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IMAGE RECONSTRUCTION USING ITERATIVE TRANSPOSE ALGORITHM FOR OPTICAL TOMOGRAPHY

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Abstract. This paper describes a transpose algorithm for use with an optical tomography system. The measurement system consisted of two orthogonal arrays, each having ten parallel views, resulting in a total of twenty sensors. The measurement section is divided into hundred equi-sized pixels. The forward problem is modelled by allocating an optical attenuation coefficient to each pixel. The attenuation of incident collimated light beams is then modelled using the Lambert-Beer law. The inverse problem is defined and the transpose of the sensitivity matrix is used to obtain an estimate of the attenuation coefficients in each pixel. The iterative method is investigated as a means of improving reconstructed image quality.

Keywords: Transponse algorithms, optical tomography, bubbles

Abstrak. Kertas kerja ini membincangkan algoritma alih yang digunakan dalam sistem tomografi optik. Sistem pengukuran terdiri daripada dua tatasusunan ortagon berjumlah 20 penderia optik. Masalah depan dimodelkan dengan menentukan pekali pelemahan optik bagi setiap piksel. Kemudiannya prinsip Lambert-Beer digunakan. Bagi masalah kebelakang sensitiviti matrik alih digunakan bagi mendapatkan pekali pelemahan optik bagi setiap piksel.

Kata kunci: Algoritma alih, tomografi optik, buih

1.0 INTRODUCTION

A basic tomography system is constructed by the combination of a sensing system, a data acquisition system, an image reconstruction system and a display unit. A good tomography sensor should have the features such as non-invasive and non-intrusive. It should not necessitate rupture of the walls of the pipeline and do not disturb the nature of the process being examined.

Optical tomography involves the use of non-invasive optical sensors to obtain vital information in order to produce images of the dynamic internal characteristics of process system [1]. It has the advantages of being conceptually straightforward and relatively

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inexpensive [2]. The optical tomography system uses a number of light emitter-receiver pairs and a wide variety of light sources such as visible light, infrared or laser light. Its working principle involves projecting a beam of light through a medium from one boundary point and detecting the level of light received at another boundary point [3]. For the type of projection used in the system, it can be parallel beam (orthogonal) projection, rectilinear projection, fan beam projection or mix projection among them. The corresponding applications are in pneumatic conveying in the industry of food processing, plastic product manufacturing and solids waste treatment. The specific measurements of it are flow concentration, flow velocity and mass flow rate determination.

2.0 GENERAL IMAGE RECONSTRUCTION SYSTEM

A general optical flow imaging system is constructed using an optical sensor, a signal measurement circuit, a data acquisition system, and a computer acting as a data processing unit and a display unit

The image reconstruction algorithm is a technique to process the data acquired from a tomographic imaging system, and depends on the physical principle of the sensing system. To reconstruct images several reconstruction methods such as iterative algorithms, artificial neural network algorithms and the linear back projection (LBP) algorithm have been developed for soft field and hard field tomography techniques [4].

Iterative methods, for example Isaksens's model-based reconstruction (MOR), Reinecke's algebraic reconstruction technique (ART), Yan's using multiple linear regression and regularisation (MLRR) algorithm and Yang's method based on Landweber's iteration method [5] can improve the images [4].

This paper describes a combination of a Transpose algorithm and an iterative algorithm for an optical tomography system. Image reconstruction for an optical tomography system containing water and gas comprises two parts. Firstly the forward problem is modelled to determine the expected measurement values for known attenuation coefficients for water and air and known distributions of gas and liquid. Then problem is solved to obtain the attenuation coefficient distribution from the measurement values; this process is known as image reconstruction.

3.0 THE FORWARD PROBLEM

In the forward problem the aim is to estimate the values of the measurements before the experiment is carried out. In this project air bubbles in the water have to be reconstructed using several algorithms. In the experimental setup the image is to be reconstructed from flow rig results based on two orthogonal projections.

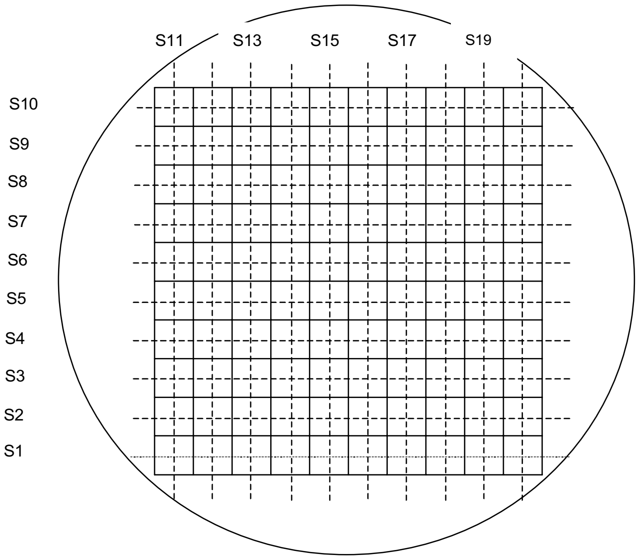

Figure 1 Imaging area inside the pipe

There are 10 optical sensors for each projection and the total number of sensors is 20. The circular pipe is projected onto a square array of 10 × 10 pixels as shown in Figure 1. The incoming light consists of twenty collimated beams, one for each sensor. Each beam passes through the pipe and so 10 pixels are associated with each sensor. The attenuation of the light is modelled by assuming each pixel has an attenuation coefficient, α as shown in Table 1.

Table 1 Table of sensors and associated attenuation coefficients

	S11	**S12**	**S13**	**S14**	**S15**	**S16**	**S17**	**S18**	**S19**	**S20**
S10	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}	α_{19}	α_{110}
S9	α_{21}	α_{22}	α_{23}	α_{24}	α_{25}	α_{26}	α_{27}	α_{28}	α_{29}	α_{210}
S8	α_{31}	α_{32}	α_{33}	α_{34}	α_{35}	α_{36}	α_{37}	α_{38}	α_{39}	α_{310}
S7	α_{41}	α_{42}	α_{43}	α_{44}	α_{45}	α_{46}	α_{47}	α_{48}	α_{49}	α_{410}
S6	α_{51}	α_{52}	α_{53}	α_{54}	α_{55}	α_{56}	α_{57}	α_{58}	α_{59}	α_{510}
S5	α_{61}	α_{62}	α_{63}	α_{64}	α_{65}	α_{66}	α_{67}	α_{68}	α_{69}	α_{610}
S4	α_{71}	α_{72}	α_{73}	α_{74}	α_{75}	α_{76}	α_{77}	α_{78}	α_{79}	α_{710}
S3	α_{81}	α_{82}	α_{83}	α_{84}	α_{85}	α_{86}	α_{87}	α_{88}	α_{89}	α_{810}
S2	α_{91}	α_{92}	α_{93}	α_{94}	α_{95}	α_{96}	α_{97}	α_{98}	α_{99}	α_{910}
S1	α_{101}	α_{102}	α_{103}	α_{104}	α_{105}	α_{106}	α_{107}	α_{108}	α_{107}	α_{1010}

S_1 to S_{20} are optical sensors, α_{ik} is the attenuation coefficient of the *i th* row, *k th* column pixel, D is length of each pixel (10 mm), I_o is intensity of light leaving the measurement section and I_i is initial intensity of light incident and the measurement section. Then based on the Lambert-Beer absorption law equation.

$$I_o = I_i e^{-\alpha x} \tag{1}$$

Where α is the attenuation coefficient and x is the length of the light path in the medium.

Equation for one beam is

$$I_{o8} = I_{i8} e^{(\alpha_{11} D + \alpha_{12} D + \cdots + \alpha_{18} D)} \tag{2}$$

$$\alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} + \alpha_{15} + \alpha_{16} + \alpha_{17} + \alpha_{18} + \alpha_{19} + \alpha_{110} = -\frac{1}{D} \ln\left(\frac{I_{o10}}{I_{i10}}\right)$$

The horizontal equations for each sensor are;

$$\alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} + \alpha_{15} + \alpha_{16} + \alpha_{17} + \alpha_{18} + \alpha_{19} + \alpha_{110} = -\frac{1}{D} \ln\left(\frac{I_{o10}}{I_{i10}}\right) \tag{3}$$

..

$$\alpha_{101} + \alpha_{102} + \alpha_{103} + \alpha_{104} + \alpha_{105} + \alpha_{106} + \alpha_{107} + \alpha_{108} + \alpha_{109} + \alpha_{1010} = -\frac{1}{D} \ln\left(\frac{I_{o1}}{I_{i1}}\right) \tag{4}$$

Simillarly the vertical equations for each sensor are

$$\alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41} + \alpha_{51} + \alpha_{61} + \alpha_{71} + \alpha_{81} + \alpha_{91} + \alpha_{101} = -\frac{1}{D} \ln\left(\frac{I_{o11}}{I_{i11}}\right) \tag{5}$$

$$\alpha_{110} + \alpha_{210} + \alpha_{310} + \alpha_{410} + \alpha_{510} + \alpha_{610} + \alpha_{710} + \alpha_{810} + \alpha_{910} + \alpha_{1010} = -\frac{1}{D} \ln\left(\frac{I_{o20}}{I_{i20}}\right) \tag{6}$$

These equations may be expressed in matrix form

$$\begin{bmatrix} 1111\ldots\ldots \\ \ldots\ldots\ldots \\ \ldots\ldots\ldots \\ 000000\ldots\ldots \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{100} \end{bmatrix} = \begin{bmatrix} -\frac{1}{D} \ln \frac{I_{o10}}{I_{i10}} \\ \cdot \\ \cdot \\ \cdot \\ -\frac{1}{D} \ln \frac{I_{o20}}{I_{i20}} \end{bmatrix} \tag{7}$$

which is shortened to

$$SR = M \tag{8}$$

where

matrix S is 20×100 matrix known as the sensitivity matrix,
matrix R is 100×1 vector of the attenuation coefficients,
matrix M is 20×1 vector of the measured values.

4.0 THE INVERSE PROBLEM

To reconstruct the image, Equation (8) has to be rearranged.

$$R = S^{-1}M \tag{9}$$

However, there are several difficulties associated with the inverse of the matrix S. The matrix is not square and so no direct inverse. Also the number of projections is smaller than the number of desired attenuation coefficients and no algebraic solution is possible. Here the method of solving the inverse problem is Transpose Algorithm.

5.0 TRANSPOSE ALGORITHM

The method used to derive the value of the absorption coefficient for each pixel in the image from sensor measurements of the cross-section flow rig is the Linear Back Projection (LBP) algorithm which was originally developed for use in X-ray tomography system [2]. This algorithm is simple, fast and widely used for image reconstruction.

In Equation (8) the measured value M is calculated by solving the forward problem. For this modelling the numerical value for an attenuation coefficient for a pixel is either the coefficient for water or air. A single bubble is modelled, one pixel (3,3) has the attenuation coefficient for air ($0.00142\ \text{mm}^{-1}$), and the rest of the pixels use water ($0.004\ \text{mm}^{-1}$). So two sensors detect an attenuation coefficient for air and seven attenuation coefficients for water. All other sensors see only water. The inverse problem is regarded as the Transpose Algorithm in which the measurements obtained at each projection are projected back along the measurement line, assigning the measured value to each point in the line. The values are smeared across the unknown density function. The transpose of matrix S can be used to estimate C [5].

$$R = S^{T}M \tag{10}$$

The calculated attenuation coefficients for water were approximately 16 times too big. In order to investigate the gain factor four further arrangements of sensors were considered. These were 4×4, 5×5, 6×6 and 7×7 pixels. Figure 3 shows the 4×4 arrangement.

	S5	**S6**	**S7**	**S8**
S4	α_1	α_2	α_3	α_4
S3	α_5	α_6	α_7	α_8
S2	α_9	α_{10}	α_{11}	α_{12}
S1	α_{13}	α_{14}	α_{15}	α_{16}

Figure 3 Arrangement of two projections each of four sensors and associated absorption coefficients

The forward and inverse problems were solved for all the sensor systems. The aim is to determine the value of *K* to make the attenuation coefficient of water 0.00287 mm^{-1}. This is achieves by only using the attenuation coefficients for water in both the forward and inverse problem using the Transpose algorithm. To determine *K* the results from the inverse problem solution have to be compared with the initial attenuation coefficient of water. From the Figure 4 the value of *K* is proportional to the number of sensors.

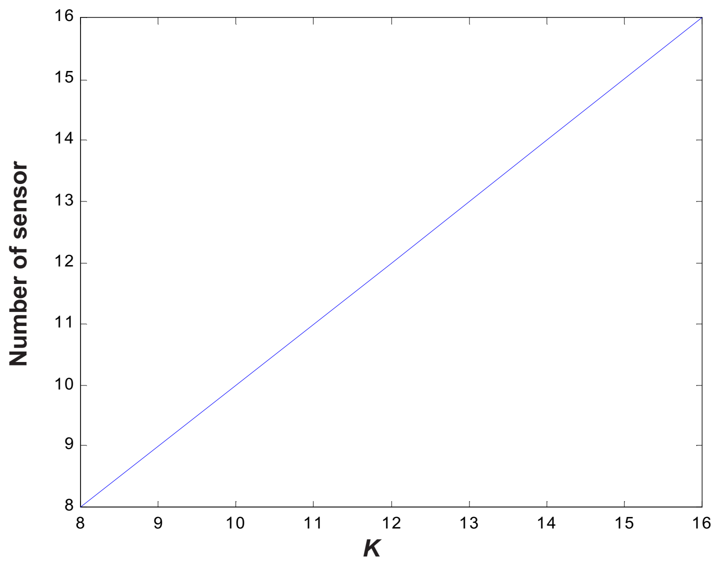

Figure 4 Numbers of sensors versus *k*

5.1 Transpose Algorithm Based on an Iterative Method

The Transpose Algorithm image is poor with little contrast. So the use of an iterative method is used to improve the image quality. The first iterative image reconstruction method is based on the use of Equations (8) and (10). The method uses both equations alternately to correct the sets of attenuation coefficients and measurement values in turn and produce a more accurate image from the measurement values.

5.1.1 The Principle of Iteration

Assume A_0 to be an initial approximation to S^{-1} then the error can be defined as

$$E = 1 - A_0 S$$
$$A_0 S = (1 - E) \tag{11}$$

$$\frac{A_0}{(1-E)} = \frac{1}{S}$$
$$A_0 (1-E)^{-1} = S^{-1} \tag{12}$$

Using binomial expansion

$$(1-E)^{-1} = 1 + E + E^2 + E^3 + \cdots E^{K-1} \tag{13}$$

where $-1 \leq E \leq 1$
From Equation (12) and k iteration

$$A_K = \left(1 + E + E^2 + E^3 + \cdots E^{K-1}\right) A_0 \tag{14}$$

Since

$$(1-E)\left(1 + E + E^2 + E^3 + \cdots E^{K-1}\right) = \left(1 - E^K\right)$$
$$A_K (1-E) = \left(1 - E^K\right) A_0$$
$$A_K - A_K E = A_0 - A_0 E^K$$

From Equation (11)

$$A_K - A_K \left(1 - A_0 S\right) = A_0 - A_0 E^K$$
$$E^K A_0 = A_0 \left(1 - S A_K\right) \tag{15}$$

For the $(k + 1)$th approximation iteration using the Transpose algorithm (Equation 10)

$$R_{K+1} = A_{K+1} M$$

From Equation (14)

$$R_{K+1} = \left(1 + E + E^2 + E^3 + \cdots E^{K-1} + E^K\right) A_0 M$$
$$R_{K+1} = \left(1 + E + E^2 + E^3 + \cdots E^{K-1}\right) A_0 M + E^K A_0 M \tag{16}$$
$$R_{K+1} = R_K + E^K A_0 M$$

From Equation (15)

$$\begin{aligned} R_{K+1} &= R_K + A_0(1 - SA_K)M \\ R_{K+1} &= R_K + A_0M - SA_0A_KM \\ R_{K+1} &= R_K + A_0M - A_0SR_K \\ R_{K+1} &= R_K + A_0(M - SR_K) \end{aligned} \tag{17}$$

Measurement error is;

$$\Delta M_K = M - SR_K \tag{18}$$

The new Transpose image after (K+1)th iteration is

$$R_{K+1} = R_K + A_0\Delta M_K \tag{19}$$

A_0 is the first estimate of S^{-1} so

$$\Delta R_K = S_T \times \Delta M_K \tag{20}$$

$$R_{K+1} = R_K + \Delta R_K \tag{21}$$

6.0 RESULTS & DISCUSSIONS

From the result the iteration images based on the Transpose are 100% better compared to the Transpose algorithm images (Figures 5 to 8). As seen in the graph, image quality increased when the number of bubbles present in the pipe was increased. The iterations shown in Figure 9 to 12 are based the Landweber method.

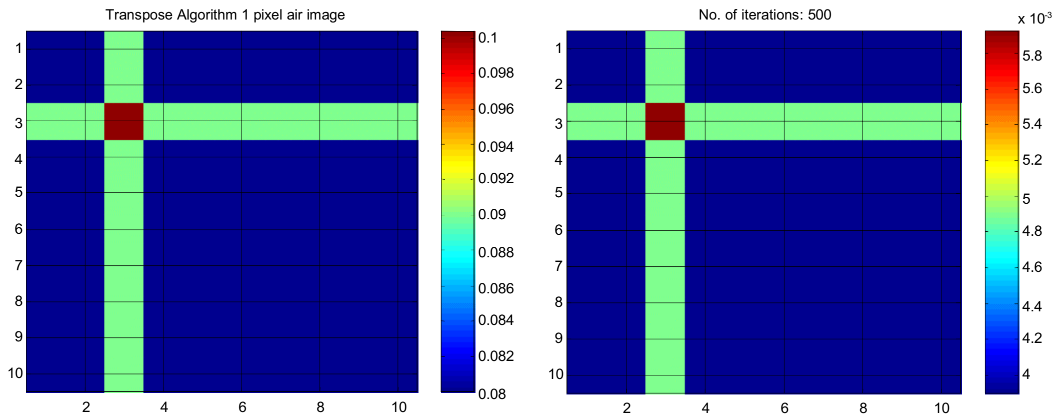

Figure 5 Transpose algorithm 1 air image and after 500 iterations

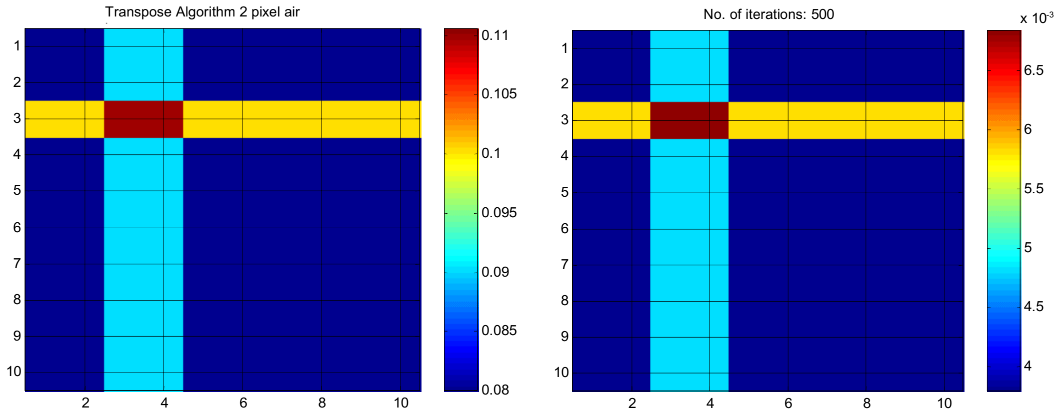

Figure 6 Transpose algorithm 2 air image and after 500 iterations

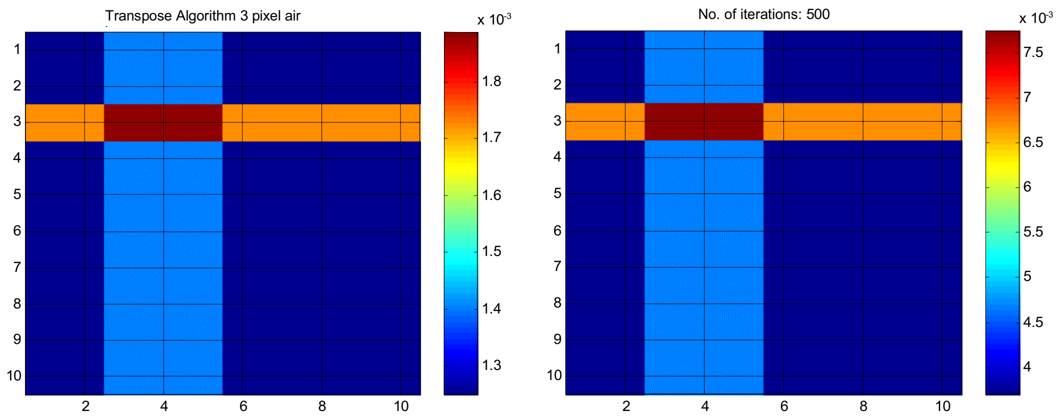

Figure 7 Transpose algorithm 3 air image and after 500 iterations

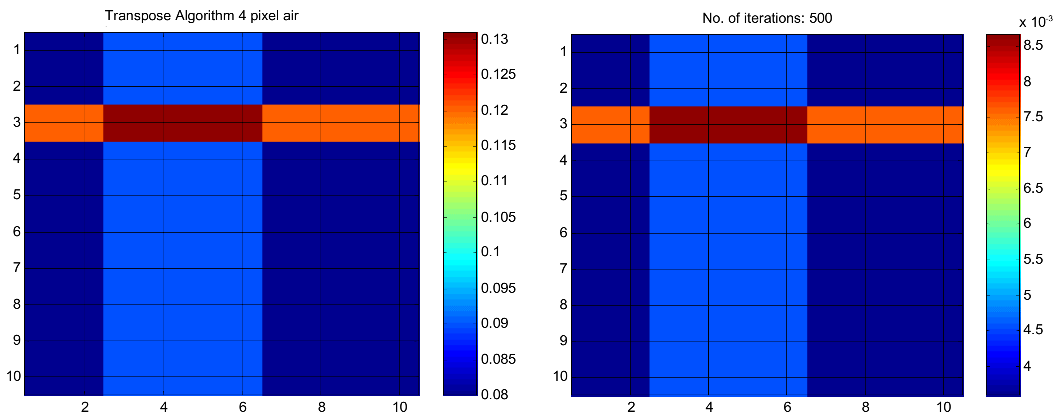

Figure 8 Transpose algorithm 4 air images and after 500 iteration

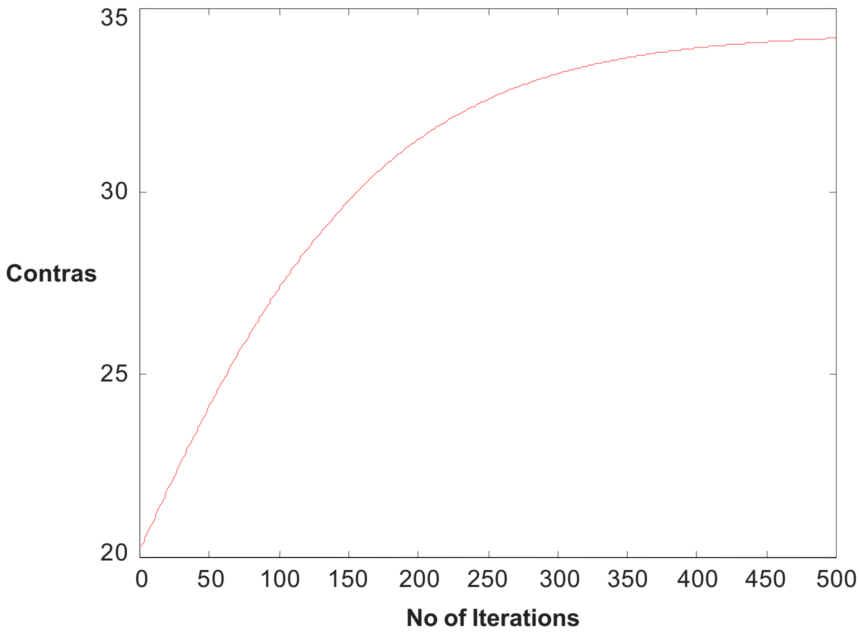

Figure 9 1 Bubble

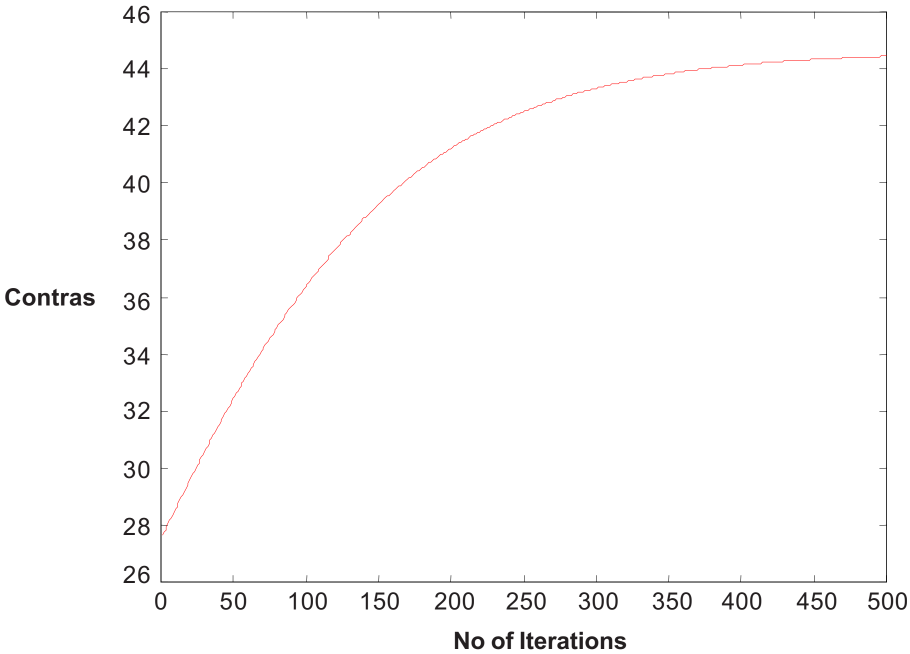

Figure 10 2 Bubbles

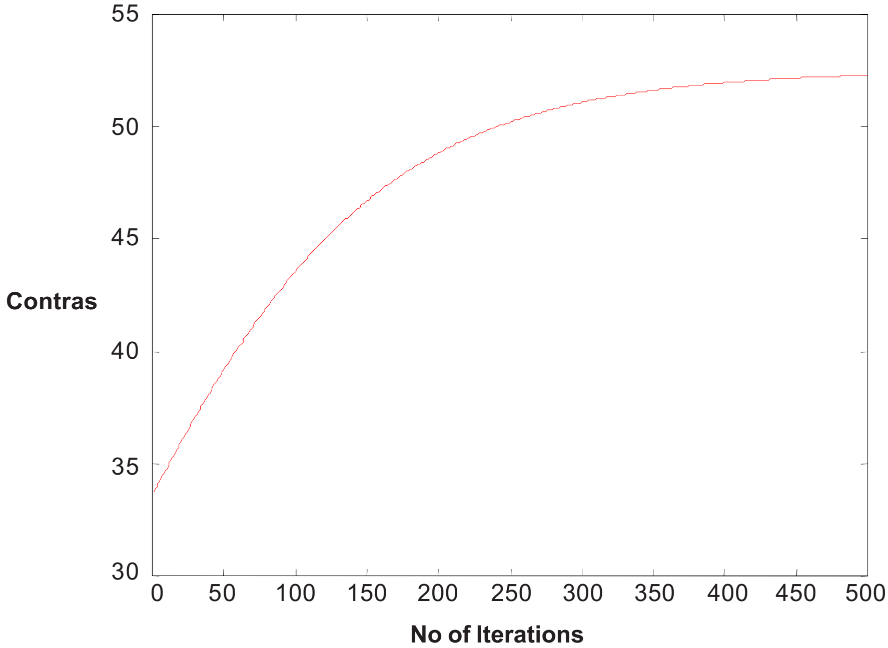

Figure 11 3 Bubbles

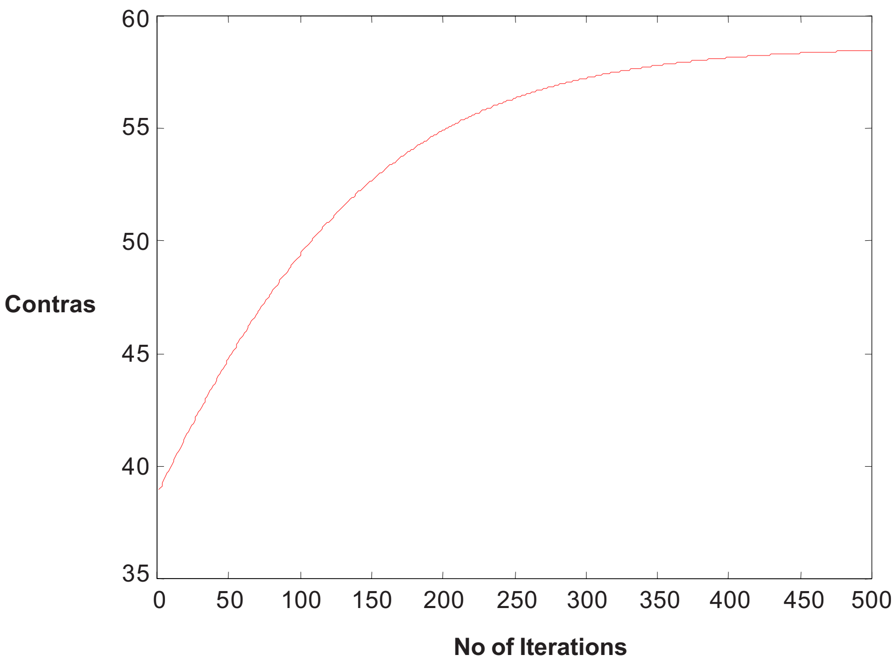

Figure 12 4 Bubbles

7.0 CONCLUSIONS

The Transpose algorithm is different compared to other algorithms because the former back projects all the ray-integrals in the object space. In other words, for each point (x,y) in the object space, one accumulates the contribution of all the ray integrals whose path intersects the point (x,y). Future work will make a comparison of the LBP and Pseudo inverse as an approximation for inverting the sensitivity matrix.

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