

AUXETIC STRUCTURES FOR ENERGY ABSORPTION APPLICATIONS

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*To my kind parents for their priceless support and motivation,*

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## ABSTRACT

Auxetic materials are new class of materials exhibiting negative Poissons ratio. This unusual behavior results in improvement of mechanical properties such as energy absorption capability. This research focuses on design of auxetic materials in order to enhance and control mechanical properties. Mechanical design of auxetic structures has been developed for both high and low stiffness applications. For high stiffness applications, auxetic structures were designed to be used for making auxetic materials. Among several auxetic structures, re-entrant structures have been selected due to their potential of modeling auxetic materials. The basic mechanical properties and impact characteristics have been determined using analytical and numerical methods. The analytical formulation has been validated by finite element analysis whereas the numerical results have been corroborated against experimental results. For validation, the basic mechanical properties and energy absorption capacity have been compared accordingly to subsequently carry out further analyses. As additional results, dynamic analysis of viscoelastic structures under impact loading was also demonstrated to examine the amount of impact resistance. For low stiffness applications, negative Poissons ratio polyurethane foam was precisely fabricated through a modified fabrication process which later involved experimental works to measure and control the mechanical properties. The effects of fabrication parameters namely hydraulic pressure, heating temperature and time on auxeticity of specimens have also been investigated. More importantly, a new method based on image processing technique has been proposed for measuring Poissons ratio of foam. In addition to this, energy absorption capability of auxetic foam was measured by using a high speed camera and falling weight system. Overall, the results highlight the pronounced effect of unit cell cross section and unit cell angle on the auxeticity and energy absorption characteristics. The primary outcome of this thesis is development of auxetic structure design for high stiffness application and modification of fabrication process of auxetic foam. Furthermore, the results demonstrated the importance of analyzing auxetic foam-filled thin-walled tubes as part of an energy absorbing system.

## ABSTRAK

Bahan-bahan auxetik adalah bahan kelas baru yang menunjukkan nisbah Poisson bernilai negatif. Keadaan perilaku luar biasa bahan ini menghasilkan pembaikan bagi sifat mekanikal seperti keupayaan penyerapan tenaga. Kajian ini memberi tumpuan kepada rekabentuk bahan auxetik bagi meningkatkan dan mengawal sifat-sifat mekanikal. Rekabentuk mekanikal bagi struktur auxetik telah dibangunkan untuk kedua-dua aplikasi kekakuan tinggi dan rendah. Untuk aplikasi kekakuan tinggi, struktur auxetik telah direkabentuk untuk diguna dalam pembuatan bahan auxetik. Di antara beberapa struktur auxetik, struktur berbentuk lekukan telah dipilih disebabkan potensinya di dalam penghasilan bahan auxetik. Sifat mekanikal asas dan karakternya terhadap hentaman telah ditentukan dengan menggunakan kaedah analitik dan berangka. Persamaan yang dianalisis telah disahkan dengan analisis unsur tak terhingga manakala keputusan berangka telah ditentusahkan terhadap keputusan eksperimen. Untuk pengesanan, sifat mekanikal asas dan kapasiti penyerapan tenaga telah dibandingkan seajarnya untuk seterusnya melakukan analisis lanjut. Sebagai keputusan tambahan, analisis dinamik bagi struktur viskoelastik di bawah bebanan hentaman juga telah ditunjukkan untuk memeriksa jumlah rintangan terhadap hentaman. Untuk aplikasi kekakuan rendah, busa auxetik telah dihasilkan secara teliti melalui proses fabrikasi yang diubahsuai yang mana kemudiannya melibatkan kerja-kerja eksperimen untuk mengukur dan mengawal sifat-sifat mekanikal. Kesan parameter pembuatan iaitu tekanan hidraulik, suhu dan masa pemanasan bagi spesimen keauxetikan juga telah disiasat. Yang lebih penting lagi, satu kaedah baru berdasarkan kepada teknik pemprosesan imej telah dicadangkan untuk mengukur nisbah Poisson bagi busa. Tambahan lagi, keupayaan penyerapan tenaga busa auxetik telah diukur dengan menggunakan sebuah kamera berkelajuan tinggi dan sistem kejatuhan berat. Secara keseluruhannya, hasil keputusan memberikan penekanan kepada kesan yang jelas daripada keratan rentas and sudut bagi sel unit terhadap keauxetikan dan ciri-ciri penyerapan tenaga. Hasil utama tesis ini adalah pembangunan rekabentuk struktur auxetik untuk penggunaan kekakuan tinggi dan pengubahsuaian proses pembuatan busa auxetik. Tambahan pula, keputusan ini telah menunjukkan kepentingan menganalisa busa auxetik yang dimasukkan ke dalam tiub berdinding nipis sebagai sebahagian daripada sistem penyerapan tenaga.

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## LIST OF ABBREVIATIONS

DPSPC	–	Deputy President in Strategic Planning and Control
E-Commerce	–	Electronic Commerce
ERA	–	Electronic Readiness Assessment
SMC	–	Small and Medium Contractor
WTO	–	World Trade Organization
USD	–	United State Dollar
GDP	–	Gross Domestic Produce
ICT	–	Information and Communication Technology
PCA	–	Principal Component Analysis
POT	–	Peak Over Threshold
ISC	–	Iran Statistic Center
IIO	–	Iran Industry Organization
DPSPC	–	Deputy President Strategy Planing & Control
PERM	–	Perceived E-Readiness Model
OTE	–	Organizational Technology & Environment
PU	–	Polyurethane
SLS	–	Selective Laser Sintering
FEM	–	Finite Element Method
IMT	–	Image Processing Technique
LCP	–	Liquid Crystalline Polymer
PVC	–	Polyvinyl chloride
FE	–	Finite Element
	–	

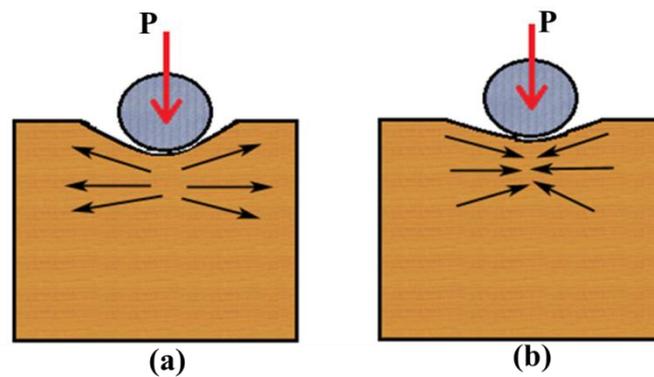
## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Background of the Research**

Modern technology requires new materials of special and improved properties. One of the reasons for interest in materials of unusual mechanical properties comes from the fact that they can be used as matrices to form composites with other materials with other required properties, e.g. electric, magnetic, etc. A new field of structural mechanics is to study materials exhibiting negative Poisson's ratio (NPR). These new types of materials are known as auxetic materials. In contrast to conventional materials (like rubber, glass, metals, etc.), auxetic materials expand transversely when pulled longitudinally and contract transversely when pushed longitudinally (Prawoto, 2012). Some of the important mechanical properties such as indentation resistance, energy absorption capacity, impact resistance, fracture toughness, fatigue toughness and shear strength are mainly dependent on Poisson's ratio (Prawoto, 2012). Due to the unique characteristic of negative Poisson's ratio in improving these properties, auxetic materials are excellent in the great number of structural applications. For instance, they have great potential to meet many needs of medical, military, automotive and textile industries where indentation resistant and energy absorbing material are required. From the mechanics of material point of views, when an object impacts an auxetic material and compresses it in one direction, the auxetic material also contracts laterally in which material flows into (compresses towards) the vicinity of the impact, as shown in Figure 1.1 (Evans and Alderson, 2000a). As such, it creates an area of denser material, which is more resistant to indentation. The previous investigation

showed that re-entrant foams have higher yield strength and less stiffness than conventional foams with the same original relative density. It has also been further proven that re-entrant foams indeed densify under indentation due to increase in shear stiffness. Figure 1.1 depicts the deformation behaviours of both conventional and auxetic materials when subjected to impact compressive loading.



**Figure 1.1:** Deformation behavior of materials. (a) Auxetic (b) Conventional

The application of auxetic material in energy absorbing structures could be divided into two main aspects; low stiffness and high stiffness applications. From automotive design aspects, auxetic materials can be used for bumpers, cushions, thermal protection, sounds and vibration absorber parts those need shear resistant, fasteners, etc. Nevertheless, for low stiffness application, auxetic materials need substantial porosity (Prawoto, 2012). Thereby, this type of material has low stiffness compared with the solids. Thus, there are some limitations in the structural applications of using materials with negative Poisson's ratio. Consequently, for applications that require substantial load-bearing, they are probably not the best choice.

In order to fabricate auxetic man-made materials from conventional ones, substantial porosity is required (Prawoto, 2012). For instance, low density polymeric foam such as polyurethane foam has great potential to be used in producing auxetic materials. Due to considerable porosity of these materials, their stiffness is considerably low. Therefore, they can be used only for low stiffness applications such as automotive bumper. To improve the energy absorption capacity in low stiffness

application, the present study has duly modified the fabrication technique to meet the higher level of energy absorption capability.

However, for high stiffness applications such as vanes for gas turbine engine, artificial skin, and artificial blood vessel, they are not proper choices. Recently, attempt has been done to design and fabricate auxetic structures experimentally. These structures in micro level can be used for making high stiffness auxetic materials. Owing to the lack design guideline for the high stiffness application, the present study has extensively investigated the mechanical properties and energy absorption capacity of auxetic materials using finite element technique in conjunction with experimental and analytical approaches.

## **1.2 Statement of Problem**

Energy absorption and impact resistance of materials are among important mechanical properties which can facilitate various industries such as automotive, aerospace and biomedical industries. In particular, lightweight materials with high energy absorption capability are desirable in designing crashworthy structures. Auxetic materials as new class of materials have the potential to absorb more energy compared to conventional materials. In point of view of automotive design, the available crush zone of frontal structure is still limited to sustain the adverse effect of impact. It is indispensable to include a unique material as supplementary material which may absorb impact load with higher capacity under limited crush zone. Due to the unusual characteristic of auxetic material, it can be one of the best candidates to be applied in automotive design.

Furthermore, there is insufficient data to understand in-depth the behaviour of auxetic material in open literature. Although a large number of experimental works have been done on auxetic materials, only few of them have focussed on the theoretical aspect and finite element modelling technique, particularly in energy absorption applications (Zi-Xing Lu, 2011). To comprehend the mechanical characteristics of auxetic material, finite element approach has been comprehensively

employed in this present research. Also, in spite of a number of continuous modified fabrication methods of auxetic foams for low stiffness application, further modification is still necessary. For the abovementioned research problems, the present research has attempted to design and develop auxetic materials analytically, numerically and experimentally. Such approaches have been adopted to develop reliable and accurate finite element model which can represent the actual behaviour of the auxetic materials. Introducing modified fabrication method and new technique in measuring mechanical properties have also been highlighted throughout the study.

### **1.3 Objectives of the Research**

This research focuses on determination of mechanical properties and energy absorption capability of auxetic structures and auxetic materials. It primarily emphasizes on the geometrical design of auxetic structures for high stiffness applications and improvement of fabrication method of auxetic foams for low stiffness applications. This present study embarks on the following objectives.

- a) To develop 3D models of auxetic structures for high stiffness applications.
- b) To determine basic mechanical properties of auxetic structures.
- c) To determine the effect of viscoelastic material on impact resistance of auxetic structures.
- d) To establish fabrication process of auxetic foams and methods for measuring important mechanical properties in low stiffness applications.

### **1.4 Scope of the Work**

In general, scopes of this research are divided into two sections namely auxetic materials for high stiffness applications and for low stiffness applications.

The scope for high stiffness applications is as follows.

- a) To determine basic mechanical properties of the auxetic structures by using energy methods: Castigliano's theorem and virtual work. Among different shapes of auxetic structures, re-entrant structures and starhoneycomb are studied.
- b) Explicit finite element code Abaqus is employed in finite element modelling of the auxetic structures.
- c) Mathematical formulation of auxetic structures is developed as a guideline for auxetic structural designers. Poisson ratios, elastic modulus, and density ratio of star honeycomb re-entrant structures are formulated. For calculating the coefficients of the formulas, finite element modelling is used.
- d) In order to increase energy absorption capacity of auxetic structures, a viscoelastic material is included into the auxetic structure. The structure is then subjected to impact loading and dynamic analysis is carried out using continuum mechanics. Maxwell and Kelvin-Voigt models are used for modelling viscoelastic material. Numerical solution and MATLAB software are employed to solve nonlinear-coupled differential equations.
- e) Different 3D re-entrant structures and conventional structures are fabricated using 3D printing machine for assessment energy absorption of auxetic and conventional structures. The material is ABS.
- f) By using Abaqus software, finite element modeling of the auxetic structures in case of impact loading and quasi-static loading is carried out to obtain specific energy absorption and impact resistance of different structures.

The scope for low stiffness applications is as follows.

- a) Fabrication of conventional polyurethane foam by mixing two components named Polyol and Isocyanate and pouring the mixture inside a mold. The weight ratio of the component is 60% polyol and 40% Isocyanate.
- b) Fabrication of auxetic foam specimens from the conventional foam through a modified fabrication process. Manufacturing parameter includes hydraulic pressure, heating time, and heating temperature. The effect of the manufacturing parameters on the auxeticity of the specimen is examined. Four hydraulic pressures and four heating temperatures are used to fabricate sixteen (16) auxetic specimens with different properties.

- c) Determination of Poisson's ratios of the auxetic specimens by using image processing technique in MATLAB software. A high speed camera model phantom V710 is used for capturing the photos.
- d) Determination of energy absorption capacity and impact resistance of the specimens by using two methods: a high speed camera and a special impactor system equipped with an accelerometer.
- e) Determination of energy absorption capacity of auxetic foam-filled tube and conventional foam-filled tube. Among the fabricated auxetic foams, the most higher energy absorption capacity is chosen for filling the aluminum tube. The auxetic foam-filled tube is subjected to uniaxial compression test to obtain energy absorption capacity and comparison is made with conventional foam-filled tube.

### **1.5 Significance of the Study**

Previous studies of auxetic materials and structures shows that analytical study and finite element work in this area is still limited and sparse, thus needing further development (Zi-Xing Lu *et al*, 2011). In this present study, a new auxetic structure named 3D re-entrant auxetic structure was developed. Analytical and finite element work has been done on the structure. In analytical work, energy methods was used which can be deemed as an alternative approach in auxetic material computation. The results provide a theoretical and imaginary basis for those interested in the research of auxeticity concept. Also, the analytical results can be used to design auxetic structures with desired mechanical properties.

In experimental section of this research, a modified fabrication method for fabricating auxetic polymeric foams was established which can contribute to fabricate more uniform auxetic specimens compared to the previous published methods. Also, a new approach based on image processing technique was used for measuring Poisson's ratio of flexible materials such as auxetic foam specimens. Since this method is economical and more accurate, this can be considered as one of the significance outcome of this research. More importantly, two new methods were

developed for measuring energy absorption of low stiff materials such as foams. This also reveals the significance of this research.

## 1.6 Contribution of the Research

The contribution of this research is highlighted as follows.

- a) The developed fabrication method of auxetic foams can be used in several industries. For instance, it can be used in automotive industry to fabricate auxetic foam for improving energy absorption capability of bumpers.
- b) Developed analytical formulation for the 3D auxetic structure can be used to design auxetic structures with controllable mechanical properties.
- c) An accurate and reliable method was established to measure Poisson's ratio of foam. The method can contribute to foam-manufacturing companies to measure Poisson's ratio of different types of foams.
- d) Two methods were used to measure energy absorption and impact resistance of foam. By changing the mass of impactor or initial height of impactor and using an image processing technique, it is possible to obtain true stress versus true strain curves for different values of strain rate. The presented experimental data can be used to be included in finite element modelling of the auxetic foam.

## 1.7 Organization of the Thesis

The present thesis is organized into seven chapters. The content of each chapter is outlined as follows.

**Chapter 1** presents an introduction of this thesis. It includes the research background, problem statement, objectives, scopes, and significance as well as contributions of the research.

**Chapter 2** focuses on a literature study of auxetic materials. Previous works on fabrication methods of these materials is addressed. Geometrical structures for modelling of these materials were then reviewed. Subsequently, previous studies of formulation, testing, mechanical properties, and application of the mentioned materials were also highlighted throughout this chapter.

**Chapter 3** describes the methodologies used in the present research. The methodologies include theoretical, experimental, and computational methodology. Also, the structure and logical flow of the research are outlined at the end of the chapter.

**Chapter 4** is devoted to analytical solution and finite element approach for calculation of basic mechanical properties of auxetic structures. Among different geometrical structures, re-entrant structure is selected because of its high potential for modelling auxetic materials. The mentioned structure is analysed theoretically and computationally in both 2D and 3D cases. In theoretical part, energy methods such as Castigliano's theorem and virtual work are used. Also, Abaqus software is used for finite element modelling.

**Chapter 5** deals with application of viscoelastic materials in auxetic industry in order to re-increase energy absorption capability. In this chapter, re-entrant star honeycomb structures are used to simulate the auxetic behavior. A viscoelastic component was then added to the structure. Maxwell and Kelvin-Voigt models were employed to model the viscoelastic part. The structure is subjected to linear and nonlinear impacts. The response of the structure is obtained for both viscoelastic models and compared.

**Chapter 6** treats the energy absorption capability of auxetic structures and materials. It includes two sections as the following:

In the first section, finite element and experimental methods were used to obtain impact resistance and energy absorption of auxetic structures. In finite element modelling, several re-entrant structures with different geometrical parameters were modelled for calculation of impact resistance and energy absorption. Also, energy absorption capability of non-auxetic structures was calculated and compared with that

of auxetic ones. In experimental section, different structures were first produced by using 3D printing machine. The models were then subjected to uniaxial compression test. Elastic modulus and energy absorption of each model was calculated.

In the second section, a modified method is employed to fabricate auxetic polymeric foams. In this method, polymeric foam specimens are compressed triaxially by hydraulic oil pressure. After compression, the specimens are put inside special molds and then placed inside an oven to heat up to a temperature around softening temperature for a special time. After that, the specimens are removed and cool down in room temperature. A number of auxetic specimens with different volumetric compression, heating time and heating temperature are fabricated by this method. To measure Poisson's ratio of the specimens accurately, image processing method is used. Experimental work is carried out to measure the impact resistance and energy absorption capacity of the specimens. The effect of hydraulic pressure, and heating temperature on impact resistance and energy absorption capability of the auxetic foams is investigated.

**Chapter 7** sums up the research project and the suggestions for future research works are also presented.

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