{2}\right)\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \frac{\partial}{\partial z}\left(V^{2}\right)
\end{array}\right\} \\
& \left.+\left\{C \pi\left(V^{2} / u^{2}\right)\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \frac{\partial}{\partial z}\left(a^{2}\right)\right] \\
& =\left\{C \pi\left[a^{2} V^{2} \frac{\partial}{\partial z}\left(1 / u^{2}\right)+\left(a^{2} / u^{2}\right) \frac{\partial}{\partial z}\left(V^{2}\right)+\left(V^{2} / u^{2}\right) \frac{\partial}{\partial z}\left(a^{2}\right)\right]\right\} \\
& x\left\{\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right\}+\left\{C \pi a^{2}\left(V^{2} / u^{2}\right) \frac{\partial}{\partial z}\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\} \tag{2.2}
\end{align*}
$$

For simplicity, the first, second and third term of the Eq.(2.2) be respectively noted as the following,

$$
\begin{equation*}
\nabla \sum_{k=(+, 0,-)}^{3} P_{k}=\{[A] x[B]\}-[C] \tag{2.3}
\end{equation*}
$$

Firstly, consider $\{[A] x[B]\}$ as a function of $u, V$, and $a$, where

$$
\left.\left.\begin{array}{l}
u^{2} \equiv\left(k^{2} n_{1}^{2}-\beta_{l m}^{2}\right) a^{2} \\
w^{2} \equiv\left(\beta_{l m^{2}}{ }^{2}-k^{2} n_{2}^{2}\right) a^{2} ; \quad \beta_{1}=k n_{1} ; \beta_{2}=k n_{2}  \tag{2.4}\\
V
\end{array}\right\}\left(u^{2}+w^{2}\right)^{1 / 2}=(2 \pi a / \lambda)\left(n_{1}^{2}-n_{2}^{2}\right)^{1 / 2}\right\}
$$

where $l$ and $m$ are the number of mode, $\beta$ is the propagation constant and $k$ is the wave number. The left hand side of Eq.(2.4) have parametric values dependent on the values of $u=u\left(a, k, n_{l}, \beta_{l m}\right), w=w\left(a, k, n_{2}, \beta_{l m}\right)$, and $V=V\left(a, n_{1}, n_{2}, \lambda\right)^{8}$. The value of $\beta_{l m}$ is calculated from the normalized propagation constant $b$, which is equal to $\left(\beta^{2}{ }_{l m^{-}} \beta_{2}\right) /\left(\beta_{1}-\beta_{2}\right)$. Since $w$ is a part of $K$ function, then it can be derived by the $K$ function itself. Evaluating these functions separately over $z$ direction we find,

$$
\begin{align*}
\nabla u= & {\left[\left(a k^{2} n_{1}^{2} d a / d z+k a^{2} n_{1}^{2} d k / d z+n_{1} k^{2} a^{2} d n_{1} / d z\right)-\left(a \beta_{l m}^{2} d a / d z+\beta_{l m} a^{2} d \beta_{l m} / d z\right)\right] /(u) } \\
\nabla \beta_{l m}= & {\left[\beta_{2}\left(d \beta_{2} / d z\right)+b_{l m}\left(\beta_{1} d \beta_{l} / d z-\beta_{2} d \beta_{2} / d z\right)\right] /\left(\beta_{l m}\right) } \\
\nabla V= & 2\left\{(\pi / \lambda)\left(n_{l}^{2}-n_{2}^{2}\right)^{1 / 2} d a / d z+\pi a\left(n_{1}^{2}-n_{2}^{2}\right)^{1 / 2}[d(1 / \lambda) / d z] d \lambda / d z\right.  \tag{2.5}\\
& \left.+(2 \pi a / \lambda)\left[1 / 2\left(n_{1}^{2}-n_{2}^{2}\right)^{-1 / 2}\right]\left(n_{1} d n_{1} / d z-n_{2} d n_{2} / d z\right)\right\}
\end{align*}
$$

where $d b_{l m} / d z$ is expected to be zero, and thus can be ignored. The first and second terms of Eq.(2.3) can be rewritten by combining Eq.(2.5) as follows:

$$
\begin{align*}
\{[A] x[B]\}= & \left\{2 C \pi\left[-u^{-3} a^{2} V^{2} \frac{\partial u}{\partial z}+V\left(a^{2} / u^{2}\right) \frac{\partial V}{\partial z}+a\left(V^{2} / u^{2}\right) \frac{\partial a}{\partial z}\right]\right\} \\
& x\left\{\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right\} \tag{2.6}
\end{align*}
$$

and, let $K$ function be derived by the first order resulting in,

$$
\begin{aligned}
{[C] } & =C \pi a^{2}\left(V^{2} / u^{2}\right) \\
& x_{l}\left[\frac{\left[\left(K_{l-1}(w)\right)^{\prime} K_{l+1}(w)+K_{l-1}(w)\left(K_{l+1}(w)\right)^{\prime}\right]\left[K_{l}^{2}(w)\right]-\left[2 K_{l}(w)\left(K_{l}(w)\right)^{\prime} K_{l-1}(w) K_{l+1}(w)\right]}{\left(K_{l}^{2}(w)\right)^{2}}\right) \\
& =C \pi a^{2}\left(V^{2} / u^{2}\right)
\end{aligned}
$$

$$
\begin{equation*}
\left.x_{l} \frac{\left[\left(K_{l-1}(w)\right)^{\prime} K_{l+1}(w)+K_{l-1}(w)\left(K_{l+1}(w)\right)^{\prime}\right]\left[K_{l}(w)\right]-\left[2\left(K_{l}(w)\right)^{\prime} K_{l-1}(w) K_{l+1}(w)\right]}{\left(K_{l}(w)\right)}\right\} \tag{2.7}
\end{equation*}
$$

Eq.(2.6) and (2.7) are then combined to have a solution of Eq.(2.3). In order to obtain a complete solution, the second kind of modified Bessel function of order $l$ is substituted by a recurrence relation for a given function as

$$
K_{n}^{\prime}(x)=\frac{n}{x} K_{n}(x)-K_{n+1}(x)
$$

Then it is finally given by,

$$
\begin{aligned}
& \nabla \sum_{k=(+, 0,-)}^{3} P_{k}=\left[2 C \pi\left\{-u^{-3} a^{2} V^{2} \frac{\partial u}{\partial z}+V\left(a^{2} / u^{2}\right) \frac{\partial V}{\partial z}+a\left(V^{2} / u^{2}\right) \frac{\partial a}{\partial z}\right\}^{\text {(I) }}\right. \\
& x\left\{\frac{{\frac{K_{l-1}}{}(w) K_{l+1}(w)}_{K_{l}^{2}(w)}^{K_{l}}}{\}]-\left[C \pi a^{2}\left(V^{2} / u^{2}\right) x\right.}\right.
\end{aligned}
$$

$$
\begin{align*}
& \text { (VIII) } \\
& \left.\left.-\left[2\left\{\frac{l}{w} K_{l}(w)-K_{l+1}(w)\right\} K_{l-1}(w) K_{l+1}(w)\right] \quad\right\}\right] \tag{2.8}
\end{align*}
$$

The Eq.(2.8) can be computed by setting a number of known parameters and evaluated within the boundary conditions of coupling region as set in Eq.(2.3). Since the total power obeys the conservation law, then $\nabla P=0$, it can also be applied for each $k$ region,

$$
\nabla \sum_{k=(+, 0,-)}^{3} P_{k}=\left.\nabla P\right|_{+}+\left.\nabla P\right|_{0}+\left.\nabla P\right|_{-}=0
$$

$$
\nabla \sum_{k=(+, 0,-)}^{3} P_{k}=\nabla\left\{C \pi a^{2}\left(V^{2} / u^{2}\right)\left[\frac{K_{l-1}(w) K_{l+1}(w)}{K_{l}^{2}(w)}\right]\right\}
$$

The subscript of $k$ is (+), ( 0 ) or (-) and then $\left.\nabla P\right|_{+}$corresponds to positive gradient where the radius of fibers is decreased and negative gradient when the radius of fibers is increased at $\left.\nabla P\right|_{-}$and $P L_{2}$, it is assumed that $\left.\nabla P\right|_{0} \approx 0$. For a simplified partial power model where the fibers are imposed by setting a temperature and the change of fiber properties as homogenous, then at $P L_{l}$ it is considered that the value of $a$ linearly changes as same as $n_{1}$ and $n_{2}$ towards the temperature. Meanwhile, the wavelength linearly depends upon $n_{1}$ and $n_{2}$. These parameter changes are the same at $P L_{3}$ but with the opposite sign. Therefore, the total power is constant, but the partial power is not zero. It can be written as the following,

$$
\left.\nabla P\right|_{+} \neq 0,\left.\nabla P\right|_{-} \neq 0, \text { but for }\left.\nabla P\right|_{0} \approx 0
$$

For the range of coupling region where $P$ will be calculated, and to correct $\frac{d P}{d z}$ for effect of change in fibers geometry, Eq.(2.8) can be derived and fix to be a constant value.

Suppose total and derivation of $P$ can be rewritten by

$$
\begin{align*}
P & =\sum_{k=(+, 0,-)}^{3} P_{k} \\
\frac{d P}{d z} & =\nabla \sum_{k=(+, 0,-)}^{3} P_{k} \tag{2.9}
\end{align*}
$$

Where total $\nabla P$ is not constant, hence

$$
\frac{1}{\nabla P} \frac{d P}{d z}=\frac{1}{z}
$$

where $z$ is the direction. Multiplying both sides with $P_{k}$ and $\frac{1}{\nabla P}$ for normalization of $\nabla P$, the Eq.(2.9) becomes

$$
\begin{equation*}
\frac{P_{k}}{\nabla P} \frac{d P}{d z}=P_{k}\left[\sum_{k=(+, 0,-)}^{3} \frac{1}{\nabla P}\right] \frac{d}{d z}\left(P_{+}+P_{o}+P_{-}\right) \tag{2.10}
\end{equation*}
$$

In order to keep total $\nabla P$ constant, we combine the two terms of Eq.(2.9) and (2.10) for $P_{k}$ obtaining,

$$
\begin{equation*}
\left(\frac{d P_{k}}{d z}\right)_{\text {correction }}=\left(\frac{d P}{d z}\right)-\frac{P_{k}}{\nabla P} \frac{d P}{d z} \tag{2.11}
\end{equation*}
$$

This formula expresses that during the power propagation at the coupling region, the total $\nabla P$ is constant even though $P_{k}$ change. For illustration, this model can then be depicted in Figure 1.

Torch Flame


Figure 1. SMF-28e ${ }^{\circledR}$ coupler fiber is heated by $\mathrm{H}_{2}$ gas at the temperature of $800-1350 \mathrm{C}$. The core and cladding reduce $75-90 \%$ in size after fusion. Total pulling of fibers to the left and right side is in the range of 7500 $9500 \mu \mathrm{~m}$ with velocity $\approx 100 \mu \mathrm{~m} / \mathrm{s}$. Pulling is stopped subject to the coupling ratio achieving a setting value.

## 3. Integration of Power and Discussion

The values of $P$ partially change at the coupling region are integrated over $z$ direction of core radius and a half pulling length. It is run in Ode45 Matlab platform with a set of
input data for refractive index of core and cladding, wave length and initial $P$. For the given values of Eq.(2.4), it shows that at $P L_{l}$, the result is as follows:

$$
\begin{align*}
&\left.\nabla(a)\right|_{+}=1044.3864 \text { to } 796.8127 \times 10^{-6}, \\
&\left.\nabla(\lambda)\right|_{+\mathrm{n} 1}=-0.0006542 \text { to }-0.0010101 \times 10^{-9}, \\
&\left.\nabla(\lambda)\right|_{\mathrm{n} 2}=-0.0008376 \text { to }-0.0012823 \times 10^{-9}, \\
&\left.\nabla\left(n_{1}\right)\right|_{+}=1.05 \text { to } 1.65 \times 10^{-6}, \\
&\left.\nabla\left(n_{2}\right)\right|_{+}=1.35 \text { to } 2.05 \times 10^{-6},  \tag{2.12}\\
& d a / d z=7.9681 \text { to } 9.0039 \times 10^{-4}, d k / d z=2.3952 \text { to } 3.6983, \\
& d n_{l} / d z=1.05 \text { to } 1.65 \times 10^{-6}, \quad d n_{2} / d z=1.35 \text { to } 2.05 \times 10^{-6} \\
& d \beta_{l} / d z=8.5516 \text { to } 13.3419, \quad d \beta_{2} / d z=9.9779 \text { to } 15.2409, \\
& d \beta_{l m} / d z=9.2064 \text { to } 14.2137, \\
& \nabla u=322.5195 \text { to } 364.4422, \\
& \nabla V=475.5291 \text { to } 537.3407
\end{align*}
$$

These parametric values are result out of the coupling ratio in the range of 1 to $75 \%$. It has a function of coupling coefficient and produces the parametric values gradients existing in that number range. The value of $\lambda_{o} / \lambda=n$ moves to decrease along $P L_{l}$ until it meets the coupling length and inversely increases along $P L_{3}$. The Eq.(2.11) is similar to $P L_{3}$ but the gradient is in the opposite sign.

The graph of $\nabla P$ at $P L_{I}$, as calculated from Eq.(2.11) is the power gradient at the first and end of the coupling region as depicted in Figure 2.


Figure 2. $V P$ along coupling region; $\nabla P=1.18$ at first of $P L_{I}$ and 1.26 at end of $P L_{I}$ at coupling region $\mathrm{z}=3.75 \times 10^{-3} \mathrm{~m}$

Comparing the two curves, it shows that the change of partial $P$ is less than that shown at the end of $P L_{l}$. It shows that end of $P L_{l}$ has decreased smoothly due to the fact of pulling length and heating of fibers at end of $P L_{l}$. It also meant that the higher gradient to reach coupling length results in the more reflection of power to the source fiber and crosstalk fiber due to the refractive indices gradient and more loss of power from the core to the cladding and to the edge cladding due to the radius gradient. This result has the same values for $P L_{3}$ but in the negative gradient.

If we assume that partial $\nabla P$ is not linear or rather is exponentially decreasing or increasing. If it is affected by the function factor $P=P\left(1+e^{-\alpha}\right)$ then the radius geometry is not proportional to the speed of pulling length and if by the function factor $P=P\left(1-e^{-\alpha}\right)$ then it means that the fibers are not precisely heated at the center of fibers and the gradient of refractive indices will be close to factor $10^{-3}$. However, these reasons are negligible, since the mechanical process of the fabrication is fixed and the radius change is much more significant than the other parameters.

Table 1. Calculation of partial $\nabla P$ in each term of Eq.(2.8)

| No. | Calculation Result | Term |
| :---: | :---: | :---: |
| 1 | $(0.2857-0.0546 \mathrm{i})$ to $(0.3229-0.006171 \mathrm{i})$ | I |
| 2 | $(0.9899+0.5456 \mathrm{i})$ | II |
| 3 | $\left(-7.3511 \times 10^{-4}+1.45057 \times 10^{-4} \mathrm{i}\right)$ | III |
| 4 | $(0.1154-0.1875 \mathrm{i})$ | IV,VII |
| 5 | $(-0.3056-0.2007 \mathrm{i})$ | V,VII |
| 6 | $(0.0177+0.6021 \mathrm{i})$ | VI |
| 7 | $(0.1154-0.1876 \mathrm{i})$ | V,VI,VII |
| 8 | $(-0.2169+0.4334 \mathrm{i})$ | VIII |
| 9 | $(0.3126+0.10181 \mathrm{i})$ to $(0.3533+0.1150 \mathrm{i})$ | I and II |
| 10 | $\left(-1.847 \times 10^{-5}-4.0113 \times 10^{-5} \mathrm{i}\right)$ | III to VIII |
| 11 | $(0.3126+0.1018 \mathrm{i})$ to $(0.3533+0.1150 \mathrm{i})$ | I to VIII |

Based on Table I, the results are significantly affected by multiplication of term I and II by factor of $10^{-1}$ rather than multiplication of term III until term VIII. Before being derived, term II is comparable towards term I in contributing the power. In fact, the order of $l$ deserves to balance of term I, but term III is too high factor by the order of $10^{-4}$, then the effect of power gradient is seemingly contributed by term I. The main influence of term III is the value of core radius by factor of $a^{2}$ which similarly occurs in term I. However, since term I is a summation operation then it disappears. Therefore, partial $\nabla P$ is reduced by the value of $a$ and $\nabla P$ is otherwise increased by $K$ function of $l$ order in term II. In other word, in summation operation, $K$ function is dominant but in reduction operation, the value of $a$ is significant. The partial power gradient at $P L_{1}$ and $P L_{3}$ make parametric
changes to reduce or to add power significantly along coupling region. This calculation can also be seen in Figure 3.

As shown by the straight lines in Figure 3, when power gradient is integrated, it describes the first $P L_{l}$ as higher than that of end $P L_{l}$. The left coupling region is set at $\mathrm{z}=0$ and lets the power curves move from $P$ input to the output at $3.75 \times 10^{-3} \mathrm{~mm}$. This phenomena expresses the change of each parameter of $P$ is set nearly linear although the actual changes are not obvious. One of the parametric values of $P$ is evaluated linearly assumption and gives a significant dependence in changing to both gradient and integral of $P$ is radius of core by order $10^{-3}$. Refractive indices and wavelength do not necessarily have linear impact since refractive indices and wavelength difference are by the order of $10^{-6}$ and $10^{-9}$ respectively. Therefore the linear effect is maintained to retain the mode at $L P_{01}$. $P$ input value changes at coupling length position from 1 mW to 0.31 mW for one core and 0.62 mW for two cores. Implicitly it explains that the partial power transmission will reduce along the coupling region as the result of refractive indices, core geometry and separation of fiber axis between the cores. This partial power results seem to be very significant, but actually it decreases or increases partially from one core source radiates to its cladding and also to another core and cladding when coupled.


Figure 3 (a)


Figure 3 （b）
Figure 3．$P$ integration over coupling region（power source of one core）；3（a） and Illustration of Power propagation；3（b）

Table 2 describes the details of parametric value changes along the coupling region．A validation of code results is maintained by initial and final $P$ ，while at the coupling region （excluding coupling length）it is assumed to change linearly．

Table 2．Power parameters of coupled SMF－28e ${ }^{\circledR}$

| Parameter | $\nabla$ at first of $P L_{I}$ | $\nabla$ at end of $P L_{l}$ | $\begin{aligned} & \quad \begin{array}{l} \text { at } \\ P L_{2} \\ \hline \end{array} ⿳ ⺈ ⿴ 囗 十 一 ~ \end{aligned}$ | $\nabla$ at first of $P L_{3}$ | $\nabla$ at end of $P L_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | 0 | $3.75 \times 10^{-3}$ | $5.42 \times 10^{-3}$ |  | $7.5 \times 10^{-3} \mathrm{~mm}$ |
| $\lambda$ | 0 to $4.5 \times 10^{-9} \quad(+)$ | 0 to $7 \times 10^{-9} \quad(+)$ | 0 | 0 to $-7 \times 10^{-9} \quad(-)$ | 0 to $-4.5 \times 10^{-9}(-)$ |
| $a$ | 0 to $1.5 \quad(+)$ | 0 to $2 \quad(+)$ | 0 | 0 to－2（－） | 0 to－1．5（－） |
| $n_{1}, n_{2}$ | 0 to $-2.55 \times 10^{-7}(-)$ | 0 to $-3.48 \times 10^{-7}(-)$ | 0 | 0 to $3.48 \times 10^{-7}(+)$ | 0 to $2.55 \times 10^{-7}(+)$ |
| $\beta_{l m}$ | 0 to $0.035 \quad(+)$ | 0 to $0.054 \quad(+)$ | 0 | 0 to $0.054 \quad(-)$ | 0 to 0.035 （－） |
| $u$ | 0 to $1.2 \quad(+)$ | 0 to 1.38 （＋） | 0 | 0 to 1.38 （－） | 0 to 1.2 （－） |
| V | 0 to $1.8 \quad(+)$ | 0 to $2 \quad(+)$ | 0 | 0 to－2（－） | 0 to－1．8（－） |

The initial core and cladding diameter are respectively $8.2 \mu \mathrm{~m}$ and $125 \mu \mathrm{~m}$
$C=6.4032 \times 10^{6}-1.2245 \times 10^{6} \mathrm{i}, P=1 \mathrm{~mW} ; \quad P_{\text {cladding }} / P_{\text {total }}=0.1702, \quad P_{\text {core }} / P_{\text {total }}=0.8298$,
After Fusion $\quad V=V_{2}=0.9761-0.3353 ; \quad n_{l}=1.4623-1.4640 ; n_{2}=1.4556-1.4577$ ；and $a=0.5$ to $1.5 \times 10^{-6} \mathrm{~m}$ （ $V, V_{l}$ and $V_{2}$ values are calculated from refractive indices known．The symbol of $(+$ ）and（－）indicates positive and negative gradient respectively and deal with along each z direction 0 to $3.75 \times 10^{-3} \mathrm{~mm}$ ）．

## 4. Conclusion

Coupling ratio range of 1 to $75 \%$ with the range of coupling coefficient at $0.6-0.9 / \mathrm{mm}$ successfully developed partial power gradient and its integration along the coupling region. Normalized frequency and power gradient give significant parametric changes over power transmission into fiber at coupling region from the power source of one core. The core radius is much more affected to $\nabla P$ rather than the refractive indices and wavelength although they change linearly.

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