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Nonlinear Smooth Transition Autoregressive (Star)–Type Modelling And Forecasting On Malaysia Airlines (Mas) Stock Returns

Siti Rohani Mohd Nor[a], Fadhilah Yusof[a*], Ibrahim Lawal Kane[b]

[a]Department of Mathematical Sciences, Faculty of Science, University Teknology Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[b]Department of Mathematics and Computer Science, Umaru Musa Yar'adua University, 2218 Katsina State, Nigeria

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*Corresponding author
fadhilahy@utm.my

Graphical Abstract

ACF of squared residuals

McLeod-Li test for AR squared residuals
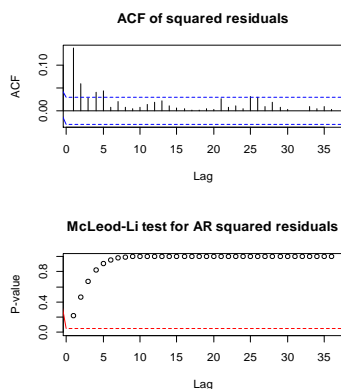
Abstract

This study aims to apply nonlinear Smooth Transition Autoregressive (STAR)-type model to the Malaysia Airlines (MAS) Stock Returns, which consists of 4450 number of observations. The data taken started from 29th August 1996 until 26th September 2014. Following the STAR strategies by Terasvirta, the diagnostic plots of linear Autoregressive (AR) model revealed that AR (3) model is adequate in modelling the MAS returns series. However, the squared residuals of Autocorrelation Function (ACF) of returns series illustrates a slight presence of correlations in the model, hence the effort to apply nonlinear model was continued. Before proceed to nonlinear STAR modelling, the identification of delay parameter in the second stage of Terasvirta need to be determined. The results of Lagrange Multiplier (LM) tests revealed that delay parameter, $d=3$ is the best to choose. In addition, the null hypothesis of linearity from LM test is rejected. Furthermore, from the sequence of nested hypothesis of delay parameter, $d=3$ indicated that LSTAR model is preferred than ESTAR model. Finally, the forecasts and comparison stages was made to compare which models are best performed in forecasting the future series of MAS returns. It proved that LSTAR model performed better in term of forecasting accuracy when compared to ESTAR and AR model.

Keywords: LSTAR, ESTAR, delay parameter, lagrange multiplier test, sequence of nested hypotheses

Abstrak

Kajian ini bertujuan untuk menggunakan model tidak linear iaitu model Pelicinan Transisi Autoregresif (STAR) dan membuat ramalan model terhadap data pulangan saham Sistem Penerbangan Malaysia (MAS). Terdapat 4450 data bertarikh di antara 29 Ogos 1996 hingga 26 September 2014. Permodelan STAR adalah berpandukan beberapa strategi yang telah disusun oleh Terasvirta. Strategi tersebut perlu dipenuhi sebelum mencapai sasaran objektif dissertasi ini, di antaranya adalah dengan menetapkan model linear Autoregresif (AR) kepada MAS data, memilih nilai parameter *d* dan menguji hipotesis nul linear, serta menentukan model yang terbaik untuk dipilih diantara model Logistik STAR (LSTAR) dan Eksponen STAR (ESTAR) model. Fasa pertama menunjukkan bahawa linear Autoregresif (AR) model adalah signifikan untuk digunakan. Namun, terdapat sedikit korelasi di dalam plot Fungsi Autokorelasi (ACF) untuk AR (3), maka usaha untuk permodelan tidak linear diteruskan. Sebelum meneruskan permodelan STAR model, transisi parameter, *d* perlu dicari dan hasil ujikaji daripada pekali Lagrange (LM) menunjukkan *d*=3 adalah yang terbaik untuk digunakan kerana mempunya nilai *p* yang paling rendah serta memperoleh nilai statistik LM yang lebih tinggi daripada yang lain. Keputusan daripada LM juga menunjukkan bahawa terdapat ciri-ciri tidak linear di dalam *d*=3. Selain itu, melalui

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hipotesis tersarang daripada $d=3$, hipotesis menunjukkan LSTAR model adalah model yang terpilih berbanding dengan ESTAR model. Akhirnya, hasil ramalan dan perbandingan antara model LSTAR, ESTAR dan AR menunjukkan bahawa LSTAR model adalah model yang paling tepat untuk dipilih kerana mempunya nilai ramalan yang paling tepat berbanding model ESTAR dan AR.

Kata kunci: LSTAR, ESTAR, transisi parameter, ujikaji pekali Lagrange, hipotesis tersarang

1.0 INTRODUCTION

Nonlinear modelling has become the centre of attraction from most of the econometrician and financer in modelling and forecasting the financial data. This is due to the complexness of the financial properties that is hard to be captured by the linear model. Maya Clayton (2011) stated that the stock returns series which is one of the major data in finance exhibited several characteristics that can be only explained by nonlinear modelling such as leptokurtosis, volatility clustering and leverage effect behaviours. However, Terasvirta (1994) said it is advisable to specify the linear model first before proceeded to the nonlinear model to check for the adequacy of linear model to the data. This is because, most of the linear model are sufficient enough to model the finance series. Thus, he suggested linearity test as a vital part in choosing nonlinear model.

Linearity test is used to check for the null hypothesis of linearity by determining the Lagrange Multiplier (LM) statistics value and *p*-values of the delay parameter. The one with the highest statistics which exceeded the Chi-Squared distributions and the lowest *p*-values indicated the existence of nonlinearity properties in it. Urrutia *et al*. (2002) had mentioned that the main reason to the weak findings of nonlinearities in the data is because of the used of aggregate data which makes the nonlinearities hard to identify. Hence, stock returns are used because of its' unique institutional behaviours that make it easier for the researches to identify the nonlinear properties inside the data. Since it is difficult to find the nonlinear properties of the series, therefore Escribano *et al*. (1994) suggested to use LM-statistics to check for the nonlinearity since it easier to compute and has a high power in determining the nonlinear Smooth Transition Autoregressive (STAR)-type model if the delay parameter *d* is chosen correctly.

Furthermore, Terasvirta and Anderson (1992) clarified that the Smooth Transition Autoregressive (STAR)-type model are used since it has the slower phase transition between the regimes switch in the model. Difference from the STAR model, Threshold Autoregressive (TAR) model, the model which was invented before the establishment of STAR model, is a discontinuous function since it causes a sudden jump between the regimes and thus make it hard for the model to have analytical computational of parameters (Luukkonen *et al*. 1988).

Finally, forecasting returns series are compared with the original series to check for the adequacy of the estimated parameters produced by the models. The complexity in the trader's behaviour that governs the movement patterns of the stock returns might be due to the limited knowledge in determining the forecast values of the future series. Maya Clayton (2011) studied that forecasting is very crucial for the financer in order to determine the outcome obtained from the futures reference. Especially for those who decided to invest in a long term investment, forecast is one of the essential parts prior to trade. More than that, the models used such as linear AR, and nonlinear-STAR type model will be compared with the results given from the test of forecast series. The purpose of the comparison is to see which of the models perform better. This is because nonlinear-STAR type model is produced under the sequence of nested hypotheses of Autoregressive (AR) model with *d* as the delay parameter. Though *d* is decided to be nonlinear, however, AR model sometimes is adequate to model the financial data.

Hence, the aim of this paper is to apply nonlinear Smooth Transition Autoregressive (STAR)-type model to the daily stock returns series of Malaysia Airlines (MAS) company by following the procedures of Terasvirta (1994).

2.0 MATERIALS AND METHODS

Following the procedures given by Terasvirta (1994), the modelling of nonlinear STAR-type model starts with the modelling of Autoregressive (AR) Linear against the STAR model. Next, the delay parameter with a symbol *d* will be chosen through Lagrange Multiplier (LM) test. Finally the two main type of STAR model will be chosen at best using a nested of sequence hypotheses. In addition, the adequacy of the models will be supported with several tests to check for the zero autocorrelation and zero heteroskedastic left in the residuals of the model. The tests mentioned for model's adequacy used in this dissertation are Breusch Godfrey test, Ljung-Box test, and McLeod-Li test. Finally, the Autoregressive (AR) Linear, Logistic STAR (LSTAR) and Exponential STAR (ESTAR) models will

be compared with the statistical statistics resulted from the forecast series of the models.

2.1 AR Modelling

Mixed autoregressive moving average (ARMA) models are the combination of Autoregressive (AR) and Moving Average (MA) models to achieve parsimony in parameterization (Box *et al.*, 1994). The model used to explain ARMA is as follows

$$y_t = \emptyset_0 + \sum_{i=0}^p \emptyset_i y_{t-i} + a_t - \sum_{i=1}^q \theta_i a_{t-1} \tag{1}$$

From (1), $\emptyset_0, \dots, \emptyset_p$ are the parameters of the autoregressive model and $\theta_i, \dots, \theta_q$ are the parameters of the moving average model, a_t is a white noise series at time t, while p and q symbolized the number of lags term in AR (p) and MA (q) model consecutively.

2.2 Serial Dependence

The model adequacy are determined by carrying several tests upon the residuals of the model chosen such as the Breusch Godfrey test, Ljung-Box test, and McLeod-Li test. These tests are used to check for the remaining correlation and heteroskedasticity in the residuals of the model.

2.2.1 Breusch-Godfrey Test

Breusch *et al.* (1979) defined Breusch-Godfrey test as a test to examine the existence of autocorrelation in the errors of a regression model y_t following Chi-Squared distribution. Breusch Godfrey is run after the model selections with order p and parameter estimates for fitted model are determined. Yusof *et al.* (2013) stated that the presence of autocorrelations in the fitted model brings a difficulty in making a statistical description of the model chosen. Therefore Breusch-Godfrey test is one of the methodologies used with the aims to encounter the problem related with the statistical inference. Estimated using OLS, the formula for Breusch-Godfrey test are given as follows:

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \rho_3 \varepsilon_{t-3} + \cdots + \rho_p \varepsilon_{t-p} + v_t \tag{2}$$

v_t is the error of ε_t following normal distribution with mean zero and variance, σ_v^2. The null and alternative hypotheses for the test with order p are:

$H_0: \rho_1 = \rho_2 = \rho_3 = \cdots = \rho_p = 0$
$H_1: \rho_1 \neq 0 \ or \ \rho_2 \neq 0 \ or \ \rho_3 \neq 0 \ or \ \dots \rho_p \neq 0$

H_0 implies that there is no serial correlation in the fitted model. The null hypothesis H_0 will be rejected if the test statistical value of Breusch Godfrey test exceeded the Chi-Squared Distribution value from the table.

2.2.2 McLeod-Li Test

McLeod-Li test was introduced by McLeod and Li at the early 1998. It was created to perform the task to check for non-zero correlation in the squared residuals (e_t^2, e_{t-k}^2) for some k of the fitted model against the Autoregressive Conditional Heteroskedasticity (ARCH) effect. Yusof *et al.*, (2013) described McLeoad-Li test as follows (3):

$$\hat{\varepsilon}(k) = \sum_{k=1}^N (e_t^2 - \hat{\sigma}^2)(e_{t-k}^2 - \hat{\sigma}^2) \Big/ \sum_{t=1}^N (e_t^2 - \hat{\sigma}^2) \tag{3}$$

where $\hat{\sigma}^2$ shows the summation from t-1 until N of $\frac{e_t^2}{N}$ and N is the number of sample size. The null hypothesis for this test is that the series targeted is an identically and independent distributed (*iid*) process. If the p-values of Breusch-Godfrey test are more than 0.05, it would be an indicator of rejection of the null hypothesis due to the presence of nonlinearity. Rejecting the null hypothesis of no ARCH effect defines that the fitted model is necessary.

$H_0: e_t^2$ is an iid process and no ARCH effect
$H_1: e_t^2$ is not an iid process

2.3 Lagrange Multiplier Test

Testing nonlinearity against the STAR model is another step suggested by Terasvirta (1994) in order to identify the nuisance parameters in different manners. Recalls the transition function in STAR model, there are two parameters that cannot be identified in the transition function of the model which are parameters γ and c. In addition, Escribano *et al.* (1994) stated that parameter Ɵ can takes any value as long as their average does not change, however the parameter Ɵ is still hard to identify. Thus, Lagrange Multiplier (LM) which has an asymptotic chi-squared distribution is tested on the model to check for the null hypothesis of linearity on the model suggested. LM-statistic is suggested since it is easy to compute and has good theoretical properties (Escribano *et al.* 1994). The null hypothesis of linearity is $H_o: \gamma = 0$. From the book entitled Non-linear time series models in empirical finance written by Franses *et al.*(1988) suggested third-order Taylor expansion approximation to replace transition function $F(y_{t-d}; \gamma, c)$ in the STAR model that is:

$$T_3(y_{t-d}; \gamma, c) \approx \gamma \left(\partial F^*(y_{t-d}; \gamma, c)/\partial \gamma \right)| \\ + (1/6)\gamma^3 \left(\partial^3 F^*(y_{t-d}; \gamma, c)/\partial \gamma^3 \right)|$$

$$= (1/4)\gamma(y_{t-d} - c) + (1/48)\gamma^3(y_{t-d} - c)^3 \tag{4}$$

where $F^*(y_{t-d}; \gamma, c)$ is the second derivative with respect to $\gamma = 0$. With this approximation, the auxillary model combined from that is given as below:

$$y_t = \beta_0 + \beta_{10} + \sum_{i=1}^p \beta_{1i} y_{t-i} + \sum_{i=1}^p \beta_{2i} y_{t-i} y_{t-d} + \\ \sum_{i=1}^p \beta_{3i} y_{t-i} y_{t-d}^2 + \sum_{i=1}^p \beta_{4i} y_{t-i} y_{t-d}^3 + \varepsilon_t$$

Where β_0 and β_i are the functions of the parameter θ, γ and c. The null hypothesis of $H_o: \gamma = 0$ now corresponds to the null hypothesis of linearity, $H_o: \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$ for $i=1,\dots, p \Leftrightarrow \gamma = 0$ which can also be tested with LM-test statistics follows an asymptotically $x^2(3p)$ distribution. The steps in computing LM statistics are based on the auxiliary model stated above are:

1) Compute the sum of square residuals $SSR_0 = \sum_{i=1}^{n} \tilde{\varepsilon}_t^2$ on x_t
2) Compute the sum of square residuals SSR_1 by estimate the auxiliary regression of $\tilde{\varepsilon}_t$ on x_t and $\tilde{x}_t y_{t-d}^j$, j=1, 2, 3.
3) LM test statistics can be tested as follows: $H_o: \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$ for $i=1,\dots, p \Leftrightarrow \gamma = 0$ is a linear model.
   H_1: Nonlinear if there exist one parameter β that is not the same with the null hypothesis.

$$LM = [(SSR\_0 - SSR\_1)]/[3p/SSR\_1/(n - 4p - 1)]$$

where 3*p* and *n-4p-1* are the degrees of freedom under the null hypothesis. The value of fixed delay parameter, *d* is determined by doing the LM-test with the different values of *d* that is bounded between 1 and *p* ($1 \le d \le p$). The delay parameter will be selected at which the *p* value is the smallest and the test statistics is the greatest.

2.4 Nonlinear (STAR)-type Modelling

There are two important models obtained from the Smooth Transition Autoregressive (STAR) model. These models are decided by the value of transition function contained in the STAR formula. The two main STAR models are the Logistic Smooth Transition Autoregressive (LSTAR) and Exponential Smooth Transition Autoregressive (ESTAR) models.

2.4.1 LSTAR Model

According to Terasvirta (1994), there are two famous transition functions that bring two different main models from STAR model. The transition function for Logistic STAR (LSTAR) model is:

$$F(y_{t-d}; \gamma, c) = [1 - \exp\{-\gamma(y_{t-d} - c)\}]^{-1}, \gamma > 0 \quad (5)$$

Since LSTAR model is a part of STAR model, thus LSTAR model allows for a smooth transition between the regime switch instead of an abrupt changes. Adopted from (Escribano *et al.* 1994), parameter γ describes the speed of the regimes adjustment and the smoothness of the transition and the size of c. Parameter γ determines the increasing ($\gamma > 0$) or decreasing ($\gamma < 0$) of a transition function by changing the sign of it and in addition the slope of the function c will become steeper as the value of γ is increasing. The steeper the transition function $F(y_{t-d}; \gamma, c)$ is, the faster the transition function $F(y_{t-d}; \gamma, c)$ will be (Zhou 2010).

Below are the steps in combining the transition function $F(y_{t-d}; \gamma, c)$ of Logistic function with nonlinear STAR-type equation model. The original STAR formula is:

$$y_t = [\pi_0 \quad \pi_1 \quad \cdots \quad \pi_p] \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} +$$
$$\left([\theta_0 \quad \theta_1 \quad \cdots \quad \theta_p] \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} \right) F(y_{t-d}) + \varepsilon_t$$

Insert the value of logistic transition function in the original formula, and thus the Logistic STAR (LSTAR) type model is defined as follows:

$$y_t = \pi' x_t + (\theta' x_t)[\{1 - \exp(-\gamma(y_{t-d} - c))\}^{-1}] + \varepsilon_t \quad (6)$$

$$y_t = \pi_0 + \sum_{i=1}^{p} \pi_i y_{t-i} + \left(\theta + \sum_{i=1}^{p} \pi_i y_{t-i} \right)$$
$$[1 - \exp(-\gamma(y_{t-d} - c))^{-1}]] + \varepsilon_t$$

2.4.2 ESTAR Model

Another model that can be acquired from the Smooth Transition Autoregressive (STAR) model is Exponential STAR (ESTAR) model. The transition function for Exponential STAR (ESTAR) model is:

$$F(y_{t-d}; \gamma, c) = 1 - \exp(-\gamma(y_{t-d} - c)^2) \quad (7)$$

ESTAR model basically has the same properties as LSTAR model. However there are certain characteristic that differ between them. If the smooth parameter γ is small, thus the transition function will switch between 0 and 1 slowly and if the parameter is large, the function will switch quickly between the bounded 0 and 1 (Zivot *et al.* 2006). The exponential function is symmetrical and ESTAR model switches between two regimes smoothly depends on the distance between y_{t-d} and c. Compared to LSTAR, ESTAR model does not matter about the sign, it is only concern about the size between y_{t-d} and c.

Below are the steps in combining the transition function $F(y_{t-d}; \gamma, c)$ of Exponential function with nonlinear STAR-type equation model. The original STAR formula:

$$y_t$$
$$= [\pi_0 \quad \pi_1 \quad \cdots \quad \pi_p] \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix}$$
$$+ \left([\theta_0 \quad \theta_1 \quad \cdots \quad \theta_p] \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} \right) F(y_{t-d}) + \varepsilon_t$$

Insert the value of exponential transition function in the original formula, and thus the Exponential STAR (ESTAR) type model is defined as follows:

$$y_t = \pi' x_t + (\theta' x_t)[1 - exp(-\gamma(y_{t-d} - c)^2] + \varepsilon_t \qquad (8)$$

$$y_t = \pi_0 + \sum_{i=1}^{p} \pi_i y_{t-i} + \left(\theta + \sum_{i=1}^{p} \pi_i y_{t-i}\right)$$
$$[1 - exp(-\gamma(y_{t-d} - c)^2] + \varepsilon_t$$

According to Escribano *et al.*, (1994), when $\theta_0 = c = 0$, this model is reduced to exponential autoregressive model (EAR). The sign of the parameter γ indicates the shape of the transition function either it is v-shaped ($\gamma > 0$) or bell-shaped ($\gamma < 0$) function. The magnitude of γ determines the speed of the switches between regimes and the sizes from c (left and right).

2.5 Decisions Rule for Selecting LSTAR and ESTAR Model

Decisions rule for selecting LSTAR and ESTAR are determined through the following sequence of nested hypothesis for the auxiliary regression model above. The sequence is run after the null hypothesis of linearity is rejected using Lagrange Multiplier (LM) test.

$$H_{01}: \beta_{4i} = 0 \qquad\qquad i = 1, ..., p$$
$$H_{02}: \beta_{3i} = 0 | \beta_{4i} = 0 \qquad i = 1, ..., p$$
$$H_{03}: \beta_{2i} = 0 | \beta_{3i} = \beta_{4i} = 0 \qquad i = 1, ..., p$$

Selection of LSTAR and ESTAR are decided by using the procedures motivated by Terasvirta (1994) above. Further elaborations about the nested hypothesis are:

1. The rejection of null hypothesis of $H_{01}: \beta_{4i} = 0$ for $i = 1, ..., p$ would imply the acceptance of LSTAR model.
2. Again, same with the number one procedure, rejection of the null hypothesis of $H_{02}: \beta_{3i} = 0 | \beta_{4i} = 0$ leads to the rejection of ESTAR model and thus accepting the null hypothesis means LSTAR model is more preferred.
3. Finally, accept the null hypothesis of $H_{03}: \beta_{2i} = 0 | \beta_{3i} = \beta_{4i} = 0$ after H_{02} are rejected would support ESTAR model. On the other hand, LSTAR model will be chosen if H_{03} is unable to accept and H_{02} are accepted.

Terasvirta stated that there is another certain condition that can help the researcher to decide which model is the best to choose that is when *p*-values of F-test of H_{01} and H_{03} are bigger than the *p*-value of H_{02}, then ESTAR model will be preferred. In addition, if H_{01} and H_{03} are rejected more strongly than H_{02}, ESTAR model would be likely to choose.

3.0 RESULTS AND DISCUSSION

There are 4450 number of observations for Malaysia Airlines (MAS) stock returns dated from 29th August 1996 until 26th September 2014. Returns series were obtained from the difference of consecutive of log

prices, $y_t = ln(P_t) - ln(P_{t-1})$. However, for forecasting purposes, we only examined 4400 data and the rest were compared with the forecasted value of the selected model. Nonlinear Smooth Transition Autoregressive (STAR)-type model was applied to the series following the procedures of Terasvirta (1994).

As proposed by Terasvirta (1994), the first stage of modelling Smooth Transition Autoregressive (STAR) strategy is by specifying the linear Autoregressive AR (*p*) model to the data. The lag length of order *p* is determined by applying the Akaike Information Criterion to the Autoregressive model, y_t. Table 1 summarizes the test's result for ordered value of AR (3) model:

Table 1 Estimated parameters of AR (p) model

Coefficients	Estimates	S.E
	-0.0003	0.0002
ar1	-0.0167	0.0151
ar2	0.0065	0.0151
ar3	0.0283	0.0151

The ACF of the residuals and the p-values for Ljung-Box statistics show that there are no correlation exists in the residuals of the estimated AR (3) parameters. To support this statement, Breusch-Godfret test and McLeod-Li test were applied to the estimated parameters of AR (3).

Table 2 Breusch-Godfrey test for fitted AR(3) residual model

	Coefficient	p-values
Breusch-Godfrey		0.8606
Intercept	-7.5270×10^{-8}	0.9997
lag(resid)_1	-4.6704×10^{-4}	0.9753
lag(resid)_2	-5.6195×10^{-4}	0.9703
lag(resid)_3	-7.8745×10^{-5}	0.9958
lag(resid)_4	1.0291×10^{-2}	0.4951
lag(resid)_5	1.8199×10^{-2}	0.2276

Breusch-Godfrey test (Table 2) provides evidence that AR (3) is sufficient enough to model the data since the null hypothesis of no serial dependence autocorrelation is accepted. According to Fadhilah *et al.* (2013), the correlations exist in the chosen model also need to be tested in the squared residuals of the models instead of only dependent on the residuals of the models.
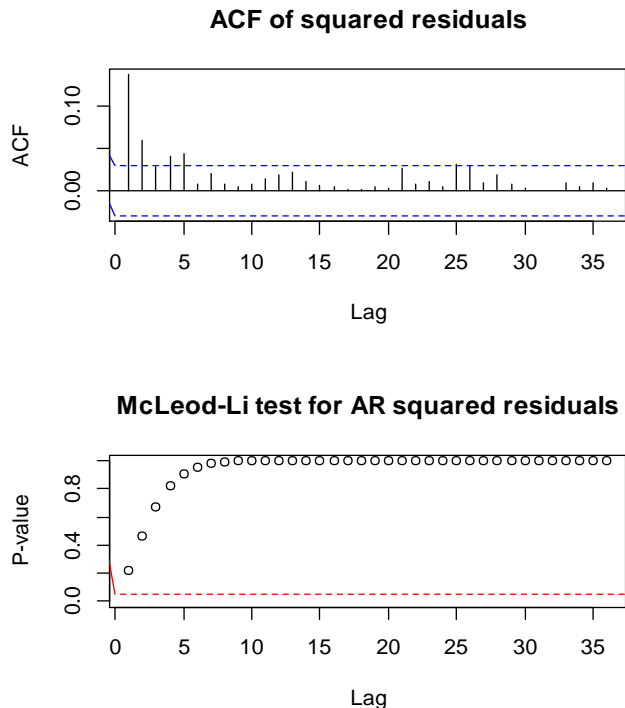
ACF of squared residuals

McLeod-Li test for AR squared residuals

Figure 1 McLeod-Li test and the squared residuals of AR (3) model

Based on Figure 1, there exists autocorrelation in the squared residuals of AR (3) model since there is several marginally significant values at a first few lags in the ACF plot. Yusof *et al.*(2013), in her case study stated that although the squared residuals of the model shows the existence of correlation, the results acquired by the Ljung-Box statistics and McLeod-Li test are more preferable in supporting the adequacy of the model compared to the diagram of the estimated models. Since McLeod-Li test illustrates the acceptance of the null hypothesis of no ARCH effect in the squared residuals, the model is adequate in modelling the series. However, since there exists a slight correlation at the first few lags of the squared residuals of ACF, the next stages of Terasvirta (1994) of testing the presence of linearity for different values of delay parameter was performed to the AR (3) model.

3.1 Nonlinear Model

Lagrange Multiplier (LM) test is used to test for the presence of nonlinearity existence in each of the delay parameter, *d* in the transition variables of AR (3) model. Table 3 below gives the estimated statistics of the delay parameters.

Table 3 LM test on AR (3) model

	d=1	d=2	d=3
LM Statistic	19.812	28.951	36.844
p-value	0.012	0.000	0.000

The null hypothesis of linearity is rejected since the value of LM statistics exceeds the value of Chi-Squared distribution which is 10.39. Consequently, the delay parameters give high power in modelling the nonlinear-Smooth Transition Autoregressive (STAR) type model since all of the LM statistics are more than the given value of Chi-Squared test. The results of the Lagrange Multiplier (LM) test indicates that the delay parameter, *d*=3 is the best parameter picked to model nonlinear STAR-type model since it has the highest LM statistic and the lowest p-value among the other delay parameters.

Table 4 Nested hypothesis with *d*=3

Parameter	Coefficient	F-Statistic	*p*-values
$\beta_{0,}$	-0.000603	28.951	0.000
$\beta_{1,1}$	-1.966	24.4790	7.794×10^{-7}
$\beta_{1,2}$	-0.4484	3.2295	0.07239
$\beta_{1,3}$	-0.1370	34.8495	3.831×10^{-9}
$\beta_{2,1}$	-7.289	4.3908	0.03619
$\beta_{2,2}$	-2.173	4.8927	0.02702
$\beta_{2,3}$	1.324	0.1547	0.69408
$\beta_{3,1}$	-38.38	0.0136	0.90729
$\beta_{3,2}$	-20.12	2.6368	0.10448
$\beta_{3,3}$	-3.651	0.3306	0.565359

The last stage on the STAR modelling strategy is to choose between LSTAR and ESTAR model through the sequence of the nested hypotheses (Table 4). According to the procedures given in the selection of LSTAR and ESTAR models motivated by Terasvirta (1994):

1. The null hypothesis of $H_{01}: \beta_{3i} = 0$ for i = 1, ... , p is accepted since *p*-values of $\beta_{3,1}$, $\beta_{3,2}$ and $\beta_{3,3}$ are more than 0.05, which would imply the acceptance of ESTAR model.
2. Again, same with the number one procedure, the null hypothesis of $H_{02}: \beta_{2i} = 0| \beta_{3i} = 0$ is rejected which eventually leads to the rejection of ESTAR model since the parameters of β_{2i} is not accepted equal to zero given the parameters $\beta_{3i} = 0$

It is concluded that LSTAR model is the most preferred model instead of ESTAR model following the decision rules outlined by Terasvirta (1994). The next step will be on choosing the best fitted LSTAR model.

3.2 Estimation and Evaluation of (LSTAR) Model

Table 5 gives the significant of the parameters value for the nonlinear and linear part of LSTAR model.

Table 5 Parameters Estimation of LSTAR (3, 3) model

Parameter	Estimate	Standard Error	t-Value	p-values
Linear Part				
X_{t-3}	0.02875	0.01641	1.75	0.0499
Non-Linear Part				
X_{t-1}	1.115	0.5846	-1.908	0.0565
X_{t-3}	-27.17	4.246	-6.398	1.73×10^{-10}
Gamma (γ)	1030	493.80	2.087	0.0370
CI	-0.03083	0.00003048	-1011.49	Less than 2×10^{-16}

The parameters estimated in the LSTAR (3, 3) model with 4400 observations for d=3 and γ = 1030 are shown in Table 5. With the given results, LSTAR (3, 3) model yields:

$$y_t = \frac{0.02875 y_{t-3} + (-1.115 y_{t-1} - 27.17 y_{t-3})^{-1}}{(1 + \exp[-1030(y_{t-3} - 0.03083)])} + \varepsilon_t$$

where the logistic transition function

$$F(y_{t-3}) = (1 + \exp[-1030(y_{t-3} - 0.03083)])^{-1}$$

and ε_t is the error for the model

Table 6 gives descriptive statistics of LSTAR model and several tests and figures were run and display to justify the adequacy of LSTAR model for MAS stock returns data.

Table 6 The descriptive statistics of LSTAR (3, 3)

| Estimation	Mean
Mean	0.0002984268
Variance	0.000178
Standard Deviation	0.013346
Skewness	0.2189704
Kurtosis	0.210448
Mean Zero	0.5442

The mean, variance and standard deviation of LSTAR model are positive and almost close to zero as that to normal distribution. In addition, the measure of kurtosis is almost to zero whereas the measure of skewness is positive and closer to zero. Therefore, the descriptive statistics in Table 6 shows that LSTAR (3,3) model follows a normal distribution.

3.2.1 Autocorrelation Residuals Test on Logistic Smooth Transition Autoregressive (LSTAR) Model

The sample ACF on Figure 2 shows that the LSTAR (3, 3) model is almost uncorrelated as most of the spikes for lag residuals and squared residuals are exponentially decreasing to 0 when the number of lags increases. While the residuals of the fitted model seem

uncorrelated, the ACF given in the Figure 2 below does not give enough justification to verify the accuracy of the model. This is because, from the visual inspection of Figure 2 (b), the time plot of the residuals are not independent and identically distributed (iid) through time. Wang (2006) stated that there is tendency of large absolute values of residuals influenced by the small absolute values of abrupt and unpredictable sign that leads to the dependency of the residuals. Hence, several formal tests are designed to check for the presence of any autocorrelation and Arch effect lefts in the model.

ACF of LSTAR residuals
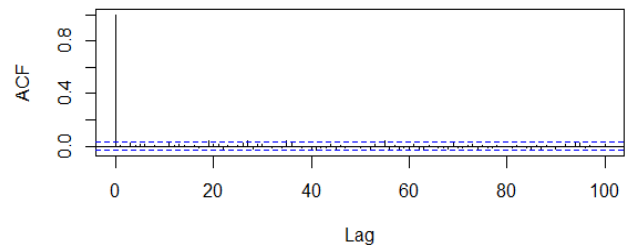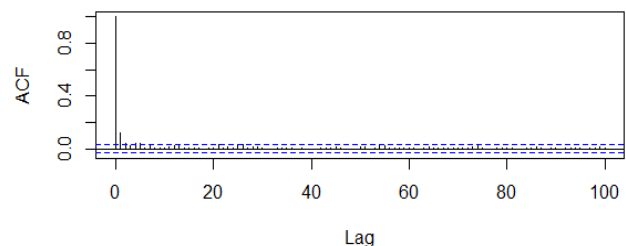

ACF of LSTAR squared residuals

Figure 2 LSTAR (3, 3) model with d=3 and γ = 1030. Panel (a) gives the ACF of LSTAR (3, 3) residuals (b) is the ACF of LSTAR (3, 3) squared residuals for lags until 4000

Table 7 Breusch-Godfrey test result for LSTAR(3,3) model

	Coefficient	p-value
Breusch-Godfrey		0.7428
Intercept	-1.7163×10^{-8}	0.9999
lag(resid)_1	5.8275×10^{-3}	0.6979
lag(resid)_2	-5.8948×10^{-3}	0.6946
lag(resid)_3	1.9579×10^{-2}	0.1922
lag(resid)_4	7.1278×10^{-3}	0.6350
lag(resid)_5	1.0238×10^{-2}	0.4954
lag(resid)_6	1.4544×10^{-2}	0.3328
lag(resid)_7	-4.4952×10^{-3}	0.7646
lag(resid)_8	2.1746×10^{-3}	0.8848
lag(resid)_9	-5.2431×10^{-3}	0.7269

Table 7 gives the result of the Breusch-Godfrey test from the first lag of the residuals until the ninth lag. From the table, the p-values of each of the lags' coefficients exceeded 0.05, hence the null hypothesis

for the correlation remains in the model assumption LSTAR (3, 3) is rejected.

3.2.2 The Heteroskedasticity Residuals Test on Logistic Smooth Transition Autoregressive (LSTAR) Model

The result from the Ljung-Box Q-test statistic in Table 8 confirms the result from the visual inspection in Figure 2. Ljung-Box test was applied to the residuals of the LSTAR (3, 3) model with difference trial of lags until 30th lag. The p-value for the residuals lag are more than 0.05 significant values, indicate that the null hypothesis of no arch effect in the model is accepted. This could be concluded that LSTAR model can be adequately capture the behaviour of the data for the daily series.

Table 8 Ljung-Box Q-test result for LSTAR(3,3) model

	Statistic	p-value
Residuals		
Up to lag 1	0.0067	0.9348
Up to lag 10	4.0848	0.9434
Up to lag 15	12.0419	0.6759
Up to lag 20	19.8737	0.4659
Up to lag 30	38.1235	0.1466
Squared Residuals		
Up to lag 1	70.8882	2.2×10^{-16}
Up to lag 10	99.7843	2.2×10^{-16}
Up to lag 15	105.1793	1.332×10^{-15}
Up to lag 20	105.3322	1.383×10^{-13}
Up to lag 30	120.433	8.633×10^{-13}

However, the Ljung-Box test shows an opposite result with the hypothesis made from the ACF of squared residuals of LSTAR model. The p-values for the number of lags for squared residuals is less than 0.05 thus indicated the presence of ARCH effect in daily series. Therefore, McLeod-Li test was applied to the residuals and squared residuals of LSTAR model to get the final conclusion for the ARCH effect in the series. Figure 3 displays the results of the test for the 4400 observed daily series of MAS stock returns where panel (a) illustrates the Mc.Leod-Li test for LSTAR residuals and (b) illustrates the result for LSTAR squared residuals. From Figure 3, both of the results from McLeod-Li test confirms the final result of no seriel dependence of autocorrelation in LSTAR model. Figure 3(a) indicates that the ARCH effect does not exist and in (b) shows that there is no heteroskedastic effect left in the series. This make LSTAR (3,2) model is adequate for the MAS stock return data since the model has zero mean, zero autocorrelation in the residuals and squared residuals and no ARCH effect or heteroskedasticity left in the model.
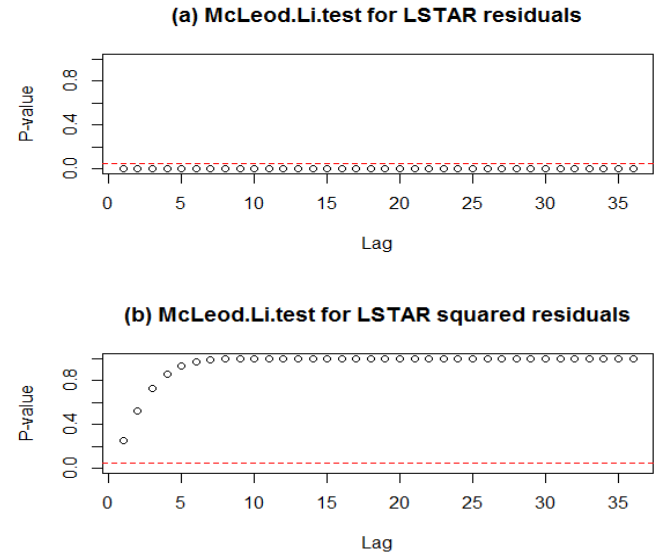

Figure 3 McLeod-Li test for the residuals from (a) LSTAR (3, 3) model and (b) squared residuals for LSTAR (3, 3) model

3.3 Forecasting Performance of the Models

Figure 4 compares the forecast series of the three models. From the figure, the series from the models looks similar thus make it hard to choose the best fitted model among AR, LSTAR and ESTAR for the returns series. Therefore, the models are compared using numerical statistics instead of diagram illustrations which is the Root Mean Square Error (RMSE) that is given in Table 9 below.
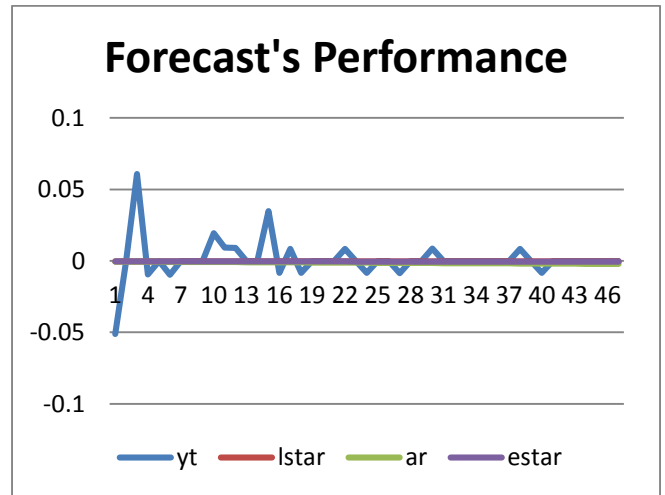

Figure 4 Forecast performance for AR, LSTAR and ESTAR models

The results from the tests indicate that LSTAR model is the best fitted model for the returns series of Malaysian Airlines (MAS) since the model has the lowest RMSE when compared to AR and ESTAR model.

The results are concluded from the forecast performances in Table 9.

Table 9 Forecasting performances of AR, LSTAR and ESTAR model

Models	RMSE
AR	0.013901
LSTAR	0.013787
ESTAR	0.013789

4.0 CONCLUSION

In this study, 4400 observed data of Malaysia Airlines (MAS) stock returns from 29th August 1996 until 26th September 2014 was analysed to check for the adequacy of nonlinear smooth transition autoregressive (STAR) model on the data. Following the procedures given by Terasvirta (1994), the data was first tested with linear Autoregressive (AR) method against the STAR model. The diagnostic plots of AR models show that the model is adequate in modelling the data and there is not exist autocorrelation in the residuals of the model. However, the squared residual of the model shows the presence of correlation in the AR model.

The study proceeded to the second stage of Terasvirta procedures which are the specification of the modelling. Lagrange Multiplier (LM) test was carried out to the AR model to determine the delay parameter, d. The result from the test shows that $d=3$ is the best delay parameter chosen since it has the lowest p-value and the highest test statistic obtained. Then, a sequence of nested hypothesis was conducted to specify the nonlinear STAR-type model on the chosen delay parameter. Table 3 illustrates the outcomes from the hypothesis, and the table indicates that Logistic Smooth Transition Autoregressive (LSTAR) model is the best fitted model compared to Exponential Smooth Transition Autoregressive model (ESTAR) model.

Further statistical tests were measured to verify that LSTAR model is adequate enough by plotting the ACF of the residuals and the squared residuals of the model. The figures indicate there is no autocorrelation in the residuals of the model. The test of autocorrelation, test of nonlinearity and ARCH-LM test were also tested on LSTAR model. The tests show that LSTAR model best fitted the data since there are no autocorrelation, no nonlinearity and no heteroskedasticy in the model. Furthermore, from the forecast series of linear AR, LSTAR, and ESTAR model,

LSTAR model has the lowest Root Mean Square Error (RMSE) when compared to AR and ESTAR model. Hence, LSTAR model is the best fitted model and forecasted model for daily 4450 observations MAS stock returns data. However, nonlinear LSTAR modelling is not accurate enough to be applied on the returns series since it does not fully capture the complex structure of nonlinearity pattern of the data. Therefore, further research on applying the combination of linear AR and LSTAR model to the additional structural break points of the series are suggested for further improvements of the model.

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