

ENHANCED EXPONENTIALLY WEIGHTED MOVING AVERAGE (EWMA)  
CONTROL CHART PERFORMANCE WITH AUTOCORRELATION

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*To my beloved mother and late father*

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## ABSTRACT

This research introduces an enhanced exponentially weighted moving average (EWMA) control chart which is effective in detecting small and unnoticed shifts in monitoring process mean for autocorrelated data. The control chart is based on extension or modification of EWMA control chart statistic. The proposed control chart is named the new EWMA (NEWMA) and is applied to simulated autocorrelated data for different autocorrelation levels (low, moderate and large) to study the performance of the chart. The run rules schemes were introduced to enhance the performance of the NEWMA chart namely; three out of three and three out of four schemes and three out of four is the best among the schemes. The NEWMA chart performs for observations that are autocorrelated. The NEWMA control chart has been tested on 100,000 simulations and it is found that it is quick in detecting process shift and able to identify the out of control points as it occurs. The performance of the technique has been evaluated using the average run length (ARL) and compared with modified exponentially weighted moving average (MEWMA) and classical exponentially weighted moving average (CEWMA) control charts and found that the NEWMA chart is faster in detecting shift. The NEWMA chart was applied to the KLSE Share index data, water quality data and Malaysian labour force data which are autocorrelated in nature and found to be effective in detecting the shifts.

## ABSTRAK

Kajian ini memperkenalkan carta kawalan purata bergerak eksponen (EWMA) yang berkesan dalam mengesan anjakan kecil dan tidak disedari yang berlaku dalam pemantauan min proses dalam data berautokorelasi. Carta ini dibina berdasarkan statistik carta kawalan EWMA lanjutan. Carta EWMA baharu ini diaplikasikan kepada data simulasi berautokorelasi bagi paras autokorelasi berlainan (rendah, sederhana dan tinggi) untuk melihat keupayaan carta ini. Petua penentuan proses luar kawalan juga diperkenalkan dalam menggunakan carta baru ini, iaitu tiga daripada tiga dan tiga daripada empat. Carta baru ini didapati berupaya meneliti proses yang berautokorelasi. Carta baru ini diuji ke atas 100,000 data simulasi yang berautokorelasi dan didapati ia pantas dalam mengenalpasti anjakan min proses. Keupayaan carta ini diukur daripada purata panjang larian (ARL) dan dibandingkan dengan carta kawalan purata bergerak eksponen terubahsuai (MEWMA) dan carta kawalan purata bergerak klasik (CEWMA). Carta kawalan baru ini didapati adalah yang terpantas dalam mengenalpasti anjakan min proses. Carta baru ini diaplikasikan kepada data indeks saham KLSE, data kualiti air dan data buruh Malaysia yang diketahui berautokorelasi dan didapati ia adalah efektif dalam mengenalpasti anjakan min.

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**LIST OF ABBREVIATIONS**

EWMA	-	Exponentially Weighted Moving Average
MEWMA	-	Multivariate Exponentially Weighted Moving Average
CUSUM	-	Cumulative Sum
ARL	-	Average Run Length
SPC	-	Statistical Process Control
AR	-	Autoregressive
MA	-	Moving Average
ARMA	-	Autoregressive Moving Average
NID	-	Normal Independent Distribution
AEWMA	-	Adaptive Exponentially Weighted Moving Average
LSL	-	Lower Signalling Limit
USL	-	Upper Signalling Limit
A	-	Upper Region
B	-	Centre Region
C	-	Lower Region
RA-EWMA	-	Risk Adjusted EWMA
GWMA	-	Generally Weighted Moving Average
i.i.d.	-	Independently Identically Distributed
ATS	-	Average Time to Signal
FIR	-	Fast Initial Response



**LIST OF SYMBOLS**

- Exponential Smoothing Constant
- Shift
- P(Type I error)
- P(Type II error)
- Autocorrelation

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## CHAPTER 1

### INTRODUCTION

#### 1.1 Background of the Study

For some time, it has been assumed in statistical process control (SPC) that the observations from the fundamental procedure are independent, however this assumption was violated in practice Montgomery DC and CM. (1991). As a result, a number of authors talked about the way the classical Shewhart, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts behave with regard to autocorrelated processes ( Harris and Ross 1991, Montgomery DC and CM. (1991, Woodall and Faltin. (1993)). This made these schemes not good when the same control limits are used as with the situation of independent variables. Because of this it is important to use time series models to create control charts. SPC is really a significant quality control issue by which data analysis is employed to find out if the process is under control. One main purpose of SPC would be to identify immediately a process shift and adopt the required corrective action to enhance process quality. Control charts tend to be the widely applied tools for monitoring processes. Harris and Ross (1991) discussed many different correlative structures and used simulations to review the impacts of those correlative structures on the traditional CUSUM as well as EWMA control charts. Padgett *et al.* (1992) studied Shewhart charts when process observations could be modelled as an autoregressive of order 1 AR (1) process with random error.

Control charts are extensively applied to evaluate manufacturing processes with the aim of detecting any kind of difference of a process parameter that could affect the quality of the result. Efficient detection of small and moderate shifts in mean and standard deviation necessitates that the control statistic somehow includes information from current and past sample statistics. Run rules that are based on patterns of points within the Shewhart chart enhance the ability of Shewhart charts to detect small and moderate shifts in mean and standard deviation Champ and Woodall (1987), Lowry and Montgomery (1995)).

A basic assumption in traditional application of SPC methods is the observations of the processes under investigation are normally and independently identically distributed (i.i.d.). When these assumptions tend to be fulfilled, conventional control charts could be applied (Woodall and Faltin, 1993). However, the independence assumptions are usually violated in practice, whether it is in discrete or continuous production process, where the data often show some autocorrelation. Even small levels of autocorrelation between successive observations might have large effects on the statistical properties of conventional control charts, because of the presence of autocorrelation within the process.

Many authors have thought about the effect of autocorrelation on the performance of SPC charts, including Johnson and Bagshaw (1974), Bagshaw and Johnson (1975), who derived approximate run length distribution for the CUSUM once the process follows an autoregressive process AR(1) or a moving average process MA(1) model.

John and Bruce (2002) presented an evaluation of traditional SPC and non-traditional methodology for controlling the effect of autocorrelated processes within production monitoring and control. They discovered that using autoregressive integrated moving average (ARIMA) control chart will give you a more regular technique for detecting assignable or special causes within the continuous production processes. However, their study is only limited to the continuous processes at low autocorrelation level.

Apley and Lee (2003), models the classical EWMA for autocorrelated processes with model uncertainty. Developing a technique for designing residual based EWMA charts under consideration of the uncertainty within the estimated model parameters. Using method of widening the EWMA control limits in line with the worst-case design approach. However, their study was limited only to the residual-based control chart for the statistical process of autocorrelated processes.

Lu and Reynolds (1999) considered the issue of detecting variations in a process where the observation could be modelled being an AR(1) processes plus a random error. The end result shows that when the level of autocorrelation is fairly high the time required to detect a shift that is a given fraction from the process standard deviation is a lot more than for the similar shift in the independence situation.

Moran and Solomon (2013), studied the effects from the data generating process for autocorrelation and seasonality which shows classical decomposition. Their results provide the monthly raw mortality series data at the intensive care unit (ICU) level shows autocorrelation, seasonality and volatility. False positive signalling with the raw mortality series data was clearly displayed by Risk Adjusted-EWMA (RA-EWMA) control limits, by controlling the specific situation using the residual control charts. However, their study only addressed identifying the presence of autocorrelation from the collected data set using the time series techniques and uses the residual control chart base to deal with the autocorrelation present in the data, which limit it to the application of RA-EWMA.

Sheu and Lu (2009) studied the Generally Weighted moving average (GWMA) control chart with autocorrelation by monitoring the process mean where the observation as in Lu and Reynolds (1999) could be modelled as AR(1) process with random error. Their study uses simulation to get the average run length of the autocorrelated GWMA control charts that turned out to be better than the autocorrelated EWMA control chart for detecting small process mean shift at lower levels of autocorrelation. However, their study could not detect the moderate shift at

different levels of autocorrelation, but it performs well for high levels of autocorrelation within 3 .

Autocorrelated observations are common within industry, particularly when data are sampled in a high frequency from processes with inertia. The classical EWMA control chart is actually non-robust to serial correlation or autocorrelation Vermaat *et al.* (2008). The MEWMA is just capable to deal with the issue of monitoring small and large shifts with high autocorrelated observations. Hence, we are motivated to further the research to enhance the performance of the EWMA control chart with low and medium autocorrelation.

Alpaben and Jyoti (2011) attempted to deal with the problem of detecting small shifts of parameter process in a small or moderate autocorrelation. However, the scheme still could not deal with the problem of detecting moderate and high shifts with large autocorrelation.

## 1.2 Problem Statement

It is known that CEWMA was established to tackle the small and moderate shifts of the process mean. But after a lot of modifications of the CEWMA to improve the performance as contained in the literature, for example see Lucas and Masarotto (1990), Harris and Ross (1992), Montgomery(2007), Woodall and Faltin(1991) and the references contained therein. Hence it was found that the CEWMA could not detect moderate shift with high autocorrelation against high exponential smoothing constant ( ) and large shifts with high autocorrelation against high exponential smoothing constant. Not until recently when Alphen and Jyoti(2011) proposed on MEWMA with correlated process, which could not detect high autocorrelation with moderate ( ) and high autocorrelation with high exponential smoothing constant. The problem at hand in this research is to construct a EWMA control chart that is able to cope with different levels of autocorrelation. Due to increasing network sensors, flow meters, video cameras which are connected

to powerful computers running sophisticated software that monitored complex process, which shows the existence of autocorrelation among them.

### **1.3 Objective of the Study**

The aim of this study is to handle the limitations of CEWMA and MEWMA with autocorrelation for detecting small, moderate and large shifts quickly as it occurs. Basically, they are as follows:

- i. To construct a EWMA control chart that can cope with autocorrelation which we called new EWMA (NEWMA), average run length is used to evaluate its performance and compared its power in detecting shift quickly with the existing classical EWMA (CEWMA) and modified EWMA (MEWMA) (2011) control charts.
- ii. To determine run rule schemes that best used with the NEWMA control chart method with autocorrelation.
- iii. To apply the NEWMA in real data sets.

### **1.4 Scope of the Study**

This study covers the following three main aspects.

- i. Theoretical aspect

The highlights of the CEWMA and MEWMA control charts were stated first, then the derivation of the asymptotic distribution of the proposed chart, using the MacLaurin series expansion to obtain the mean and variance. The results are used to construct the control limits.

ii. Evaluation aspects

In order to evaluate the performance of the NEWMA control chart, we investigate its average run length and compared it with the CEWMA and MEWMA control charts.

iii. Practical aspect.

We apply the NEWMA to real problem to illustrate the advantages of the New EWMA control chart method, and compare it to the CEWMA and MEWMA.

## 1.5 Contributions of the Study

This study provides few contributions, which includes:

- i. A new EWMA control chart to handle autocorrelation with reduced false alarm i.e. reduced Type I error.
- ii. Identification and evaluation of the best run rule to be used with the new control chart.

## 1.6 Thesis Organization

The organization of the thesis is as follows. Chapter 1 briefly overviews the SPC, and the effects of autocorrelation on the CEWMA and MEWMA control charts. These lead to the problem statement and the objectives of the research. The scope of the study is presented and the contribution of the research is stated at the end of the Chapter.

Chapter 2 analyse the univariate and existing control charts in the presence of autocorrelation. Chapter 3 presents the methodology of the new control chart construction.



In Chapter 4, the results of the evaluation of the new EWMA control chart to deal with autocorrelation were simulated and presented. Chapter 5 presents the application of the new control chart to several data sets. Lastly, Chapter 6 concludes the research results while recommending future research for improvement.

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