

UNSTEADY BOUNDARY LAYER FLOW OF A MICROPOLAR FLUID NEAR A STAGNATION POINT WITH UNIFORM SUCTION OR INJECTION

LOK YIAN YIAN¹, NORSARAHADA AMIN^{2*} & IOAN POP³

Abstract. This paper studies the effect of suction or injection on unsteady boundary layer flow of a micropolar fluid near the forward stagnation point of a permeable wall. The velocity of the external flow as well as that of suction or injection are assumed to start impulsively from rest and are maintained thereafter. The model is governed by a set of coupled partial differential equations that is solved numerically for some values of the suction, injection and material parameters. Profiles for the velocity, microrotation and skin friction coefficient are determined and presented in tables and graphical forms. The results show that suction increases the skin friction while the opposite behaviour is observed for the case of injection. It is also found that an increase in the material parameter will enhance the flow near the wall. On the other hand, it can be seen that in the case of injection, longer time is required to approach the steady flow than in the case of suction.

Keywords: Unsteady boundary layer flow, micropolar fluid, forward stagnation point, suction, injection

Abstrak. Kertas kerja ini mengkaji kesan sedutan dan suntikan ke atas aliran lapisan sempadan tak mantap di titik genangan depan dinding boleh telap bagi bendalir mikropolar. Halaju aliran luar serta halaju sedutan dan suntikan diandaikan bermula secara dedunyt, seterusnya menjadi malar selepas aliran mula bergerak. Masalah ini dimodelkan oleh persamaan terbitan separa yang perlu diselesaikan secara berangka untuk beberapa nilai sedutan, suntikan dan parameter bahan. Taburan halaju, mikroputaran dan pekali geseran permukaan dipersembahkan dalam bentuk jadual dan gambar rajah. Hasil kajian menunjukkan kesan sedutan dapat meningkatkan geseran permukaan, manakala keputusan sebaliknya bagi kes suntikan. Seterusnya, didapati juga dengan meningkatkan nilai sedutan dan suntikan, kesan mikroputaran berdekatan dinding dapat diperlambatkan. Di samping itu, suntikan didapati memerlukan lebih banyak masa untuk mencapai aliran mantap berbanding sedutan.

Kata kunci: Aliran lapisan sempadan tak mantap, bendalir mikropolar, titik genangan depan, sedutan, suntikan

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1.0 INTRODUCTION

The drag force due to skin friction is a fluid dynamic resistive force, which is a consequence of the fluid and the pressure distribution on the surface (Zheng *et al.* [1]). Investigations into the problems of how to predict the drag force behaviour of the fluid in motion have received considerable attention due to the important roles they play in boundary layer control and thermal protection in high energy flow by means of mass transfer. In these problems, the fluid can be sucked or injected, where this additional mass transfer is very efficient in altering the skin friction. Non-Newtonian fluids that contain suspended particles, dust and metal particles are called micropolar fluids.

Eringen [2, 3] was the first to formulate the theory of micropolar fluids and derive constitutive laws for fluids with microstructure. The theory includes the effects of local rotary inertia and couple stresses, and provides a mathematical model for the non-Newtonian behaviour that can be observed in certain liquids such as polymers, colloidal suspensions, fluids with additives, animal blood, liquid crystals etc. A thorough review of the subject and application of micropolar fluid mechanics was provided in the review articles by Ariman *et al.* [4, 5] and in the recent books by Lukaszewicz [6] and Eringen [7].

The problems that deal with micropolar fluid with suction or injection have been studied by Subhardra Ramachandran *et al.* [8], Lien *et al.* [9], Kelson and Farrell [10], Agarwal and Dhanapal [11] and Takhar *et al.* [12] for the curved surface, sphere, porous stretching sheet, porous disc and enclosed rotating disc cases, respectively. However, not much work has been done for the above problem near a stagnation point flow in a micropolar fluid. Hassanien and Hady [13] studied the boundary layer flow in a moving wall with suction and injection. Bhargava *et al.* [14] presented a numerical solution for the mixed convection flow on a horizontal cylinder with suction effects. Recently, Elbarbary and Elgazery [15] studied the effects of variables properties on a horizontal cylinder with suction numerically.

The studies mentioned above deal with steady flows, but many problems of practical interest may be unsteady. The problem of unsteady forced convection flow has long been a major subject in fluid mechanics because of its great importance both from a theoretical and practical view points. In fact there is no actual flow situation, natural or artificial, which does not involve some unsteadiness. Unsteady flows are frequently encountered in such technological and environmental situations as geophysical and biological flows, the processing of materials, the spread of pollutants and fires (Telionis [16]). Literature on unsteady stagnation point flow of a micropolar fluid with suction or injection is not as extensive as steady case. Lok *et al.* [17,18] studied the problem near the forward and rear stagnation point on a plane wall but without suction or injection. Agarwal *et al.* [19] investigated the problem with hard blowing on a plane wall.

The present paper studies the unsteady boundary layer flow of a micropolar fluid with uniform suction or injection near the forward stagnation point of a permeable infinite plane wall. The governing equations are solved numerically using a very efficient

implicit-finite difference scheme, in conjunction with Newton linearization, namely the Keller-box method which is described in the books by Cebeci and Bradshaw [20], and Cebeci [21]. Profiles for the velocity, microrotation and skin friction coefficient are presented for a range of the suction and material parameters. It should be mentioned here that Katagiri [22] considered the steady magnetohydrodynamic boundary layer flow near a stagnation point with suction or injection. Later, the same author [23] extended the problem to steady and unsteady cases for a Newtonian fluid.

2.0 GOVERNING EQUATIONS

Consider the unsteady two-dimensional boundary layer flow of a micropolar fluid near a plane stagnation point O . The external stream towards the wall and the suction or injection through the wall are assumed to start impulsively at time $t = 0$ from rest. The flow configuration is shown schematically in Figure 1. Cartesian coordinates (x, y) are used, in which x and y are taken parallel and perpendicular to the wall, respectively. Under the boundary layer approximation, the governing equations can be written as, see Lok *et al.* [17],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \left(\frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y}, \quad (2)$$

$$\rho j \left(\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = -\kappa \left(2N + \frac{\partial u}{\partial y} \right) + \gamma \frac{\partial^2 N}{\partial y^2}, \quad (3)$$

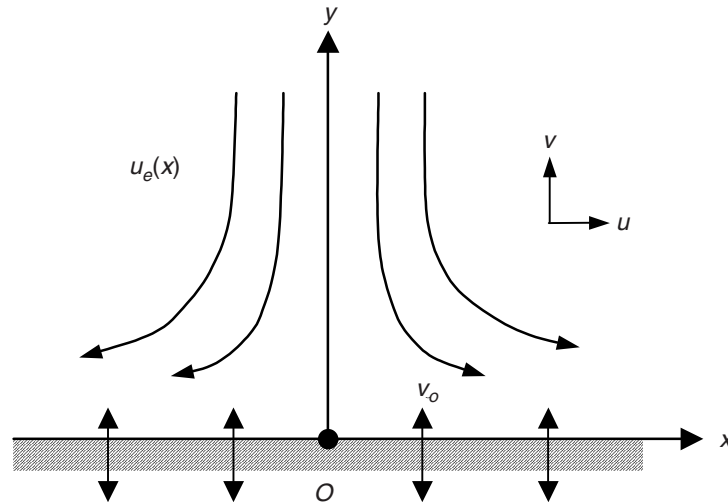


Figure 1 Physical model and coordinate system

where u and v are the velocity components along the x and y axis, respectively, N is the component of the microrotation vector normal to the x - y plane, ρ is the density, μ is the absolute viscosity, κ is the vortex viscosity, γ is the spin-gradient viscosity and j is the microinertia density. It is assumed that all physical quantities ρ , μ , κ , γ and j are constants.

The initial and boundary conditions are:

$$\begin{aligned} t < 0 : u(t, x, y) &= 0, & v(t, x, y) &= 0, & N(t, x, y) &= 0, \\ t \geq 0 : u &= 0, & v &= v_o, & N &= -n \frac{\partial u}{\partial y} & \text{at } y = 0, \\ u &\rightarrow u_e(x), & N &\rightarrow 0 & \text{as } y &\rightarrow \infty \end{aligned} \quad (4)$$

where $v_o (< 0)$ is the constant velocity of suction or $v_o (> 0)$ is the constant velocity of injection, and n is a constant such that $0 \leq n \leq 1$. It is noteworthy that the case when $n = 0$ is called strong concentration by Guram and Smith [24]. In this case, $N = 0$ near the wall implying that concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena and Mathur [25]). The case corresponding to $n = 1/2$ indicates the vanishing of anti-symmetrical part of the stress tensor and denotes weak concentration (Ahmadi [26]). The case $n = 1$, as suggested by Peddieson [27], is used for the modeling of turbulent boundary layer flows. We shall only consider the case $n = 0$.

We assume that $u_e(x)$ is the free stream velocity given by

$$u_e(x) = ax \quad (5)$$

where a is a positive constant ($a > 0$) for the forward stagnation point of dimension (time)⁻¹.

Introducing the following dimensionless variables (Lok *et al.* [17])

$$\psi = 2\sqrt{v}taxf(\tau, \eta), \quad N = \frac{ax}{2\sqrt{v}t}g(\tau, \eta), \quad \eta = \frac{y}{2\sqrt{v}t}, \quad \tau = 2\sqrt{at} \quad (6)$$

where v is the kinematic viscosity and ψ is the stream function which is defined as $u = -\partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, satisfying (1) identically.

Following Rees and Bassom [28], and Rees and Pop [29], we assume

$$\gamma = (\mu + \kappa / 2)j = \mu(1 + K / 2)j, \quad (7)$$

where $K = \kappa / \mu$ is the material parameter. Substituting expressions (5) to (7) into Equations (2) to (4), we obtain

$$(1 + K) \frac{\partial^3 f}{\partial \eta^3} + 2\eta \frac{\partial^2 f}{\partial \eta^2} - 2\tau \frac{\partial^2 f}{\partial \tau \partial \eta} + \tau^2 \left\{ 1 - \left(\frac{\partial f}{\partial \eta} \right)^2 + f \frac{\partial^2 f}{\partial \eta^2} \right\} + K \frac{\partial g}{\partial \eta} = 0, \quad (8)$$

$$\left(1 + \frac{K}{2}\right) \frac{\partial^2 g}{\partial \eta^2} + 2\eta \frac{\partial g}{\partial \eta} - 2\tau \frac{\partial g}{\partial \tau} + 2g + \tau^2 \left\{ f \frac{\partial g}{\partial \eta} - \frac{\partial f}{\partial \eta} g - K \left(2g + \frac{\partial^2 f}{\partial \eta^2} \right) \right\} = 0, \quad (9)$$

subject to the boundary conditions:

$$\begin{aligned} f &= \frac{f_w}{\tau}, \quad \frac{\partial f}{\partial \eta} = 0, \quad g = -n \frac{\partial^2 f}{\partial \eta^2}, \quad \text{on } \eta = 0, \\ \frac{\partial f}{\partial \eta} &\rightarrow 1, \quad g \rightarrow 0, \quad \text{as } \eta \rightarrow \infty, \end{aligned} \quad (10)$$

where the parameter f_w is positive ($f_w > 0$) for suction and negative ($f_w < 0$) for injection that is defined by

$$f_w = -\frac{v_0}{\sqrt{a_v}}. \quad (11)$$

The wall skin friction τ_w and the skin friction coefficient C_f are defined as (Lok *et al.* [17])

$$\tau_w = \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}, \quad C_f = \frac{(u_e / \sqrt{a_v}) \tau_w}{\rho u_e^2}. \quad (12)$$

Therefore, the coefficient of skin friction is given by

$$C_f^{\text{unsteady}} = \frac{1}{\tau} \left[1 + (1-n)K \right] \frac{\partial^2 f}{\partial \eta^2} \Big|_{(\tau,0)}, \quad (13)$$

for the unsteady-state flow. Although this paper is concerned with the flow for $\tau > 0$, the solution at $\tau = 0$ is necessary in solving the differential Equation (8) and (9). The solution at $\tau = 0$ is that of unsteady micropolar flow without suction or injection, i.e. $v_0 = 0$. Thus at $\tau = 0$, Equations (8) and (9) become

$$(1 + K) f''' + 2\eta f'' + Kg' = 0, \quad (14)$$

$$(1 + K/2) g'' + 2\eta g' + 2g = 0, \quad (15)$$

subject to

$$\begin{aligned} f(0) &= f'(0) = 0, \quad g(0) = -nf''(0), \\ f' &\rightarrow 1, \quad g \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \end{aligned} \quad (16)$$

where primes denote the differentiation with respect to η .

For the steady-state flow ($\partial/\partial t = 0$), we introduce the variables, see Lok *et al.* [17]

$$\psi = ax(v/a)^{1/2} f_s(\xi), \quad N = ax(a/v)^{1/2} g_s(\xi), \quad \xi = (a/v)^{1/2} y, \quad (17)$$

so that Equations (2) to (3) become

$$(1+K) f_s''' + f_s f_s'' + 1 - f_s'^2 + K g_s' = 0, \quad (18)$$

$$(1+K/2) g_s'' + f_s g_s' - f_s g_s' - K(2g_s + f_s'') = 0, \quad (19)$$

and subject to the boundary conditions

$$\begin{aligned} f_s(0) &= f_w, & f_s'(0) &= 0, & g_s(0) &= -n f_s''(0), \\ f_s' &\rightarrow 1, & g_s &\rightarrow 0 & \text{as } \xi &\rightarrow \infty, \end{aligned} \quad (20)$$

where primes denote the differentiation with respect to ξ .

The skin friction coefficient for the steady-state flow is given by

$$C_f^{\text{steady}} = [1 + (1-n)K] f_s''(0). \quad (21)$$

We shall solve Equations (8) - (9) and (18) - (19) for $n = 0$ (strong concentration) and different values of material parameter K and suction or injection parameter f_w .

3.0 NUMERICAL METHOD

Equations (8)-(9) and (18)-(19) have been solved numerically for different values of the material parameter K and the suction ($f_w > 0$) or injection ($f_w < 0$) parameter when 0 (strong concentration) using the Keller-box method. This method has been successfully used in several papers by Lok *et al.* [17, 18, 30]. In this approach, the differential equations are first reduced to a system of first order equations which are then expressed in finite difference forms using central differences. This system of equations is linearized using Newton's method before putting them in matrix-vector form. The resulting linear system is solved along with their boundary conditions by the block-tridiagonal-elimination method. The details of this method can be seen from the books by Cebeci and Bradshaw [20], and Cebeci [21].

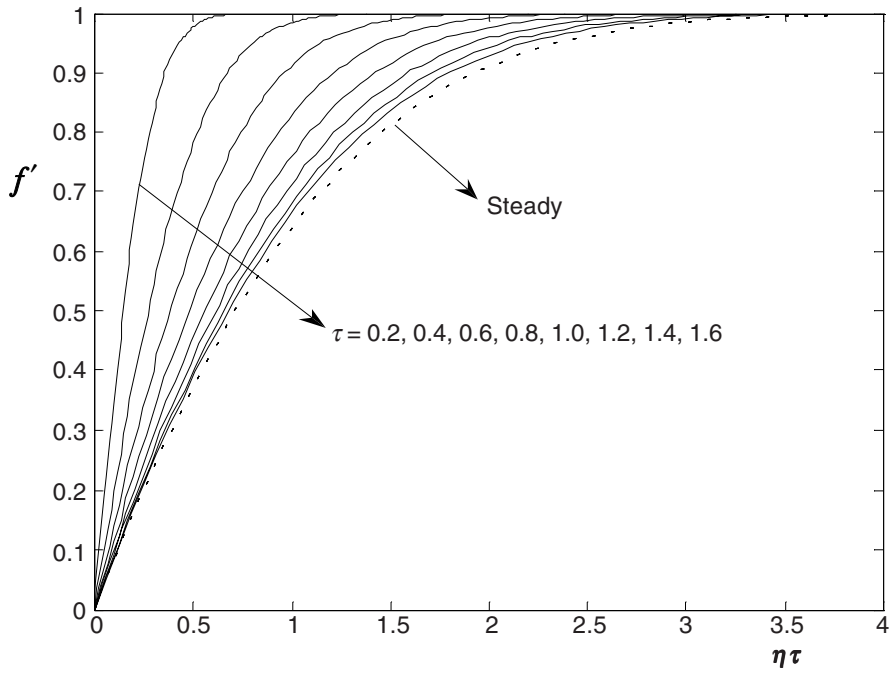
To start the computation, the initial Equations (14) and (15) along with the boundary conditions Equation (16) has to be used. The system of equations is solved iteratively until the required τ value is reached. The computation is started by using a small value of η_∞ and then successively increase the value of η_∞ until a suitable η_∞ is obtained. In most laminar boundary layer flows, a step size $\Delta\eta = 0.02$ or even 0.04 is sufficient to provide accurate numerical results while the step size $\Delta\xi$ can be arbitrary as it did not appreciably affect the converged results (Chen [31]). A convergence

criterion based on the relative difference between the current and previous iterations has been used. From the calculations, we found that the greatest error appears in the wall-shear parameter, $f''(\tau, 0)$, so it is used as the convergence criterion. Therefore, in this paper we use the grid size in η of 0.04, $\eta_\infty = 12$, grid size in τ of 0.02, and the convergence criterion is 5×10^{-7} .

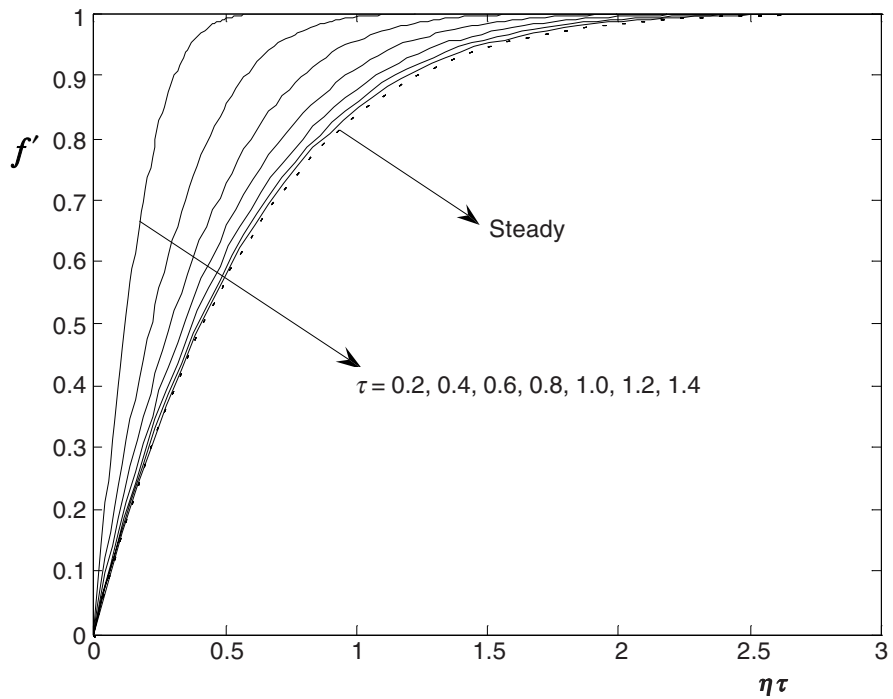
4.0 RESULTS AND DISCUSSION

Representative results for the skin friction coefficient, the velocity and microrotation profiles have been obtained for several values material parameter K , and suction or injection parameter f_w . Tables 1 and 2 show the skin friction coefficient, C_f as function of the suction or injection parameter f_w for different values of material parameter K at certain time up to the steady flow case. As a validation of our code, the results for $K = 0$ (Newtonian fluid) for different values of f_w compare reasonably well with that of Katagiri [22], who used the difference-differential method. The results obtained by Lok *et al.* [17] for $K = 0$ and $f_w = 0$ (without suction or injection) are also included. It is found that the suction parameter enhances the skin friction. Opposite effect is observed when injection is present. It is also noticed that increasing the values of the material parameter results in an increase in values of C_f which in turn causes a decrease in $f''(0)$. Therefore, imposition of fluid suction at the plane surface causes decrease in the skin friction. This behaviour is useful in controlling the flow to some desired way.

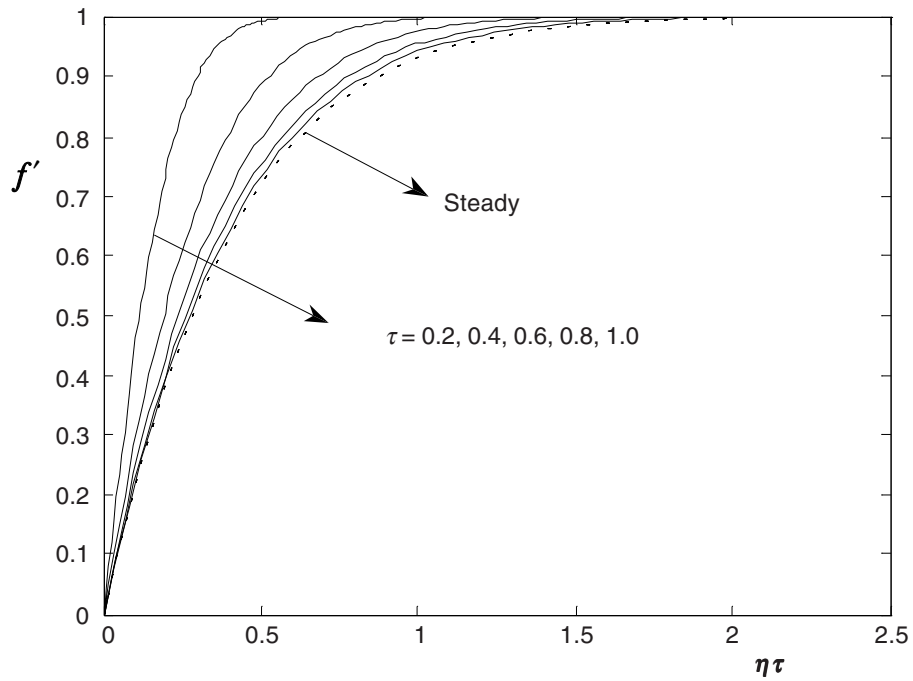
The velocity and microrotation profiles versus a similar variable of steady flow case $\eta\tau (= y\sqrt{a/\nu})$ for $f_w = 0, 2, 4, -2$ and -4 when $K = 1$ are presented in Figures 2 and 3. Our results describe the smooth transition from the unsteady flow (small time) to the steady flow. Both velocity and microrotation profiles develop rapidly from rest as τ increases from zero and they attain those of the steady flow case (dot lines) obtained by solving Equations (18) and (19) subject to boundary conditions Equation (20). It is noticed that as the suction parameter increases, the boundary layer thickness decreases. The opposite trend is observed for the case of injection. This is because more and more fluid is withdrawn from the wall when injection increases. This phenomenon reflects the fact that the boundary layer thickness is reduced due to suction and increased due to blowing, thus suction implies an increase in skin friction, as the wall shear stress is inversely proportional to the boundary layer thickness (Mohanty [32]). This explains the behaviour that we observed from Tables 1 and 2. For the case of injection, longer times are required to approach the steady flow than for the case of suction since suction improves stability. The graphs in Figure 3 show that microrotation velocity has a parabolic profile, reaching a maximum value near the wall and then decreases monotonically to zero as $\eta\tau$ increases. For the case of injection, it shows that close to the wall the unsteady-state flow overshoots the steady-state flow. This overshoot increases as the injection parameter f_w increases. Finally, Figure 4 presents the microrotation profiles for some values of K and f_w when the flow is in steady-state. It is



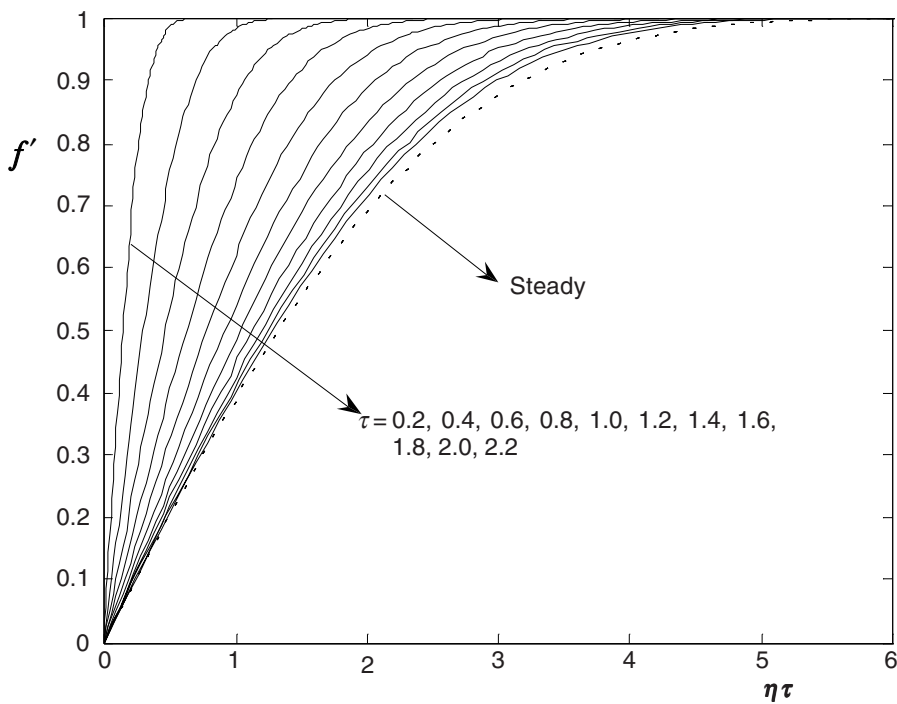
(a) $f_w = 0$



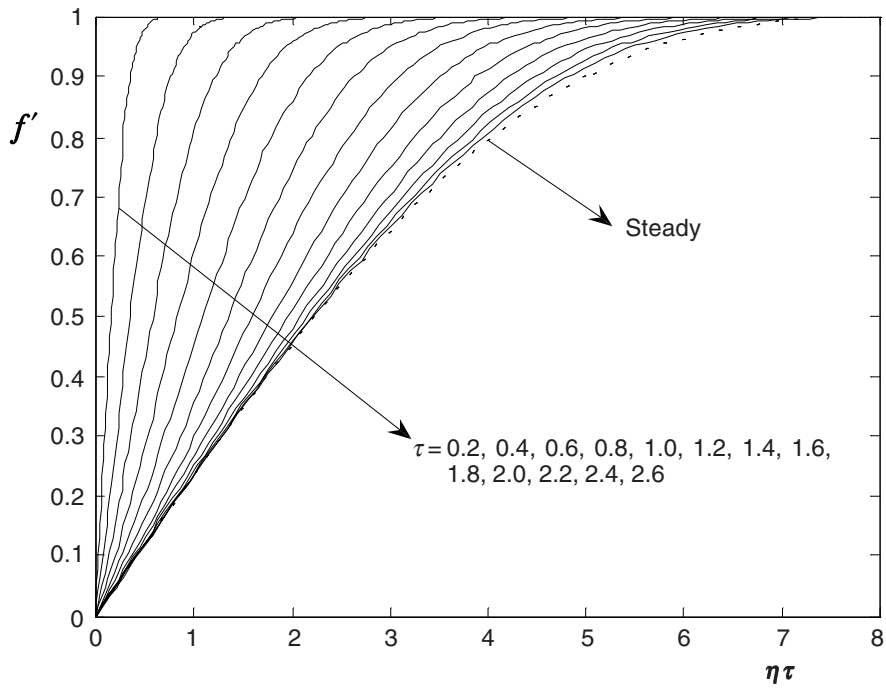
(b) $f_w = 2$



(c) $f_w = 4$

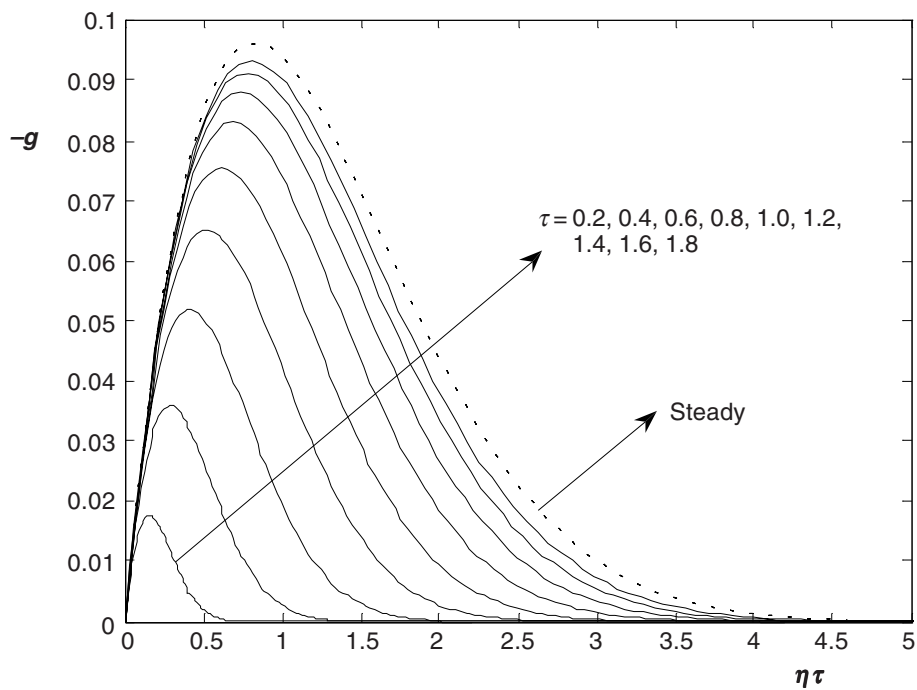


(d) $f_w = -2$

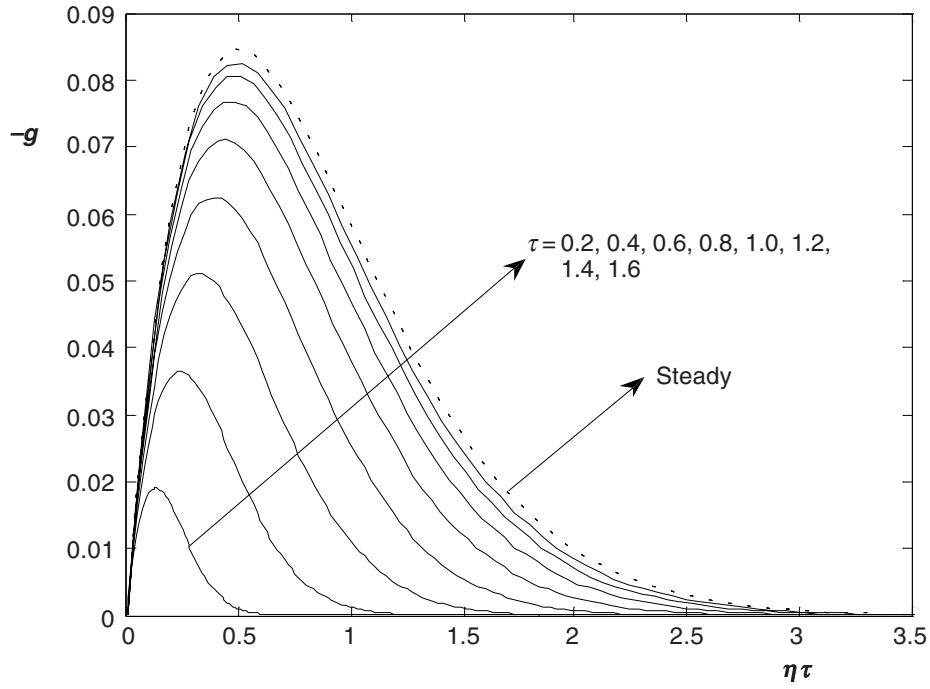


(e) $f_w = -4$

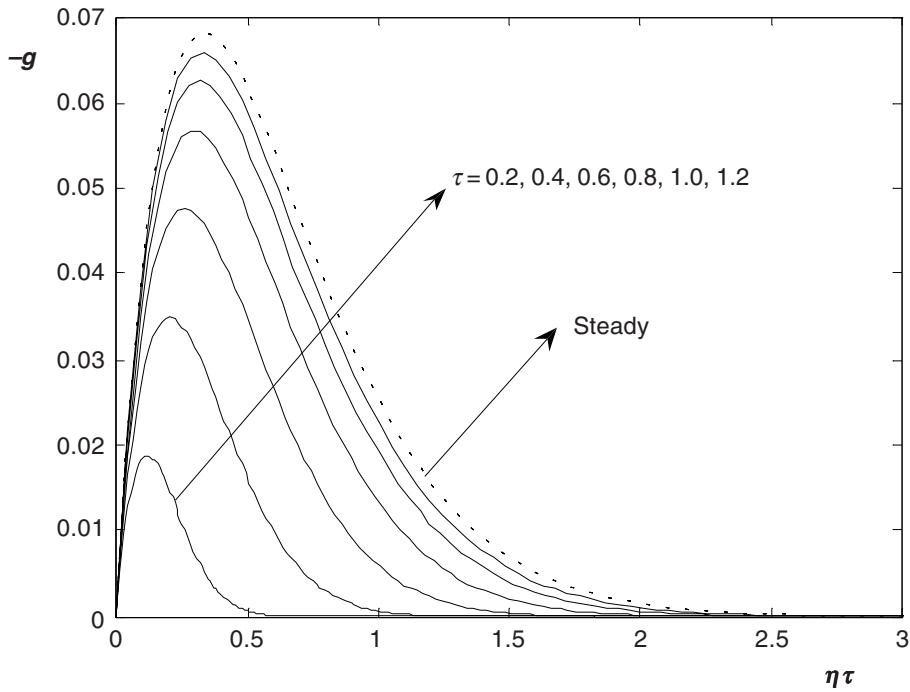
Figure 2 The velocity profiles for various values of f_w when $K = 1$



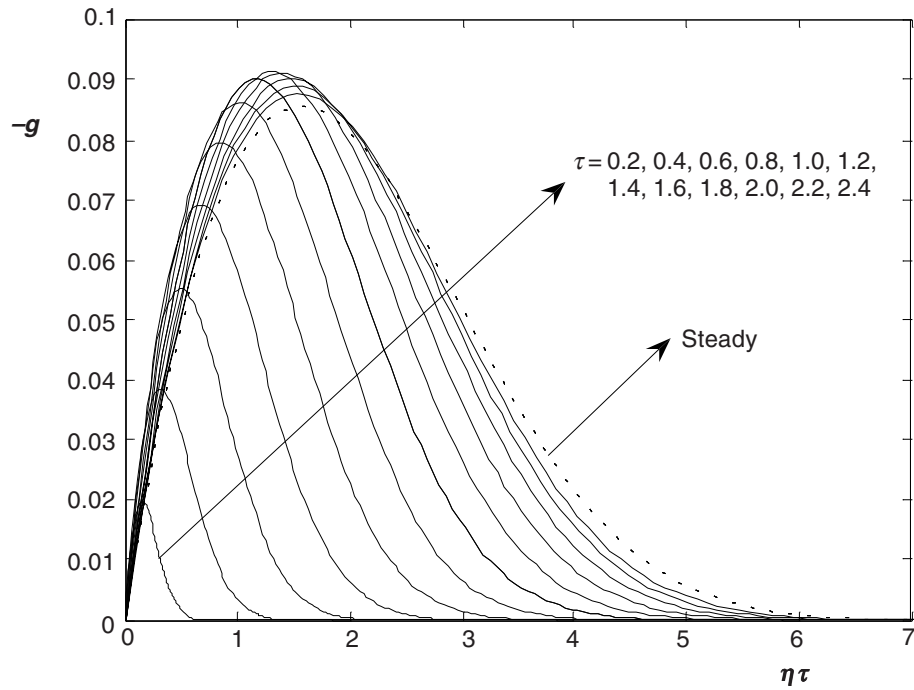
(a) $f_w = 0$



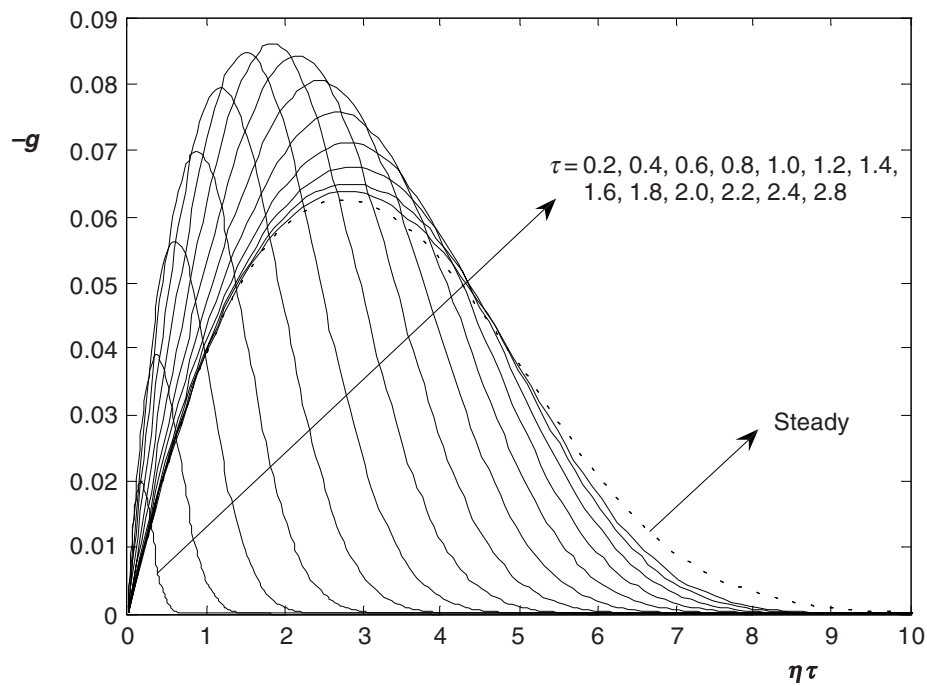
(b) $f_w = 2$



(c) $f_w = 4$



(d) $f_w = -2$



(e) $f_w = -4$

Figure 3 The microrotation profiles for various values of f_w when $K = 1$

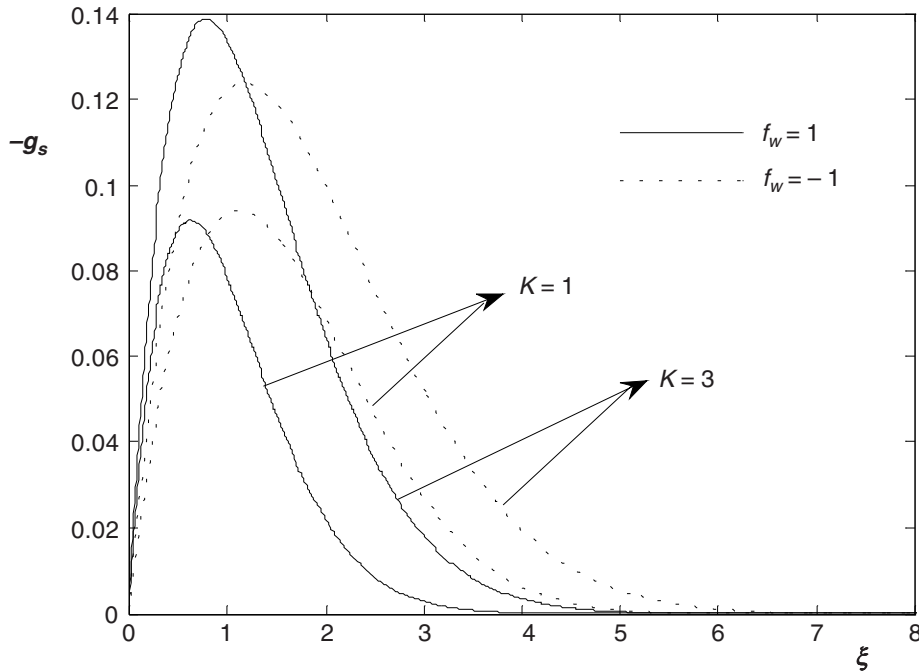


Figure 4 The microrotation profiles for various values of f_w and K (steady case)

found that for both cases of injection and suction, the boundary layer thickness increases when K increases. This means the micropolar fluid will help in reducing skin friction thus enhance the fluid flow.

5.0 CONCLUSIONS

The growth of the boundary layer flow of a micropolar fluid, with the velocity of the external flow and that of suction or injection assumed to be started impulsively from rest, and are kept steady thereafter, is studied theoretically. The non-linear coupled partial differential equations governing the unsteady boundary layer flow are solved numerically using a very efficient implicit finite-difference method. The presented results show the temporal development of the momentum and micropolar boundary layer characteristics. The numerical results indicate that suction increases the skin friction, while the opposite effect is observed for the injection case, thus enabling the boundary layer flow to be controlled. It is also found that an increase in the material parameter K will increase the velocity of the flow near the wall.

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Table 1 Values of the skin friction coefficient C_f when $n = 0$ (strong concentration) for some values of material parameter, K and suction parameter, at certain time τ

f_w	τ	C_f			
		$K = 0$ (Newtonian fluid)	$K = 1$	$K = 2$	$K = 3$
0	0.5	2.4563	3.4462	4.1640	4.7389
	1.0	1.5151	2.0954	2.4831	2.7824
	1.5	1.3033	1.7856	2.1019	2.3523
	2.0	1.2484	1.7045	2.0063	2.2483
	2.5	1.2363	1.6865	1.9860	2.2264
	Steady	1.2327	1.6823	1.9815	2.2216
		(1.232627) [1.232588]	(1.682170)	(1.981438)	(2.221496)
1	0.5	2.9985	3.9786	4.6923	5.2650
	1.0	2.1043	2.6691	3.0517	3.3482
	1.5	1.9303	2.3968	2.7089	2.9565
	2.0	1.8962	2.3387	2.6370	2.8758
	2.5	1.8915	2.3297	2.6259	2.8628
	Steady	1.8895	2.3277	2.6237	2.8604
		[1.889303]			
2	0.5	3.6076	4.5593	5.2610	5.8267
	1.0	2.8072	3.3292	3.6952	3.9822
	1.5	2.6874	3.1126	3.4093	3.6468
	2.0	2.6726	3.0789	3.3635	3.5923
	2.5	2.6719	3.0759	3.3591	3.5864
	Steady	2.6703	3.0746	3.3578	3.5851
		[2.670006]			
3	0.5	4.2804	5.1871	5.8692	6.4234
	1.0	3.6026	4.0659	4.4068	4.6790
	1.5	3.5326	3.9088	4.1846	4.4082
	2.0	3.5284	3.8927	4.1597	4.3760
	2.5	3.5281	3.8922	4.1585	4.3740
	Steady	3.5268	3.8910	4.1575	4.3730
		[3.526497]			
4	0.5	5.0120	5.8600	6.5157	7.0544
	1.0	4.4663	4.8665	5.1777	5.4317
	1.5	4.4310	4.7619	5.0160	5.2247
	2.0	4.4305	4.7558	5.0042	5.2079
	2.5	4.4302	4.7556	5.0042	5.2075
	Steady	4.4291	4.7546	5.0032	5.2066
		[4.428637]			

() Results by Lok *et al.*, [17]

[] Results by Katagiri [22]

Table 2 Values of the skin friction coefficient C_f when $n = 0$ (strong concentration) for some values of material parameter, K and injection parameter, at certain time τ

f_w	τ	C_f			
		$K = 0$ (Newtonian fluid)	$K = 1$	$K = 2$	$K = 3$
-1	0.5	1.9827	2.9628	3.6762	4.2486
	1.0	1.0510	1.6134	1.9933	2.2877
	1.5	0.8375	1.2959	1.6013	1.8450
	2.0	0.7775	1.2057	1.4947	1.7291
	2.5	0.7617	1.1821	1.4681	1.7006
	3.0	0.7583	1.1771	1.4627	1.6948
	Steady	0.7567	1.1753	1.4609	1.6931
		[0.756574]			
-2	0.5	1.5769	2.5281	3.2291	3.7942
	1.0	0.7.99	1.2232	1.5826	1.8647
	1.5	0.5326	0.9307	1.2103	1.4376
	2.0	0.4888	0.8514	1.1112	1.3264
	2.5	0.4783	0.8312	1.0862	1.2982
	3.0	0.4766	0.8268	1.0807	1.2920
	Steady	0.4759	0.8255	1.0792	1.2902
		[0.475810]			
-3	0.5	1.2363	2.1412	2.8221	3.3753
	1.0	0.4764	0.9190	1.2474	1.5108
	1.5	0.3546	0.6765	0.9208	1.1247
	2.0	0.3332	0.6213	0.8439	1.0328
	2.5	0.3300	0.6101	0.8271	1.0117
	3.0	0.3296	0.6082	0.8240	1.0074
	Steady	0.3295	0.6077	0.8232	1.0063
		[0.329456]			
-4	0.5	0.9563	1.8005	2.4541	2.9912
	1.0	0.3268	0.6905	0.9814	1.2214
	1.5	0.2568	0.5088	0.7158	0.8936
	2.0	0.2496	0.4781	0.6657	0.8280
	2.5	0.2491	0.4740	0.6575	0.8159
	3.0	0.2491	0.4736	0.6565	0.8140
	Steady	0.2491	0.4734	0.6562	0.8140

[] Results by Katagiri [22]

interesting comments.

REFERENCES

- [1] Zheng, L. C., X. X. Zhang, and J. C. He. 2003. Drag Force of Non-Newtonian Fluid on a Continuous Moving Surface with Strong Suction/Blowing. *Chin. Phys. Lett.* 20(6): 858-861.
- [2] Eringen, A. C. 1966. Theory of Micropolar Fluids. *J. Math. Mech.* 16: 1-18.
- [3] Eringen, A. C. 1972. Theory of Thermomicropolar Fluids. *J. Math. Analysis & Appl.* 38: 480-496.
- [4] Ariman, T., M. A. Turk, and N. D. Sylvester. 1973. Microcontinuum Fluid Mechanics - A Review. *Int. J. Engng. Sci.* 11: 905-930.
- [5] Ariman, T., M. A. Turk, and N. D. Sylvester. 1974. Application of Microcontinuum Fluid Mechanics. *Int. J. Engng. Sci.* 12: 273-293.
- [6] Lukaszewicz, G. 1999. *Micropolar Fluids: Theory and Application*. Basel: Birkhäuser.
- [7] Eringen, A. C. 2001. *Microcontinuum Field Theories. II: Fluent Media*. New York: Springer.
- [8] Subhardra Ramachandran, P., M. N. Mathur, and S. K. Ojha. 1979. Heat Transfer in Boundary Layer Flow of a Micropolar Fluid Past a Curved Surface with Suction and Injection. *Int. J. Engng. Sci.* 17: 625-639.
- [9] Lien, F. S., C. K. Chen, and J. W. Cleaver. 1986. Analysis of Natural Convection Flow of Micropolar Fluid about a Sphere with Blowing and Suction. *J. Heat Transfer.* 108: 967-970.
- [10] Kelson, N. A. and T. W. Farrell. 2001. Micropolar Flow over a Porous Stretching Sheet with Strong Suction or Injection. *Int. Comm. Heat Mass Transfer.* 28: 479-488.
- [11] Agarwal, R. S. and C. Dhanapal. 1988. Stagnation Point Micropolar Fluid Flow Between Porous Discs with Uniform Suction. *Int. J. Engng. Sci.* 26(3): 293-300.
- [12] Takhar, H. S., R. Bhargava, and R. S. Agarwal. 2001. Finite Element Solution of Micropolar Fluid from an Enclosed Rotating Disc with Suction and Injection. *Int. J. Engng. Sci.* 39: 913-927.
- [13] Hassanien, I. A. and F. M. Hady. 1988. Micropolar Boundary Layer Flow at a Stagnation Point on a Moving Wall with Suction and Injection. *Revue Roumaine des Sciences Techniques, Serie de Mecanique Appliquee.* 33: 439-447.
- [14] Bhargava, R., L. Kumar, H. S. Takhar, and R. S. Agarwal. 2004. Quasi Linearization of Mixed Convection Micropolar Flow Near Stagnation Point with Suction. *Heat and Technology.* 22: 103-111.
- [15] Elbarbary, E. M. E. and N. S. Elgazery. 2005. Flow and Heat Transfer of a Micropolar Fluid in an Axisymmetric Stagnation Flow on a Cylinder with Variable Properties and Suction (Numerical Study). *Acta Mechanica.* 176: 213-219.
- [16] Telionis, D. P. 1981. *Unsteady Viscous Flows*. New York: Springer.
- [17] Lok, Y. Y., P. Phang, N. Amin, and I. Pop. 2003. Unsteady Boundary Layer Flow of a Micropolar Fluid Near the Forward Stagnation Point of a Plane Surface. *Int. J. Engng. Sci.* 41: 173-186.
- [18] Lok, Y. Y., N. Amin, and I. Pop. 2003. Unsteady Boundary Layer Flow of a Micropolar Fluid Near the Rear Stagnation Point of a Plane Surface. *Int. J. Thermal Sci.* 42: 995-1001.
- [19] Agarwal, R. S., R. Bhargava, and S. Jain. 1995. Non-steady Plane Stagnation Point Flow of a Micropolar Fluid with Hard Blowing. *Z. Angew. Math. Mech.* 75(4): 313-315.
- [20] Cebeci, T. and P. Bradshaw. 1984. *Physical and Computational Aspect of Convective Heat Transfer*. New York: Springer.
- [21] Cebeci, T. 2002. *Convective Heat Transfer*. California: Horizons Publishing Inc.
- [22] Katagiri, M. 1969. Magnetohydrodynamic Flow with Suction or Injection at the Forward Stagnation Point. *J. Phy. Soc. Japan.* 27(6): 1677-1685.
- [23] Katagiri, M. 1971. Unsteady Boundary Layer Near the Forward Stagnation Point with Uniform Suction or Injection. *J. Phys. Soc. Japan.* 31: 935-939.
- [24] Guram, G. S. and C. Smith. 1980. Stagnation Flows of Micropolar Fluids with Strong and Weak Interactions. *Comp. & Maths. with Appls.* 6: 213-233.
- [25] Jena, S. K. and M. N. Mathur. 1981. Similarity Solutions for Laminar Free Convection Flow of a Thermomicropolar Fluid past a Nonisothermal Flat Plate. *Int. J. Engng. Sci.* 19: 1431-1439.
- [26] Ahmadi, G. 1976. Self-Similar Solution of Incompressible Micropolar Boundary Layer Flow over a Semi-

- Infinite Plate. *Int. J. Engng. Sc.* 14: 639-646.
- [27] Peddieson, J. 1972. An Application of the Micropolar Fluid Model to the Calculation of Turbulent Shear Flow. *Int. J. Engng. Sc.* 10: 23-32.
- [28] Rees, D. A. S. and A. P. Bassom. 1996. The Blasius Boundary-Layer Flow of a Micropolar Fluid. *Int. J. Engng. Sc.* 34: 113-124.
- [29] Rees, D. A. S. and I. Pop. 1998. Free Convection Boundary-Layer Flow of a Micropolar Fluid from a Vertical Flat Plate. *IMA J. Appl. Math.* 61: 179-197.
- [30] Lok, Y. Y., N. Amin, and I. Pop. 2006. Non-orthogonal Stagnation Point Flow Towards a Stretching Sheet. *Int. J. Non-Linear Mechanics.* 41: 622-627.
- [31] Chen, T. S. 1988. Parabolic Systems: Local Nonsimilarity Method. In *Handbook of Numerical Heat Transfer*. Edited by W. J. Minkowycz. Canada: John Wiley and Son Inc.
- [32] Mohanty, A. K. 2002. *Fluid Mechanics*. New Delhi: Prentice-Hall of India.