SEQUENCES AND CUBES OF FINITE VERTICES OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING

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To my beloved parents Mohd Yunus Razali and Ozairah Zakaria who are a great source of joy and inspiration to me

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ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) was first developed by Fuzzy Research Group (FRG) of UTM. FTTM is a novel method for solving neuromagnetic inverse problems to determine the current source, i.e. epileptic foci in epilepsy disorder patient. FTTM consists of four components which are connected by three algorithms. FTTM is specially designed to have equivalent topological structures between its components. In addition, FTTM was generalized as a set of vertices which led to infinitely many forms of FTTM. This includes the possibilities of finite vertices of FTTM. In this research, the structure for finite vertices of FTTM, namely FK where K represents the number of vertices is established. Firstly, the sequences of FK, given by FK_n are constructed as sequences of polygons. In this process, geometrical and algebraic structures for some FK_n are obtained and proven in this thesis. Some patterns on FK_n are observed and defined recursively. Several new features for sequences of FK_n are introduced, such as sequence of vertices, sequence of faces, and sequence of cubes. Consequently, some theorems are proven in order to describe patterns for the sequence of cubes for FK_n. Interestingly, the cube of FK_n appears to be an example of generalized Fibonacci sequence, namely the k-Fibonacci sequence. Furthermore, the number of new elements produced from the combination of sequences of FK_n can be expressed as a combination of cubes of FK_n.

ABSTRAK

Pemetaan Topologi Topografi Kabur (FTTM) telah dibangunkan oleh Kumpulan Penyelidikan Kabur (FRG), UTM. FTTM merupakan kaedah untuk menyelesaikan masalah neuromagnetik songsang untuk menentukan kedudukan titik tumpuan arus di dalam otak bagi pesakit epilepsi. FTTM terdiri daripada empat komponen yang dihubungkan oleh tiga algoritma. FTTM direka khas untuk mempunyai struktur topologi yang setara di antara komponennya. Sebagai tambahan, FTTM kemudian diklasifikasikan sebagai satu set bucu dan ini membawa kepada pelbagai bentuk FTTM termasuklah bilangan bucu yang terhingga FTTM. Dalam kajian ini, pelbagai struktur bagi bucu terhingga FTTM iaitu FK dengan K mewakili bilangan bucu diperkenalkan. Pertama, jujukan bagi FK, yang dinamakan sebagai FK_n dibina sebagai suatu jujukan poligon. Dalam proses itu, beberapa struktur geometri dan aljabar jujukan FK_n diperolehi dan dibuktikan dalam thesis ini. Sebilangan corak pada FK_n telah diperhatikan dan ditakrifkan secara jadisemula. Beberapa gambaran baharu bagi jujukan FK_n diperkenalkan seperti jujukan bucu, jujukan muka, dan jujukan kiub. Seterusnya, beberapa teorem dibuktikan untuk menggambarkan corak jujukan kiub bagi FK_n. Menariknya, kiub FK_n didapati menyerupai contoh bagi jujukan teritlak Fibonacci, iaitu jujukan k- Fibonacci. Selain daripada itu, bilangan elemen baharu yang dihasilkan daripada gabungan jujukan FK_n boleh diungkapkan sebagai satu gabungan daripada kiub bagi FK_n.

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LIST OF ABBREVIATIONS

UTM	-	Universiti Teknologi Malaysia
FTTM	-	Fuzzy Topographic Topological Mapping
FRG	-	Fuzzy Research Group
MEG	-	Magnetoencephalography
EEG	-	Electroencephalogram
BM	-	Base Magnetic Plane
FM	-	Fuzzy Magnetic Field
MC	-	Magnetic Contour Plane
TM	-	Topographic Magnetic Field
BMI	-	Base Magnetic Image Field
FMI	-	Fuzzy Magnetic Image Field
MI	-	Magnetic Image Field
TMI	-	Topographic Magnetic Image Field
FK	-	Finite vertices of FTTM

LIST OF SYMBOLS

$\cong$	-	Homeomorphism
$\leq$	-	Less than or equal
$\geq$	-	Greater than or equal
$\Rightarrow$	-	Implies
$\in$	-	Element of
$vFTTM$	-	Vertices of FTTM
$eFTTM$	-	Edges of FTTM
$fFTTM$	-	Faces of FTTM
$\text{FTTM}_{2/n}$	-	Cubes with combination of two number of FTTM in FTTM_n
$\text{FTTM}_{3/n}$	-	Cubes with combination of three number of FTTM in FTTM_n
$\text{FTTM}_{4/n}$	-	Cubes with combination of four number of FTTM in FTTM_n
$\text{FTTM}_{k/n}$	-	Cubes with combination of k number of FTTM in FTTM_n
FK_n	-	Sequence of finite vertices of FTTM
vFK_n	-	Vertices of finite vertices of FTTM
eFK_n	-	Edges of finite vertices of FTTM
fFK_n	-	Faces of finite vertices of FTTM
$FK_{r/n}$	-	Cubes with combination of r number of FK in FK_n

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

The human brain (see Figure 1.1) is the most important structure in our body. It is also the most complexly organized structure known to exist [1]. There are four lobes in both halves of the cortex: frontal, pariental, temporal and occipital.

The outermost layer of the brain is called the cerebral cortex. The cerebral cortex has a total surface area of about $2500cm^2$, folded in a complicated way, so that it fits into the cranial cavity formed by the skull of the brain. There are at least 10^{10} neurons in the cerebral cortex. These neurons are the active units in a vast signal-handling network [1]. When information is being processed, small currents flow in the neural system, producing a weak magnetic field (see Figure 1.2).

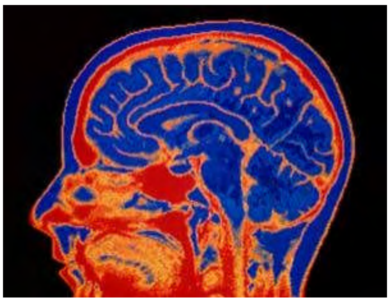

Figure 1.1: Human brain (source from http://www.newscientist.com)

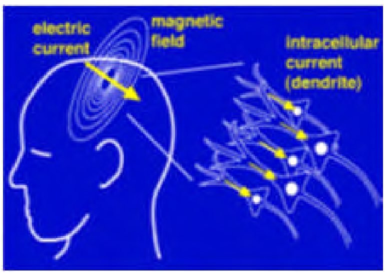

Figure 1.2: Neuromagnetic field

Different parts of brain produce different patterns of magnetic fields [2]. The small area of brain tissues that triggers epileptic seizures is called epileptic foci. It is very important to accurately locate the epileptic foci in the cortical region for a successful surgery [1]. Both invasive and noninvasive methods of locating the epileptic foci have been used in the past, but only the invasive pathway has yielded necessary results for surgical removal.

Magnetoencephalography (MEG) is one of the noninvasive neuroimaging techniques used to identify epileptic foci (see Figure 1.3). This study was first conducted by the University of California [3]. MEG is the study of magnetic field generated by currents in the neurons [4]. MEG consists of the superconducting quantum interface device (SQUID) detectors coupled with flux transformers. The recorded magnetic fields help in determining where the electrical currents originate and the strength of currents. MEG is completely noninvasive and non-hazardous. The recorded magnetic field gives information in the process to determine location, direction and magnitude of a current source. Estimating the cereberal current sources underlying a measured distribution of the magnetic field is called the neuromagnetic inverse problem [1].

There is a method for solving this problem, called Bayesian, that needs a priori information (data based model), and it is time consuming [5]. By using Bayesian, forward calculation is used to calculate the magnetic field caused by the current dipole at every possible point. The best location of the current source is determined by minimizing the sum of the squares of the difference between the measured and the

Figure 1.3: MEG Systems (source from http://infocenter.nimh.nih.gov)

calculated value similar to the least squares method. Then, a point with a minimum least square is the location of a current source. On the other hand, Fuzzy Topographic Topological Mapping (FTTM) is a model for the solving neuromagnetic inverse problem. It does not need priori information and is less time consuming [6].

1.2 Research Background

FTTM was first developed by Fuzzy Research Group (FRG) group in 1999 in order to determine the location of epileptic foci in epilepsy disorder patients [6]. The model consists of four components, which are magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic plane (FM), topographic magnetic field (TM) and three mathematical algorithms (see Figure 1.5). In 2002, Zakaria has developed FTTM version 1 (see Figure 1.4) to present a 3-D view of an unbounded single current source [7], and later, Rahman developed FTTM version 2 (see Figure 1.5) to present a 3-D view of a bounded multi current source [8]. The structure of FTTM will be discussed in detail in Chapter 2.

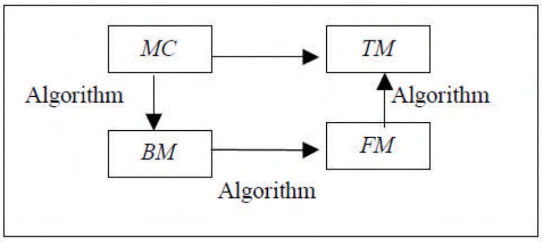

Figure 1.4: FTTM version 1

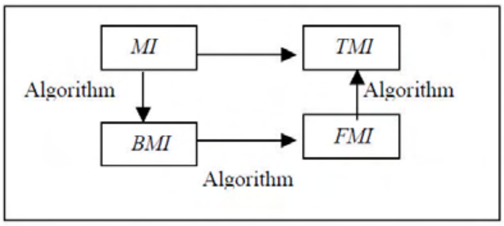

Figure 1.5: FTTM version 2

1.3 Problem Statement

FTTM version 1 and FTTM version 2 (See Figure 1.6) are specially designed to have equivalent topological structures between its components. This was proven by Yun [9]. In other words, FTTM version 1 and FTTM version 2 are homeomorphic component-wise. Yun also noticed that if there are two elements of FTTM that are homeomorphic to each other component-wise, it would generate more homeomorphisms [10]. The number of generating new elements of FTTM is

$$\left[\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \times \binom{2}{1}\right] - 2 = 14 \text{ elements.} \tag{1.1}$$

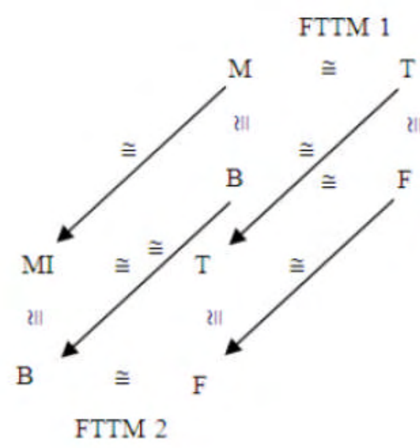

Figure 1.6: Equivalance of Structure of $FTTM$ Componentwise

Consequently, Yun proposed a conjecture such that if there exist n elements of FTTM, then the number of new elements are $n^4 - n$ [10]. This conjecture is proven by Jamaian [11]. Consequently, Jamaian proposed an open problem as follows:

given cube of two, three, and four FTTM given by FTTM$_{2/n}$, FTTM$_{3/n}$ and FTTM$_{4/n}$. Thus for every nonzero of FTTM$_{2/n}$, FTTM$_{3/n}$ and FTTM$_{4/n}$ appears in the third, fourth, and fifth main diagonal of Pascals triangle respectively, therefore for every nonzero sequence of $FK_{2/n}$, $FK_{3/n}$, $FK_{4/n}$,...,$FK_{l/n}$ they also obey the third, fourth, fifth, until $(l+1)^{th}$ main diagonal of Pascals triangle with K representing the number of components [12]. The number of new elements of FK_n can be written as,

$$Fk_n = C_1\binom{n}{2} + C_2\binom{n}{3} + C_3\binom{n}{4} + \cdots + C_p\binom{n}{k} \tag{1.2}$$

where $n \geq k$ with k the number of component and C_1, C_2, C_3,..., C_p are the coefficient for each combination. Since FTTM exists in a sequence, therefore the need to analyse the sequence of finite vertices of Fuzzy Topographic Topological Mapping (FK_n) is paramount.

1.4 Research Objectives

The aims of this research are as follows:

(i) To develop sequences of finite vertices of Fuzzy Topographic Topological Mapping (FK_n).

(ii) To prove the theorem on sequences of FK_n using difference equation.

(iii) To find the relation between sequences of cubes of FK_n and k-Fibonacci sequences.

(iv) To prove the conjecture proposed by Jamaian.

1.5 Scope of the Study

This research focuses on the goal to prove the conjecture proposed by Jamaian in [12] and the relation between generalized FTTM and k-Fibonacci sequence. This form of sequence for FK_n was only limited to the form that were adopted by Jamaian in [12].

1.6 Significance of Findings

By proving the conjecture, other versions of FTTM can be introduced. In other words, a new version of FTTM besides FTTM version 1 and version 2 can be developed. The relation between sequences of FTTM, Pascal's Triangle and also Fibonacci sequences are obtained.

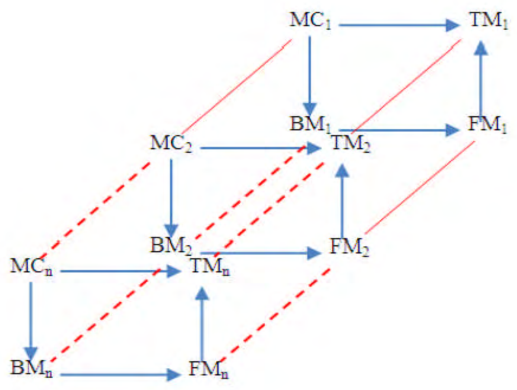

Figure 1.7: Sequences of FTTM$_n$

1.7 Research Methodology

The research starts by studying different types of FTTM and the geometry for sequences of FTTM. There are three methods used in order to prove the theorem which are constructive, difference equation, and mathematical induction.

1.7.1 Constructive

According to Hein [13], a constructive proof is a method of proving that demonstrate the existence of a mathematical object with certain properties by creating or providing a method for creating such an object. In addition, the constructive method can be identified by certain keywords that appear in the statement such as there is, there are, there exist, for all, for each and for every [14]. Furthermore, this method never puts any condition that the statement of a problem should be identified first.

1.7.2 Difference Equation

A **difference equation** (also called a recurrence equation) is the discrete analog of a differential equation [13]. A difference equation involves an integer function $f(n)$

in a form, such as the following:

$$f(n) - f(n+1) = g(n). \tag{1.3}$$

For non-homogenous equation

$$U_{n+1} + aU_n = f(n) \tag{1.4}$$

where a is constant, the solution U_n is given by

$$U_n = \begin{pmatrix} \text{general solution of} \\ \text{associated homogeneous} \\ \text{equation} \end{pmatrix} + \begin{pmatrix} \text{one particular solution} \\ \text{of the non} - \text{homogeneous} \\ \text{equation} \end{pmatrix} \tag{1.5}$$

1.7.3 Mathematical Induction

Mathematical induction is a way to prove statements for all positive integers [15]. There are two steps in mathematical induction: the basis and the inductive steps.

(i) The basis (base case): showing that the statement holds when n is equal to the lowest value that n is given in the question. Usually, $n = 0$ or $n = 1$.

(ii) The inductive step: showing that if the statement holds for some n, then the statement also holds when $n + 1$ is substituted for n.

1.8 Thesis Organization

In general, the thesis contains seven chapters. The first chapter serves as an introduction to the whole thesis. This chapter includes the background of the research, problem statements, objectives, scope and importance of the research.

Chapter 2 presents the literature review of this research. Various works by different researchers regarding FTTM are discussed in this chapter. Some definitions on k-Fibonacci sequence are also presented in this chapter.

Chapter 3 consists the geometrical features of FK_n. It consists of generalization of FK_n. The geometrical feature of FK_n is discussed in this chapter. Several definitions on sequence of FK_n are also provided.

Chapter 4 provides the theorems on sequences of FK_n. The proofs for the theorems are provided in this chapter.

In Chapter 5, the proofs of sequence of cubes FK_n are provided. It reveals the relation between cube of FK_n, Pascal's triangle, and k-Fibonacci sequence. This chapter covers the proof to the theorem and corollaries of cube FK_n.

Chapter 6 covers on the elements of FK_n and the relation to cubes of FK_n. Finally, Chapter 7 consists of conclusions and recommendations for future work.

The framework of this research is summarized in Figure 1.8.

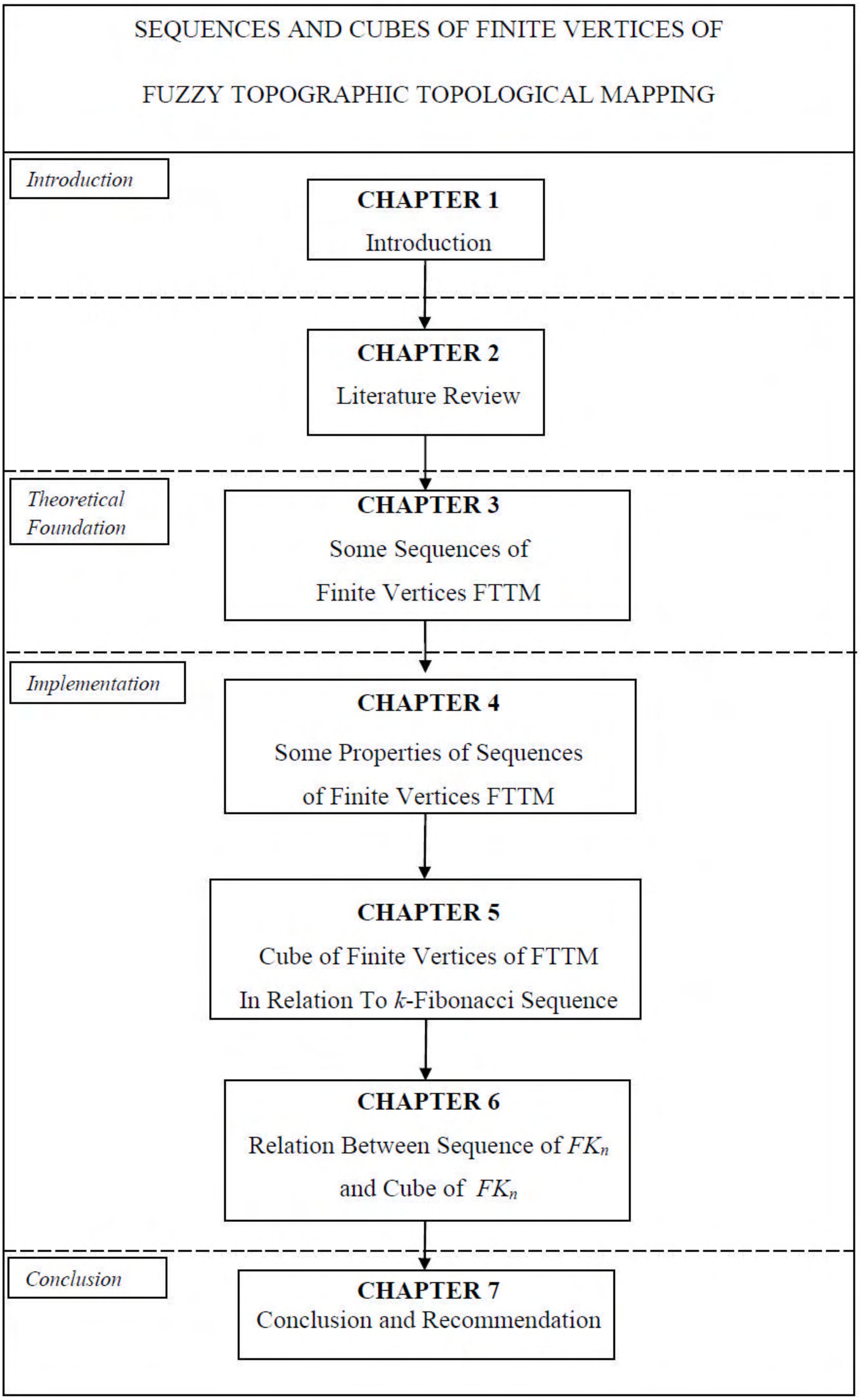

Figure 1.8: Research Framework

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