DYNAMIC EPILEPTIC ACTIVITY ANALYSIS BASED ON GEOMETRIC STRUCTURE OF ELECTROENCEPHALOGRAPHIC SIGNALS

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for Tata

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ABSTRACT

Epilepsy is a chronic brain disorder that affects people all over the world. It is characterized by recurring seizures which are caused by sudden and brief excessive electrical discharges in the brain. Epileptic seizures are notoriously difficult to model due to their erratic behavior and limited availability of clean data to work with. In this research, an emulated probability measure called the Delia measure is developed to normalize raw EEG data before being mapped to the surface of a unit hypersphere and modeled using a von-Mises Fisher distribution. By computing the parameters for the distribution using genetic algorithms, it is determined that seizures can be sorted in order of their spread, which is an indicator of seizure violence. The Delia measure values are also used in conjunction with information theory to yield the self-information and entropy of seizures. Based on the information content obtained, a Type-2 fuzzy graph and a crisp graph are generated to describe the information flow and electrode interconnectivity respectively. These graphs show that there is a distinct difference between focal and generalized seizures, and that seizure data can be segregated into multiple communities. Gödel's incompleteness theorem is also used in conjunction with non-Euclidean geometry to prove that no two seizures are the same. Together, these results verify that there exists a governing pattern for epileptic seizures.

ABSTRAK

Epilepsi merupakan gangguan otak yang kronik yang melibatkan ramai orang di serata dunia. Ia dicirikan dengan serangan secara tiba-tiba dan berulang. Ini disebabkan oleh pelepasan cas elektrik yang berlebihan di dalam otak. Dalam penyelidikan ini, ukuran kebarangkalian yang dipanggil ukuran Delia dibangunkan untuk menormalkan data EEG sebelum dipetakan kepada permukaan hipersfera menggunakan ukuran taburan von-Mises Fisher. Dengan mengira parameter ukuran taburan tersebut menggunakan algoritma genetik, serangan sawan dapat disusun mengikut sebaran data yang merupakan petunjuk kepada tahap serangan sawan. Nilai ukuran Delia tersebut juga digunakan bersama dengan teori informasi untuk menghasilkan entropi dan maklumat tentang serangan sawan. Berdasarkan kandungan maklumat yang dicerap, graf kabur Jenis-2 dan graf klasik dijanakan untuk menerangkan aliran maklumat dan sambungan antara elektrod. Graf-graf ini menunjukkan bahawa terdapat perbezaan ketara antara serangan sawan fokal dan umum. Data serangan sawan dapat diasingkan kepada beberapa kelompok. Teorem ketaklengkapan Gödel juga digunakan bersama geometri bukan-Euklidean untuk membuktikan bahawa tidak ada dua serangan sawan yang sama. Hasil penyelidikan ini mengesahkan bahawa wujud corak tertentu dalam sesuatu serangan sawan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BM	–	Base Magnetic Plane
dEEG	–	Digital Electroencephalography
EEG	–	Electroencephalography
FACS	–	Fuzzy Autocatalytic Set
FM	–	Fuzzy Magnetic Field
FTTM	–	Fuzzy Topographic Topological Mapping
GNU	–	GNU's Not Unix!
HUSM	–	Universiti Sains Malaysia Teaching Hospital
KS	–	Kolmogorov-Smirnov
MC	–	Magnetic Contour Plane
MEG	–	Magnetoencephalography
MLE	–	Maximum Likelihood Estimate
PDF	–	Probability Density Function
PMF	–	Probability Mass Function
SI	–	Self-information
SPS	–	School of Postgraduate Studies
SPSS	–	Statistical Package for the Social Sciences
TM	–	Topographic Magnetic Field
USM	–	Universiti Sains Malaysia
UTM	–	Universiti Teknologi Malaysia
vMF	–	von Mises-Fisher

LIST OF SYMBOLS

C_{EEG}	–	Cerebral EEG.
μ	–	Delia EEG measure.
β	–	Emulated baseline.
δ	–	Jitter.
κ	–	Concentration parameter.
I	–	Self-information
H	–	Shannon entropy.

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

In 1968, an electrical engineer named Shun'ichi Amari postulated the use of geometric tools to supplement existing methods in statistical analysis of real-world models [1]. Forty years down the road, Amari's method has bloomed into a mathematical field of it's own, now widely known as information geometry. Utilizing tools that were previously exclusive to differential geometry, it provides new insight to the extraction and analysis of statistical data, particularly those relating to the behavior of statistical distributions itself. This research, titled Dynamic Epileptic Activity Analysis Based On Geometric Structure Of Electroencephalographic Signals represents an initial foray into the application of said tools to modeling the behavior of epileptic seizures as viewed from their electroencephalographic signals.

This first chapter aims to acquaint readers with the background knowledge as to why this project was undertaken. Discussions will cover the related literature that motivates this project, followed by a formal statement of the issue being tackled. This, together with a set of well-defined research objectives and their respective limitations underline the structure of this thesis, right down to the organization of the chapters. Kicking things off is a short introduction to epilepsy and some prior work that has been done to address the issue.

1.1 Problem Background

Epilepsy is a chronic disorder of the brain that affects people in every country of the world. It is characterized by recurrent seizures, which are physical reactions to sudden, usually brief, excessive electrical discharges in a group of brain cells [2].

Part of understanding the way in which epilepsy affects the brain is the analysis of the aforementioned electrical discharges. These discharges are commonly monitored using electroencephalography (EEG) or magnetoencephalography (MEG) apparatus. These machines however, only provide neurologists with simple details about the seizure, such as the time and duration of the seizure, as well as its magnitude at the point of detection. Details such as the origin of the electrical discharge and pattern of propagation through the brain are not obtainable from simply viewing the EEG or MEG printouts. Consequently, mathematics was introduced as a research tool to identify, categorize, and predict the pattern in which these electrical discharges occur.

Years of research have not resulted in marked advancements in interpretation of EEG/MEG output. Despite being the most cost-efficient method of viewing the brain's activity, neurologists are still expected to spend years of training learning to decipher the brainwave patterns that are shown on said output. Many researchers chose to embark on the more ambitious project of predicting the occurrence of epileptic seizures, with varying amounts of success (more on this in Chapter 2).

In 1999, Tahir et. al. developed the Fuzzy Topographic Topological Mapping (FTTM) technique, which transports EEG/MEG signals to a fuzzified space with the aim of identifying the location of the epileptic foci, i.e. the points at which a seizure emanates from [3]. Ten years of research under the FTTM banner has yielded promising results which further suggest the existence of a pattern to the electrical discharge propagation of epileptic seizures [4]. This study, although it does not specifically employ the FTTM algorithm, is part of the FTTM project, and represents the next stage of the group's research into epileptic seizures.

1.2 Research Hypothesis

The problem addressed by this research is the construction and analysis of a geometric model for the electrical potential data of EEG signals obtained during an epileptic seizure. Simply put, the research seeks to answer the following question:

> ”*Can the electrical potential data of EEG signals obtained during an epileptic seizure be embedded in some geometrical space?*”

1.3 Research Objectives

The objectives of this research are as follows:

1. Construct a manifold to model the electrical potential data sampled from EEG data of an epileptic seizure
2. Employ elements of information geometry to identify the existence of a governing pattern for epileptic seizures
3. Validate the progression of an epileptic seizure as a mathematical graph

1.4 Limitations

The limitations of this research is as follows:

1. To construct a generalized manifold that can be used to compare the EEG signals of three epileptic seizures of varying length and category
2. To analyse the distribution of data points of the EEG signals on the manifold, resulting in a statistical distribution that describes the progression of epileptic seizures
3. To provide physical interpretation of the results obtained

1.5 Research Significance

The results of this research are crucial to further understanding the dynamics of epileptic seizures. As opposed to the FTTM algorithm, the data structure developed in this research does away with fuzzification of the raw data, resulting in a more *natural* model. This research also bridges the gap between the FTTM and Fuzzy Auto-Catalytic Set (FACS) projects.

Essentially, this research has one main focus: to try and provide a geometric view of EEG signals of epileptic seizures. It is hoped that this geometric view will reveal some visible pattern to the distribution of EEG signals during epileptic seizures, further corroborating existing proof of seizures being deterministic.

1.6 Dissertation Organization

The organization of this thesis is somewhat different compared to the general ”template” that UTM adheres to as it has been tailored to best represent the work done in this research. Rest assured, it still conforms to the guidelines set by the School of Postgraduate Studies (SPS). In total, there are eight chapters to this thesis, namely:

1. Chapter 1: Introduction
2. Chapter 2: Literature Review
3. Chapter 3: Research Methodology
4. Chapter 4: Measuring EEG Signals of Epileptic Seizures
5. Chapter 5: Manifold Construction
6. Chapter 6: The vMF Distribution of Epileptic Seizures
7. Chapter 7: Visualization of Epileptic Seizures via Graph Theory
8. Chapter 8: Conclusion & Discussion

The organization of the chapters are such that they remain concise, with minimal overlap. Some additional chapters were included as part of the research being extended from a period of three years to five. Figure 1.1 illustrates the dependencies of one chapter on another.

Now that the project has been formally introduced, the next chapter will proceed to acquaint readers with the pre-requisite literary background of the project.

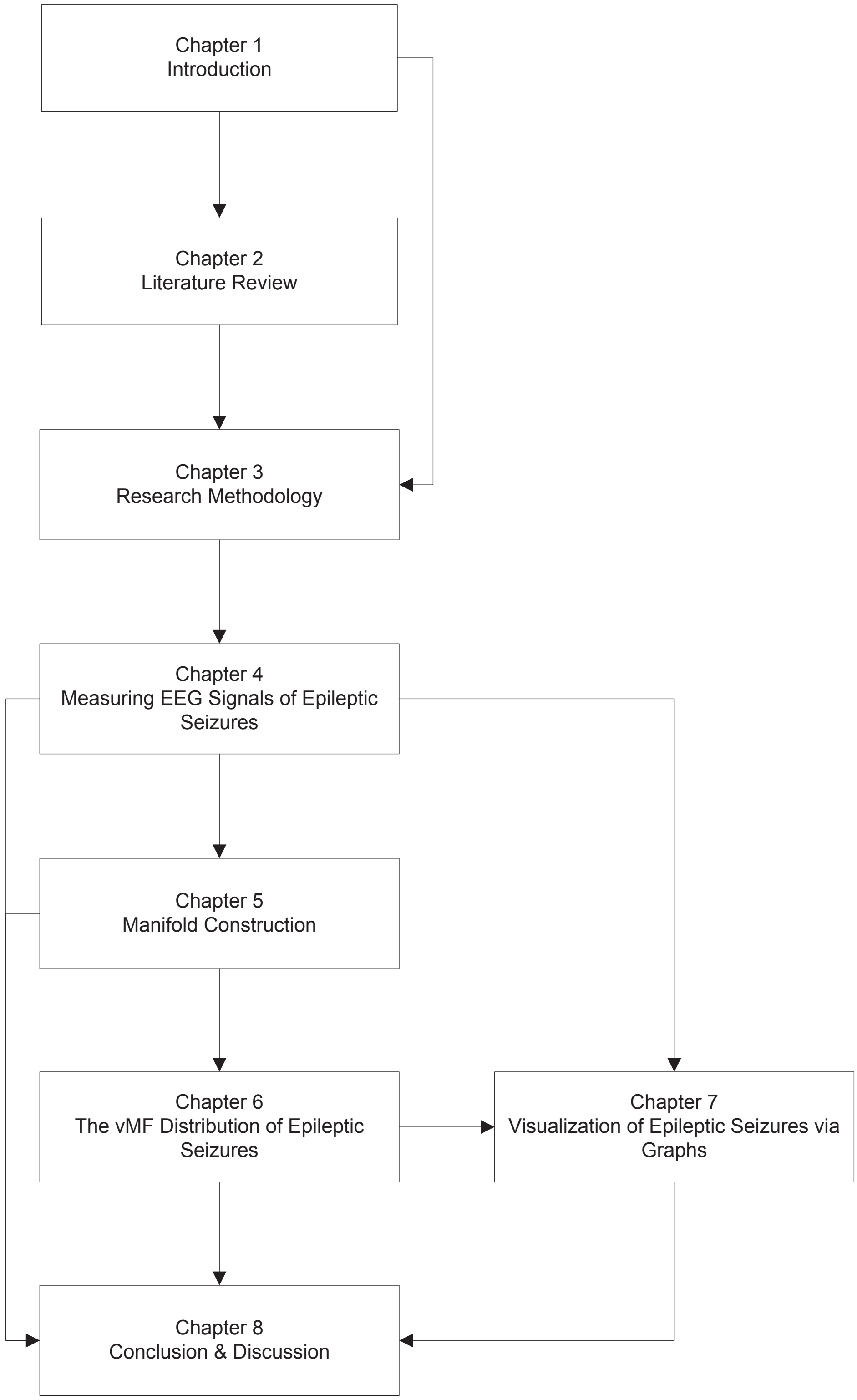

Figure 1.1: Chapter dependencies for this thesis.

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