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# OPTIMAL CONTROL OF VECTOR-BORNE DISEASE WITH DIRECT TRANSMISSION

Nurul Aida Nordin, Rohanin Ahmad*, Rashidah Ahmad

Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310, UTM, Johor Bahru, Johor, Malaysia

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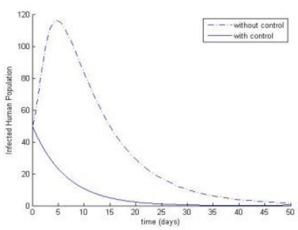
*Corresponding author
rohanin@utm.my

**Graphical abstract**




## Abstract

This paper introduces the usage of three controls as a way to reduce the occurrence of vector-borne disease. The governing equation of the dynamical system used in this paper describes both direct and indirect transmission mode of vector-borne disease. This means that the disease can be transmitted in two different ways. First, it can be transmitted through mosquito bites and the other is through human blood transfusion. The three controls that are incorporated in the dynamical system include a measurement of basic practice for blood donation procedure, self-prevention effort and vector control strategy by health authority. The optimality system of the three controls is characterized using optimal control theory and the existence and uniqueness of the optimal control are established. Then, the effect of the incorporation of the three controls is investigated by performing numerical simulation.

*Keywords*: Epidemic model, vector-borne disease, direct transmission, optimal control

## Abstrak

Kertas kerja ini memperkenalkan penggunaan tiga kawalan sebagai satu cara untuk mengurangkan kejadian penyakit bawaan vektor. Sistem dinamik yang digunakan dalam kertas kerja ini merangkumi kedua-dua mod penyebaran secara langsung dan tidak langsung bagi penyakit bawaan vektor. Ini bermakna, penyakit ini boleh disebarkan dalam dua cara yang berbeza. Pertama, ia boleh disebarkan melalui gigitan nyamuk dan cara kedua adalah melalui pemindahan darah manusia. Tiga kawalan yang diperkenalkan dalam sistem dinamik tersebut adalah pengukuran amalan asas untuk prosedur derma darah, usaha pencegahan oleh setiap individu dan strategi kawalan vektor oleh pihak berkuasa kesihatan. Sistem optimaliti untuk tiga kawalan tersebut dicirikan menggunakan teori kawalan optimum dan juga kewujudan dan keunikan kawalan optimum ditunjukkan. Kemudian, kesan penggabungan tiga kawalan tersebut dikaji dengan melakukan simulasi berangka.

*Kata kunci*: Model epidemic, penyakit bawaan vector, penyebaran secara langsung, kawalan optimum



## 1.0 INTRODUCTION

Vector–borne disease is a kind of disease which is transmitted by organisms that carry infectious pathogen from one host to another. Some of the organisms which serve as vectors include mosquitoes, fleas, biting flies, bugs, mites and ticks. There are several type of diseases that are caused by this kind of transmission including malaria, dengue fever, yellow fever, chikungunya and several others. According to the World Health Organization (WHO), 17% of the estimated global burden of all infectious diseases are caused by vector-borne diseases.

As far as mathematical modelling is concerned, vector-borne disease is often being modelled as an interaction between the hosts population, particularly between human and vector population. The population for each of them is then being classified into several distinct classes or also known as compartment with each portrays the host and the mosquito disease status. Some of the early studies on mathematical model for vector-borne disease include Feng and Velasco-Hernández [1], Esteva and Vargas [2], Ferguson *et al*. [3], Ngwa and Shu [4], and Esteva and Matias [5].

Though vector-borne diseases are primarily transmitted via vectors, there are several findings which indicated the occurrence of virus transmission through blood transfusion. This is reported in several articles including the one written by Kelly *et al*. [6], Kitchen and Chiodini [7], Tambyah *et al*. [8], Punzel *et al*. [9] and Stramer *et al*. [10]. Furthermore, Harif *et al*. [11] did an experimentation to investigate the existence of viremia among blood donors, where a random serum samples from 360 donors were selected from blood donated during the period of December 2009 to January 2010, in which the period mentioned was the time of dengue fever outbreak. Their results indicated that 15 of the donors may be in the carrier stage of the dengue virus which may eventually lead to the possibility of transmitting the virus through blood transfusion.

In consideration of this fact, several studies had addressed the situation of vector-borne transmission through blood transfusion from the mathematical point of view. One of them is Wei *et al*. [12], who considered this situation and denoted it as direct transmission mode of vector-borne disease. A mathematical model for the transmission was formulated in [12], which include both populations, hosts and vectors, and also presented a differential-delay model with a discrete time delay which accounts for the incubation period of the vectors. Then, there is Cai and Li [13] who also addressed this issue and formulated two mathematical models. Lashari and Zaman [14] extended the model in [13] by introducing exposed classes to both host and vector populations. Whereas, Cai *et al*. [15] extended the study of [12] with modification on the incidence rate of the model.

On the formulation of optimal control problem, it is originally part of the basis in modelling infectious disease to understand transmission mechanism and investigate the appropriate control strategy [16]. For example, one can determine how to optimally manage limited resources of vaccines, or treatment facilities when there is an outbreak of infectious disease. Specifically in the case of vector-borne disease, the main objectives can be to reduce the number of infected people as well as to eradicate, if possible, the vector population so that the virus cannot be further circulated. Hence, this paper will investigate the usage of optimal control in achieving the mentioned objectives. The optimal control problem is formulated by modifying the model in [15] with the incorporation of three controls. The first control is on the practice of blood screening procedure as a way to prevent the virus to be transmitted through blood transfusion. Second is on the self-prevention effort, which is to clean up house compound, to use insect repellent, and others, particularly the one who lives at the area of high disease occurrence. The third is on the effort by health authority, which is on the usage of adulticide, larvacide and others. This type of optimal control problem with the said controls had been addressed in [17] and in [18]. However, the underlying dynamical system presented in this paper is slightly different from theirs, which then lead to a different results. In [17], the authors presented seven classes of disease status, in which they introduced an exposed class for both host and vector. Whereas, in [18], density dependent mortality rates are used for both vector and host populations.

This paper is organized as follows. In Section 2, the formulation of optimal control problem is presented, in which its existence and characterization are also shown. Then, in Section 3, discussion and the results from numerical simulation is presented. Finally, the conclusion is given in Section 4.

## 2.0 THE FORMULATION OF OPTIMAL CONTROL PROBLEM

In this section, a vector-host epidemic model which characterizes both direct transmission and the conventional way of transmission through vector is presented. The dynamical system for both host and vector population is governed by a model which is formulated in [15]. The dynamical system is as presented in (1). $S_h(t)$, $I_h(t)$ and $R_h(t)$ represent susceptible, infected and recovered host population size at time *t*, respectively. On the other hand vector population is classified into two subpopulations. These are $S_v(t)$ and $I_v(t)$. Accordingly, $S_v(t)$ and $I_v(t)$ represent the number of susceptible and infected vectors at time *t* respectively. All parameters are non-negative, where $b_1$ and $b_2$ are the recruitment rate of host and vector respectively. The terms $\beta_1 S_h(t) I_h(t)$, $\beta_2 S_h(t) I_v(t)$ and $\beta_3 S_v(t) I_h(t)$ denote the occurrence of new incidence through blood transfusion, infection from infected vector to susceptible host and infection from infected host to susceptible vector respectively. $\mu_h$ and $\mu_v$ represent the natural death rate for host and vector respectively. The host recovery rate is denoted by $\gamma$ and $\alpha$ represent the disease induced death rate.

$$
\begin{aligned}
\frac{dS_h(t)}{dt} &= b_1 - \beta_1 S_h(t) I_h(t) - \beta_2 S_h(t) I_v(t) - \mu_h S_h(t) \\
\frac{dI_h(t)}{dt} &= \beta_1 S_h(t) I_h(t) + \beta_2 S_h(t) I_v(t) \\
&\quad -(\gamma + \mu_h + \alpha) I_h(t) \\
\frac{dR_h(t)}{dt} &= \gamma I_h(t) - \mu_h R_h(t) \\
\frac{dS_v(t)}{dt} &= b_2 - \beta_3 S_v(t) I_h(t) - \mu_v S_v(t) \\
\frac{dI_v(t)}{dt} &= \beta_3 S_v(t) I_h(t) - \mu_v I_v(t)
\end{aligned} \tag{1}
$$

Accordingly, in this paper the dynamical system of (1) is modified by the inclusion of three control variables. The three controls introduced are, cost of basic prevention practice in blood donation procedure, $u_1(t)$, self-prevention effort (repellent, clean up possible vector breeding sites such as vase, pail and others), $u_2(t)$, and the cost to reduce the vector population by health authority such as adulticide and larvacide, $u_3(t)$. Also, the recruitment rate in both host and vector susceptible population is modified so that it will be density dependent [1, 16]. This is denoted as follows.

$b_1 \to b_1 + \rho N_h$ , and $b_2 \to b_2 N_v$ ,

which then, transformed the system of (1) to the one in (2). Note that $\rho$ represents the proportionality constant showing the impact of density to host recruitment rate. If there is no new host recruitment, then $\rho$ will be the per capita birth rate of host [19]. $N_h(t)$ denotes the size of total host population at time $t$, in which it is the sum of $S_h(t)$, $I_h(t)$ and $R_h(t)$. On the other hand, $N_v(t)$ is the sum of $S_v(t)$ and $I_v(t)$, which represents the size of total vector population.

$$
\begin{aligned}
\frac{dS_h(t)}{dt} &= b_1 + \rho N_h - (1-u_1)\beta_1 S_h(t) I_h(t) \\
&\quad -(1-u_2)\beta_2 S_h(t) I_v(t) - \mu_h S_h(t) \\
\frac{dI_h(t)}{dt} &= (1-u_1)\beta_1 S_h(t) I_h(t) + (1-u_2)\beta_2 S_h(t) I_v(t) \\
&\quad -(\gamma + \mu_h + \alpha) I_h(t) \\
\frac{dR_h(t)}{dt} &= \gamma I_h(t) - \mu_h R_h(t) \\
\frac{dS_v(t)}{dt} &= b_2 N_v (1-u_3) - \beta_3 S_v(t) I_h(t) - r_0 u_3 S_v \\
&\quad -\mu_v S_v(t) \\
\frac{dI_v(t)}{dt} &= \beta_3 S_v(t) I_h(t) - r_0 u_3 I_v - \mu_v I_v(t)
\end{aligned} \tag{2}
$$

It should be noted that there is an addition of new rate constant, $r_0$ in (2) since the mortality rate of both susceptible and infected vectors may increase in proportion to the third control, $u_3(t)$ [19]. The rate of $r_0$ is always positive. Then, the objective functional is defined as:

$$
J(u_1,u_2,u_3) = \int_0^T \begin{bmatrix} A_1 I_h(t) + A_2 N_v(t) + B_1 u_1^2(t) \\ +B_2 u_2^2(t) + B_3 u_3^2(t) \end{bmatrix} dt \tag{3}
$$

which is subject to the system of (2). The parameters $A_1$, $A_2$, $B_1$, $B_2$ and $B_3$ are positive weight constants. The terms $A_1 I_h(t)$, $A_2 N_v(t)$ denote the cost associated in reducing the infected host and vector population respectively. Also, $B_1 u_1^2(t)$, $B_2 u_2^2(t)$ and $B_3 u_3^2(t)$ represent the cost associated with the basic practice of blood donation procedure, self-prevention effort by host, and vector control, respectively. The purpose is then to find an optimal control triplet $u_1^*$, $u_2^*$ and $u_3^*$ which satisfy:

$$
J(u_1^*,u_2^*,u_3^*) = \min_{(u_1,u_2,u_3)\in \mathbf{U}} J(u_1,u_2,u_3)
$$

where

$$
\mathbf{U} = \begin{Bmatrix} (u_1,u_2,u_3) \mid u_i(t) : 0 \le u_i(t) \le m_i,\ 0 \le t \le T, \\ i = 1,2,3,\ \mathrm{u}_i(t) \text{ is Lebesgue measurable} \end{Bmatrix}
$$

Accordingly, the existence and characterization of its optimal control for the above formulation will be shown in the following section. The Hamiltonian function, $H$ with respect to $u_1$, $u_2$ and $u_3$ is defined as follows:

$$
\begin{aligned}
H &= A_1 I_h(t) + A_2 N_v(t) + B_1 u_1^2(t) + B_2 u_2^2(t) + B_3 u_3^2(t) \\
&+ \lambda_1 \begin{bmatrix} b_1 + \rho N_h(t) - (1-u_1(t))\beta_1 S_h(t) I_h(t) \\ -(1-u_2(t))\beta_2 S_h(t) I_v(t) - \mu_h S_h(t) \end{bmatrix} \\
&+ \lambda_2 \begin{bmatrix} (1-u_1(t))\beta_1 S_h(t) I_h(t) + (1-u_2(t))\beta_2 S_h(t) I_v(t) \\ -(\gamma + \mu_h + \alpha) I_h(t) \end{bmatrix} \\
&+ \lambda_3 \left[\gamma I_h(t) - \mu_h R_h(t)\right] \\
&+ \lambda_4 \left[b_2 N_v(t)(1-u_3(t)) - \beta_3 S_v(t) I_h(t) - r_0 u_3(t) S_v(t)\right] \\
&+ \lambda_5 \left[\beta_3 S_v(t) I_h(t) - r_0 u_3(t) I_v(t) - \mu_v I_v(t)\right]
\end{aligned}
$$

## 2.1 Existence of Optimal Control

**Theorem 2.1**: Consider the objective functional of (3) with $(u_1,u_2,u_3) \in \mathbf{U}$ subject to the controlled system of (2). There exists $u^* = (u_1^*,u_2^*,u_3^*) \in \mathbf{U}$ such that

$$
\min_{(u_1,u_2,u_3)\in \mathbf{U}} J(u_1,u_2,u_3) = J(u_1^*,u_2^*,u_3^*)
$$

**Proof**: Stated in [20], the following conditions should be satisfied as to ensure the existence of optimal control.

i. the set of controls and corresponding state variables is nonempty
ii. the control set $\mathbf{U}$ is convex and closed
iii. the right hand side of the state system is bounded by a linear function in the state and control variables
iv. the integrand of the objective functional is convex on $\mathbf{U}$
v. the integrand of the objective functional is bounded below by $c_1\left(|u_1|^2 + |u_2|^2 + |u_3|^2\right)^{\frac{b}{2}} - c_2$, where $c_1$ and $c_2$ are positive constants and $b > 1$.

To verify these properties, the result from Lukes [21] is used to give the existence of solutions for the state system of (2) with bounded coefficients, which gives Condition i. The control set is closed and convex by definition, hence satisfies Condition ii. The right hand side of system (2) satisfies Condition iii since the state solutions are bounded. The integrand of our objective functional is clearly convex on $\mathbf{U}$, which then gives Condition iv. Also, there are $c_1, c_2 > 0$ and $\beta > 1$ satisfying

$$
A_1 I_h(t) + A_2 N_v(t) + B_1 u_1^2(t) + B_2 u_2^2(t) + B_3 u_3^2(t) \ge c_1\left(|u_1|^2 + |u_2|^2 + |u_3|^2\right)^{\frac{b}{2}} - c_2
$$

because the state variables are bounded. □

### 2.2 Characterization of Optimal Control

Pontryagin's maximum principle [22] is used to derive the necessary conditions for the optimal control triplet. The Lagrangian, which is the Hamiltonian augmented with penalty terms for the control constraints is defined as follows.

$$\begin{aligned}L &= H - w_{11}(t)u_1(t) - w_{12}(t)(m_1 - u_1(t)) - w_{21}(t)u_2(t)\\ &\quad - w_{22}(t)(m_2 - u_2(t)) - w_{31}(t)u_3(t) - w_{32}(m_3 - u_3(t))\\ &= A_1 I_h(t) + A_2 N_v(t) + B_1 u_1^2(t) + B_2 u_2^2(t) + B_3 u_3^2(t)\\ &\quad + \lambda_1\begin{bmatrix} b_1 + \rho N_h(t) - (1-u_1(t))\beta_1 S_h(t) I_h(t)\\ -(1-u_2(t))\beta_2 S_h(t) I_v(t) - \mu_h S_h(t)\end{bmatrix}\\ &\quad + \lambda_2\begin{bmatrix}(1-u_1(t))\beta_1 S_h(t) I_h(t) + (1-u_2(t))\beta_2 S_h(t) I_v(t)\\ -(\gamma + \mu_h + \alpha) I_h(t)\end{bmatrix}\\ &\quad + \lambda_3\left[\gamma I_h(t) - \mu_h R_h(t)\right]\\ &\quad + \lambda_4\left[b_2 N_v(t)(1-u_3(t)) - \beta_3 S_v(t) I_h(t) - r_0 u_3(t) S_v(t)\right]\\ &\quad + \lambda_5\left[\beta_3 S_v(t) I_h(t) - r_0 u_3(t) I_v(t) - \mu_v I_v(t)\right]\\ &\quad - w_{11}(t)u_1(t) - w_{12}(t)(m_1 - u_1(t)) - w_{21}(t)u_2(t)\\ &\quad - w_{22}(t)(m_2 - u_2(t)) - w_{31}(t)u_3(t) - w_{32}(m_3 - u_3(t))\end{aligned}$$

where $w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32} \ge 0$ are penalty multipliers satisfying

$$\begin{aligned}&w_{11}(t)u_1(t) = 0, \quad w_{12}(t)(m_1 - u_1(t)) = 0 \quad \text{at } u_1^*,\\ &w_{21}(t)u_2(t) = 0, \quad w_{22}(t)(m_2 - u_2(t)) = 0 \quad \text{at } u_2^* \text{ and}\\ &w_{31}(t)u_3(t) = 0, \quad w_{32}(t)(m_3 - u_3(t)) = 0 \quad \text{at } u_3^*.\end{aligned}$$

**Theorem 2.2**: Given optimal controls of $u_1^*, u_2^*, u_3^* \in \mathbf{U}$ and corresponding state solutions $S_h^*, I_h^*, R_h^*, S_v^*$ and $I_v^*$ of the corresponding state system, there exist adjoint functions $\lambda_i$ for $i = 1,2,\ldots,5$ satisfying

$$\begin{aligned}\lambda_1' &= \lambda_1\left[-\rho + (1-u_1)\beta_1 I_h + (1-u_2)\beta_2 I_v + \mu_h\right]\\ &\quad - \lambda_2\left[(1-u_1)\beta_1 I_h + (1-u_2)\beta_2 I_v\right]\\ \lambda_2' &= -A_1 + \lambda_1\left[-\rho + (1-u_1)\beta_1 S_h\right]\\ &\quad + \lambda_2\left[-(1-u_1)\beta_1 S_h + (\gamma + \mu_h + \alpha)\right]\\ &\quad - \lambda_3\gamma + \lambda_4\beta_3 S_v - \lambda_5\beta_3 S_v \qquad (4)\\ \lambda_3' &= \lambda_3\mu_h - \lambda_1\rho\\ \lambda_4' &= -A_2 + \lambda_4\left[-b_2(1-u_3) + \beta_3 I_h + \mu_v + r_0 u_3\right] - \lambda_5\beta_3 I_h\\ \lambda_5' &= -A_2 + (1-u_2)\beta_2 S_h(\lambda_1 - \lambda_2) - \lambda_4 b_2(1-u_3)\\ &\quad + \lambda_5(\mu_v + r_0 u_3)\end{aligned}$$

with the transversality condition of

$$\lambda_i(T) = 0, \text{ for } i = 1,2,\ldots 5. \qquad (5)$$

The optimal controls are given by,

$$\begin{aligned}u_1^* &= \max\left\{0, \min\left\{m_1, \frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1)}{2B_1}\right\}\right\},\\ u_2^* &= \max\left\{0, \min\left\{m_2, \frac{\beta_2 S_h I_v(\lambda_2 - \lambda_1)}{2B_2}\right\}\right\} \text{ and} \qquad (6)\\ u_3^* &= \max\left\{0, \min\left\{m_3, \frac{\lambda_4(b_2 N_v + r_0 S_v) + \lambda_5 r_0 I_v}{2B_3}\right\}\right\}.\end{aligned}$$

**Proof**: The form of the adjoint functions and transversality conditions are standard results from Pontryagin's Maximum Principle [22]. The Lagrangian is differentiated with respect to states, $S_h$, $I_h$, $R_h$, $S_v$ and $I_v$ respectively, which resulted in the following adjoint functions.

$$\begin{aligned}\lambda_1' &= -\frac{\partial L}{\partial S_h} = \lambda_1\left[-\rho + (1-u_1)\beta_1 I_h + (1-u_2)\beta_2 I_v + \mu_h\right]\\ &\quad - \lambda_2\left[(1-u_1)\beta_1 I_h + (1-u_2)\beta_2 I_v\right]\\ \lambda_2' &= -\frac{\partial L}{\partial I_h} = -A_1 + \lambda_1\left[-\rho + (1-u_1)\beta_1 S_h\right]\\ &\quad + \lambda_2\left[-(1-u_1)\beta_1 S_h + (\gamma + \mu_h + \alpha)\right]\\ &\quad - \lambda_3\gamma + \lambda_4\beta_3 S_v - \lambda_5\beta_3 S_v\\ \lambda_3' &= -\frac{\partial L}{\partial R_h} = \lambda_3\mu_h - \lambda_1\rho\\ \lambda_4' &= -\frac{\partial L}{\partial S_v} = -A_2 + \lambda_4\left[-b_2(1-u_3) + \beta_3 I_h + \mu_v + r_0 u_3\right] - \lambda_5\beta_3 I_h\\ \lambda_5' &= -\frac{\partial L}{\partial I_v} = -A_2 + (1-u_2)\beta_2 S_h(\lambda_1 - \lambda_2) - \lambda_4 b_2(1-u_3)\\ &\quad + \lambda_5(\mu_v + r_0 u_3)\end{aligned}$$

with zero transversality conditions. The characterization of the optimal control (6) is obtained by solving

$$\begin{aligned}\frac{\partial L}{\partial u_1} &= 2B_1 u_1^*(t) + \beta_1 S_h I_h(\lambda_1 - \lambda_2) - w_{11} + w_{12} = 0, \text{ at } u_1^*,\\ \frac{\partial L}{\partial u_2} &= 2B_2 u_2^*(t) + \beta_2 S_h I_v(\lambda_1 - \lambda_2) - w_{21} + w_{22} = 0, \text{ at } u_2^* \text{ and}\\ \frac{\partial L}{\partial u_3} &= 2B_3 u_3^*(t) - \lambda_4(b_2 N_v + r_0 S_v) - \lambda_5 r_0 I_v - w_{31} + w_{32} = 0, \text{ at } u_3^*.\end{aligned}$$

Solving for each of the optimal control,

$$\begin{aligned}u_1^* &= \frac{\beta_1 S_h I_h(\lambda_1 - \lambda_2) + w_{11} - w_{12}}{2B_1},\\ u_2^* &= \frac{\beta_2 S_h I_v(\lambda_1 - \lambda_2) + w_{21} - w_{22}}{2B_2},\\ u_3^* &= \frac{\lambda_4(b_2 N_v + r_0 S_v) + \lambda_5 r_0 I_v + w_{31} - w_{32}}{2B_3}\end{aligned}$$

Then, to determine an explicit expression for $u_1^*$ without $w_{11}$ and $w_{12}$, the following three cases should be considered.

i. On the set $\{t \mid 0 < u_1^* < m_1\}$ we have

$$w_{11}(u_1^*) = w_{12}(m_1 - u_1^*) = 0 \Rightarrow w_{11} = w_{12} = 0$$

Hence, the optimal control is

$$u_1^* = \frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1)}{2B_1}$$

ii. On the set $\{t \mid u_1^*(t) = m_1\}$, we have

$$w_{11}(u_1^*) = w_{12}(m_1 - u_1^*) = 0 \Rightarrow w_{11} = 0$$

Hence, the optimal control is

$$m_1 = u_1^* = \frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1) - w_{12}}{2B_1}$$

which implies that

$$\frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1)}{2B_1} \ge m_1 \text{ since } w_{12}(t) \ge 0$$

iii. On the set $\{t \mid u_1^*(t) = 0\}$, we have

$$w_{11}(u_1^*) = w_{12}(m_1 - u_1^*) = 0 \Rightarrow w_{12} = 0$$

Hence

$$0 = u_1^* = \frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1) + w_{11}}{2B_1}$$

which implies that

$$\frac{\beta_1 S_h I_h(\lambda_2 - \lambda_1)}{2B_1} \le 0 \text{ since } w_{11}(t) \ge 0$$

Combining these three cases, the optimal control of $u_1^*$ can be characterized by

$$u_1^* = \max\left\{0, \min\left\{m_1, \frac{\beta_1 S_h I_h (\lambda_2 - \lambda_1)}{2B_1}\right\}\right\}$$

Using similar argument, the optimal control of $u_2^*$ and $u_3^*$ are obtained as follows

$$u_2^* = \max\left\{0, \min\left\{m_2, \frac{\beta_2 S_h I_v (\lambda_2 - \lambda_1)}{2B_2}\right\}\right\} \text{ and}$$

$$u_3^* = \max\left\{0, \min\left\{m_3, \frac{\lambda_4 (b_2 N_v + r_0 S_v) + \lambda_5 r_0 I_v}{2B_3}\right\}\right\} \quad \square$$

## 3.0 NUMERICAL RESULTS AND DISCUSSION

In this section, the optimal solution to the vector host model (2) is solved numerically using forward-backward sweep method introduced in [23]. The optimality system, which consisted of the state equations of (2), the adjoint equations (4) and controls characterization (6), is solved using iterative method of fourth-order Runge-Kutta scheme. The algorithm started with the initial guess for the control variables. The state variables are solved forward in time using fourth-order Runge-Kutta method by considering the initial conditions. With the output of the state variables, and also considering the transversality conditions of (5) , the adjoint variables are solved backward in time also using the fourth-order Runge-Kutta method. The control variables were then being updated with the insertion of the current value of the state and adjoint variables. The process is repeated until they converge sufficiently.

The initial conditions, which represent the initial population size of each classes are assumed to be $S_h(0) = 200$, $I_h(0) = 50$ and $R_h(0) = 80$, where they are the susceptible, infected and recovered population for host respectively. On the other hand, for the population of vectors, they are assumed to be $S_v(0) = 500$ and $I_v(0) = 100$. These values of initial conditions are extracted from [24]. Table 1 presents the value of parameters used in the numerical simulation.

Other parameters values are taken from [17, 18, 19] in which they are arbitrarily assumed. These are, $\rho = 0.00285$, $\beta_1 = 0.0004$, $\beta_2 = 0.0006$, $\beta_3 = 0.009$, $\mu_v = 0.15$ and $r_0 = 0.02$. As for the weights in the objective functional of (3), they are assumed with the value of $A_1 = 0.1$, $A_2 = 0.05$, $B_1 = 50$, $B_2 = 20$ and $B_3 = 40$. The value of $A_1$ and $A_2$ are chosen in that particular way so as to show that the minimization of the infected host population is given more importance as compared to the reduction of vector population. Whereas, the weights of $B_1$, $B_2$ and $B_3$ indicate the cost associated with the controls, considering the fact that the cost for $u_2(t)$ is lower than $u_3(t)$ and the cost of $u_3(t)$ might be lower than $u_1(t)$. Also, the control variables, $u_1(t)$, $u_2(t)$ and $u_3(t)$ may vary from 0 to 1.

Considering the estimated value of parameters and initial conditions mentioned, the results from the numerical simulation is presented in Figure 1 – 6. Figure 1 shows the dynamic of infected host population with and without controls. It can be seen from the figure, without the controls, the size of population will increase and eventually reduced to 0. On the other hand, with the controls, the population size of infected host never increase and eventually reached 0 in a time faster than the one without the controls. Whereas, the population dynamic of susceptible vector is shown in Figure 2. From the figure, it can be seen that the population size with the controls reached the size of 0 in a slower time as compared to the one without the controls. This means that the controls allow less vector to be infected with the virus and causing the size of susceptible vector population to be slowly reduced to 0. Figure 3 shows the population dynamic of infected vector with and without the controls. Rationally, Figure 2 and Figure 3 are directly related since once susceptible vector acquired the virus, it will move to the class of infected vectors. As can be seen from Figure 3, the population size without the controls increased higher than with the controls. Eventually, both populations, which are with and without the controls reached 0 but the one with the controls reached to 0 in a faster time. Also, the rate of each of the three controls over time are shown in Figure 4, Figure 5 and Figure 6. From Figure 4, during the initial outbreak of the disease, full rate of control for blood screening procedure need to be enforced. This enforcement need to be undertaken until it reaches the time between day 30-35, in which afterward the rate of control can gradually be reduced. In relation with the disease status, as what is presented in Figure 1, this is the time when the infected host population started to be zero. Full enforcement rate of control also need to be implemented for self-prevention effort and pesticide control during initial disease outbreak. This enforcement need to be implemented until it reaches the time between day 40-45. Afterward, both rate of control can steadily be reduced and eventually reached to the point of no controls are needed.

**Table 1** Estimated value of parameters

| Parameter | Description | Estimated Value | Reference |
|---|---|---|---|
| $b_1$ | Recruitment rate of susceptible host | 2.5 per day | [25] |
| $b_2$ | Recruitment rate of susceptible vector | 0.4 per day | [26] |
| $\mu_h$ | Host natural death rate | 0.0000457 per day | [2] |
| $\gamma$ | Host recovery rate | 0.1428 per day | [2] |
| $\alpha$ | Disease induced death rate | 0.01 per day | [26] |

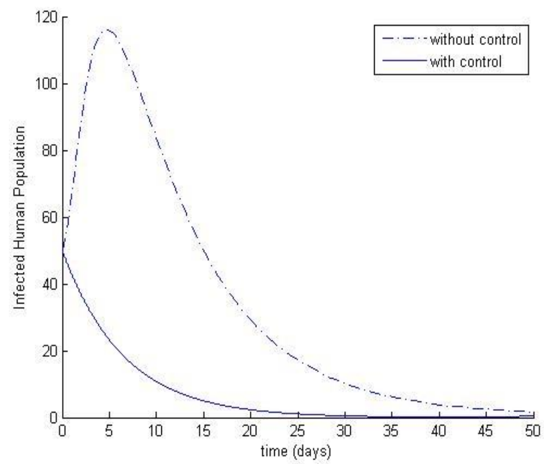



**Figure 1** Infected host population

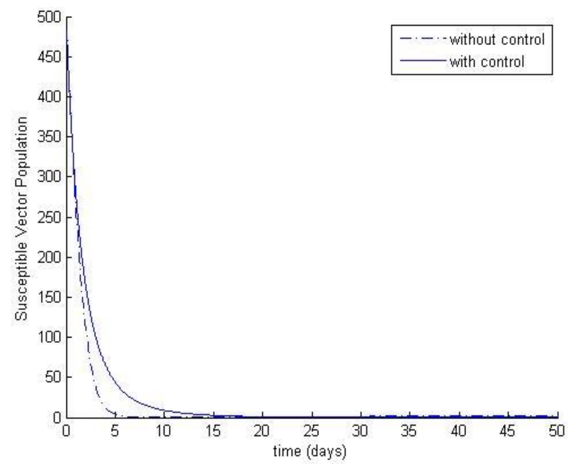



**Figure 2** Susceptible vector population

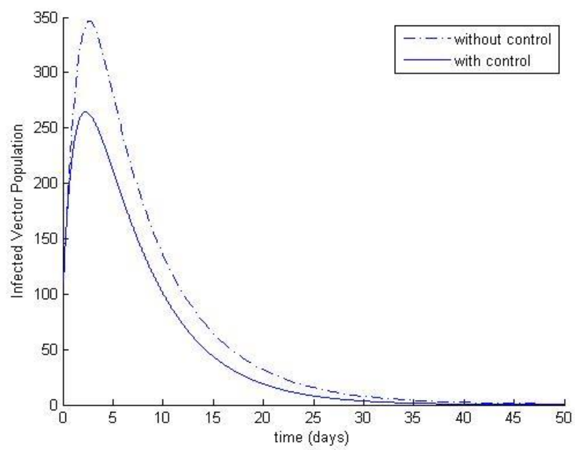



**Figure 3** Infected vector population

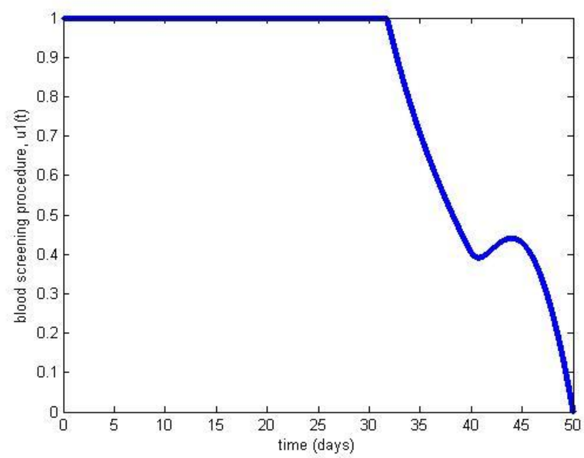



**Figure 4** Blood screening procedure

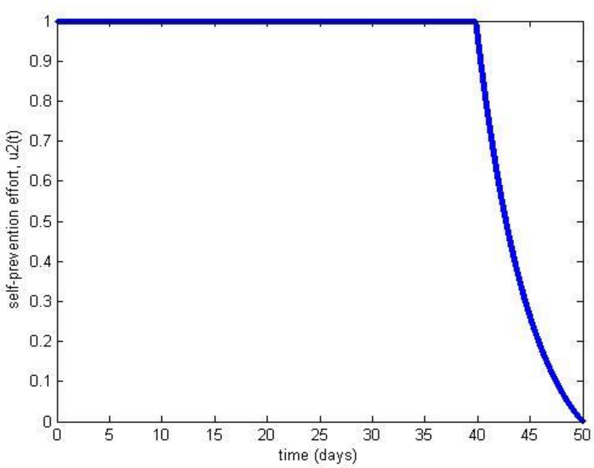



**Figure 5** Self-prevention effort

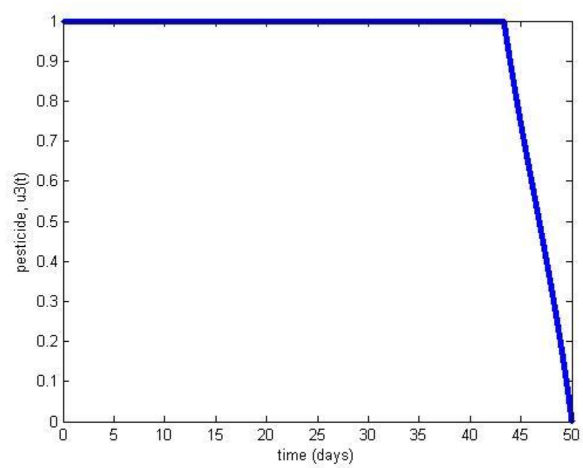



**Figure 6** Pesticide control

## 4.0 CONCLUSION

This paper has introduced an optimal control problem to a dynamical system of vector-borne disease with direct transmission. By direct transmission, it means that the disease can also be transmitted directly from one host to another apart from the conventional way of its transmission through vector. This particularly occurred through blood transfusion in which real life cases have been reported in several studies. As a way to reduce or if possible to eradicate the disease, three controls have been introduced in this study. One is to perform blood screening procedure during blood transfusion process. The other control is to do self-prevention effort, for example by cleaning up house compound and wearing insect repellent. The last control is the responsibility of the health authority to provide adulticide and lavarcide as a method to reduce the size of vector population. With the said controls, and its underlying dynamical system, the optimal control problem is formulated, with the objective to reduce the size of infected host population and also the size of vector population. Eventually, the optimality system is derived using the Pontryagin's maximum principle. Then, numerical simulation is performed to observe the impact of the controls to the population dynamic of both host and vector. Based on the results, it can be said that the introduction of the controls have caused the population dynamic of host and vector to reform in a positive way. It should be noted that several modification can be done to the dynamical system. Firstly, instead of using constant population for both host and vector, one can change it to variable population such as the one done by [4] and [27]. Also, the incidence rate can be changed to bilinear and saturation incidence as were done by [5] and [28]. This subject matter is also under consideration by the authors.

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