OMEGA ALGEBRA MODEL OF CLINICAL WASTE INCINERATION PROCESS

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To my beloved mother and father.

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ABSTRACT

This research shows how the Fuzzy Autocatalytic Set (FACS) is transformed into a semigroup. Omega algebra is used as the main aspect to relate the FACS to the transformation semigroup of omega algebra of the FACS. Clinical waste incineration process is used as an example in supporting the developed theorems and lemmas. C++ programming is used to develop a customized program that computes the necessary data in supporting new theorems and lemmas. First, the FACS is defined in terms of omega algebra, giving the omega algebra of FACS. Next, the semigroup of omega algebra of FACS is constructed using the omega algebra of FACS. The membership value of fuzzy edge connectivity is also determined to complete the definition of the omega algebra of FACS. New definitions and terminologies are used to model the clinical waste incineration process in terms of omega algebra. As a result, the new model is shown to be able to comprehensively explain the catalytic relation amongst the chemical elements in the clinical waste incineration process. The established semigroup of omega algebra of FACS is then defined to be a transformation semigroup of omega algebra of FACS. The results show that FACS of the clinical waste incineration process and the structure of transformation semigroup of omega algebra of the clinical waste incineration process are "compatible". In addition, the manifold representation of the transformation semigroup of omega algebra of FACS is proposed for further studies.

ABSTRAK

Kajian ini menunjukkan bagaimana Set Automangkinan Kabur (FACS) ditransformasi kepada semikumpulan. Aljabar omega digunakan sebagai aspek utama untuk menghubungkan FACS kepada transformasi semikumpulan aljabar omega. Proses pembakaran sisa buangan klinikal digunakan sebagai satu contoh untuk menyokong teorem dan lema yang telah dibangunkan. Pengaturcaraan C++ digunakan untuk membangunkan satu pengaturcaraan bagi mengira data yang diperlukan dalam menyokong teorem dan lema baharu tersebut. Pada permulaanya, FACS ditakrifkan dalam sebutan aljabar omega, yang memberikan aljabar omega FACS. Seterusnya, semikumpulan aljabar omega FACS dibangunkan dengan menggunakan aljabar omega FACS. Nilai keahlian kabur juga ditentukan untuk menyempurnakan definisi aljabar omega FACS. Definisi dan istilah baharu ini digunakan untuk memodelkan proses pembakaran sisa buangan klinikal. Natijahnya, model baharu yang dibangunkan berupaya memaparkan hubungan pemangkinan antara unsur kimia dalam proses pembakaran sisa buangan klinikal dengan lebih menyeluruh. Semikumpulan aljabar omega FACS yang telah diperkenalkan kemudiannya ditarifkan sebagai transformasi semikumpulan aljabar omega FACS. Keputusan menunjukkan bahawa FACS dari proses pembakaran sisa buangan klinikal dan struktur transformasi semikumpulan aljabar omega adalah “serasi”. Sebagai tambahan, perwakilan manifold kepada transformasi semikumpulan aljabar omega FACS telah dicadangkan untuk kajian pada masa hadapan.

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LIST OF ABBREVIATIONS

ACS	-	Autocatalytic Set
FACS	-	Fuzzy Autocatalytic Set
$FACS_{G_F}$	-	Fuzzy autocatalytic set of the clinical waste incineration process
max	-	Maximal
max-av	-	Maximal average
max-prod	-	Maximal product
min	-	Minimum
$PF(Q)$	-	Partial function of Q
sup	-	Supreme
ts	-	Transformation semigroup

LIST OF SYMBOLS

$A_{\tilde{r}}$	-	Assignment function of gradual number $\tilde{r}$
$A_{\tilde{n}^-}$	-	Assignment function of left profile over fuzzy interval N
$A_{\tilde{n}^+}$	-	Assignment function of right profile over fuzzy interval N
$[a,b]$	-	Closed interval
$(a,b]/[a,b)$	-	Semi-open interval
C	-	Set of fuzzy edge connectivity
$C_{F_{ij}}$	-	Fuzzy edge connectivity between node i and node j
E	-	Set of edges
$E(w_F)$	-	Edges have fuzzy weights
$E(x, y)$	-	Strength/ membership value of arc or edge between vertex x and vertex y
e_i	-	The i^{th} edge/arc
G	-	Graph
G_d	-	Crisp graph representation of clinical waste incineration process
G_F	-	Fuzzy graph representation of clinical waste incineration process
G_F^i	-	i^{th} type fuzziness of fuzzy graph
g	-	Mapping between fuzzy interval

$\bar{g}$	-	United extension of *g*
$h(e_i)$	-	Fuzzy head of the i^{th} edge
M	-	Manifold
N	-	Fuzzy interval
$\tilde{n}^{+}$	-	Fuzzy upper bound/ right profile over fuzzy interval *N*
$\tilde{n}^{-}$	-	Fuzzy lower bound/ left profile over fuzzy interval *N*
$\Re$	-	Real number
$\tilde{r}$	-	Gradual number
S,T	-	Semigroup
(S,Θ)	-	Semigroup with semigroup operation
$S(N)$	-	Subset of fuzzy interval *N*
$t(e_i)$	-	Fuzzy tail of the i^{th} edge
$ts\Omega_{G_F}$	-	Transformation semigroup of clinical waste incineration process
V	-	Set of vertices
$V(x)$	-	Strength / membership value of vertex *x*
V_{FACS}	-	Set of vertices for FACS
v_i	-	The i^{th} vertex
W_k	-	Walk with *k*-number of vertices
$\omega_{k(v_i,v_j)}$	-	Omega operation of *k*-operand from v_i to v_j
ω_n	-	Omega operation of *n*-operand
$\underline{X}$	-	Left endpoint of interval *X*
$\overline{X}$	-	Right endpoint of interval *X*
$+$	-	Addition function of vector space

$\bullet$	-	Multiple function of vector space
α -cut	-	Alpha-cut
o	-	Composition of fuzzy relation
$*$	-	Omega operation of clinical waste incineration process
$\circ$	-	Semigroup action
Θ	-	Semigroup operation
$\in$	-	Element of
$\geq$	-	Equal or greater than
$\forall$	-	Every/ each
$>$	-	Greater than
$\leq$	-	Less or equal
$<$	-	Less than
$\vee$	-	Maximal
$\wedge$	-	Minimal
$\underset{+}{o}$	-	Max-av composition
$\underset{\bullet}{o}$	-	Max-prod composition
$\mu_N(x)$	-	Membership function of fuzzy interval N
$\mu(\omega_{ij})$	-	Membership value of fuzzy edge connectivity between vertex v_i and vertex v_j
$\mu(e_i)$	-	Membership value for fuzzy edge connectivity for edge i
Ω -algebra	-	Set of omega algebra
Ω_{G_F}	-	Omega algebra of clinical waste incineration process
Ω_{FACS}	-	Omega algebra of FACS
$\bigcup$	-	Union

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CHAPTER 1

INTRODUCTION

1.0 Background of the Research

From the time when Zadeh (1965) brought the inspiration of fuzzy set theory by utilizing the concept of grade membership, many researchers and mathematicians have been concerned with the properties and applications of fuzzy sets (Gitman & Levine,1970; Tamura *et al.*, 1971; Kandel and Yelowitz, 1974; Pathak and Pal, 1986; Keller and Tahani, 1992). This is due to the fact that most crisp mathematical models are always unable to model some details of reality including complexity and ill-defined circumstances. Therefore the crisp models are generally insufficient in describing the whole process of a system. On the other hand, the fuzziness concept is proficient in modelling, explaining as well as predicting real life issues, such as weather prediction (Riordan and Hansen, 2002). As a result, more and more studies have been carried out by utilizing this idea.

The concept of fuzzy sets provide a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of a random variable. The fuzzy graph was one of the examples in fuzzy theory extensive application in its relation to graph theory. Fuzzy graph theory was first introduced by Azriel Rosenfeld in 1975. Since then, it has been quickly expanding and has several applications in various fields.

Modeling an incineration process is one of its applications to the real life problem. Any waste or remaining supplies from medical, dental, veterinary, pharmaceutical, treatment, hospital that may result in contamination to any individual after exposure is defined as clinical waste. The waste incineration process is a waste treatment process which underwent some controlled conditions which included operating temperature, oxygen level, fuel supply and residence time.

Sabariah *et al*. (2002) embarked to model an incineration process of a regional clinical waste incinerator facility in Malacca that is owned by Pantai Medivest Sdn Bhd. Even though the incineration process seems simple, but as the research was carried out in the year 2002, there was no essential mathematical model to give details of the underlying principles involved in the process and operational control of the incinerator. The schematic diagram of the facility is shown in Figure 1.1.

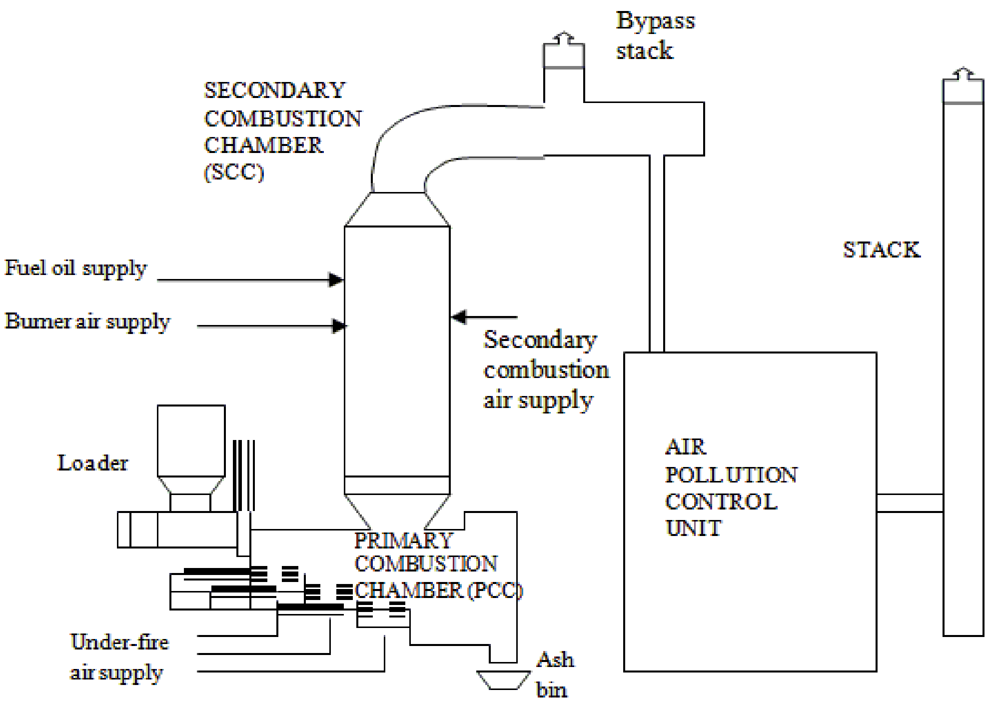

Figure 1.1 Illustration of a clinical waste incinerator (Sabariah *et al.*, 2002)

A graphical representation in Figure 1.2 demonstrating the relation between input-output variables of the incineration process was successfully achieved using a graph. However, this graph model in Figure 1.2 was not able to describe the incineration in detail.

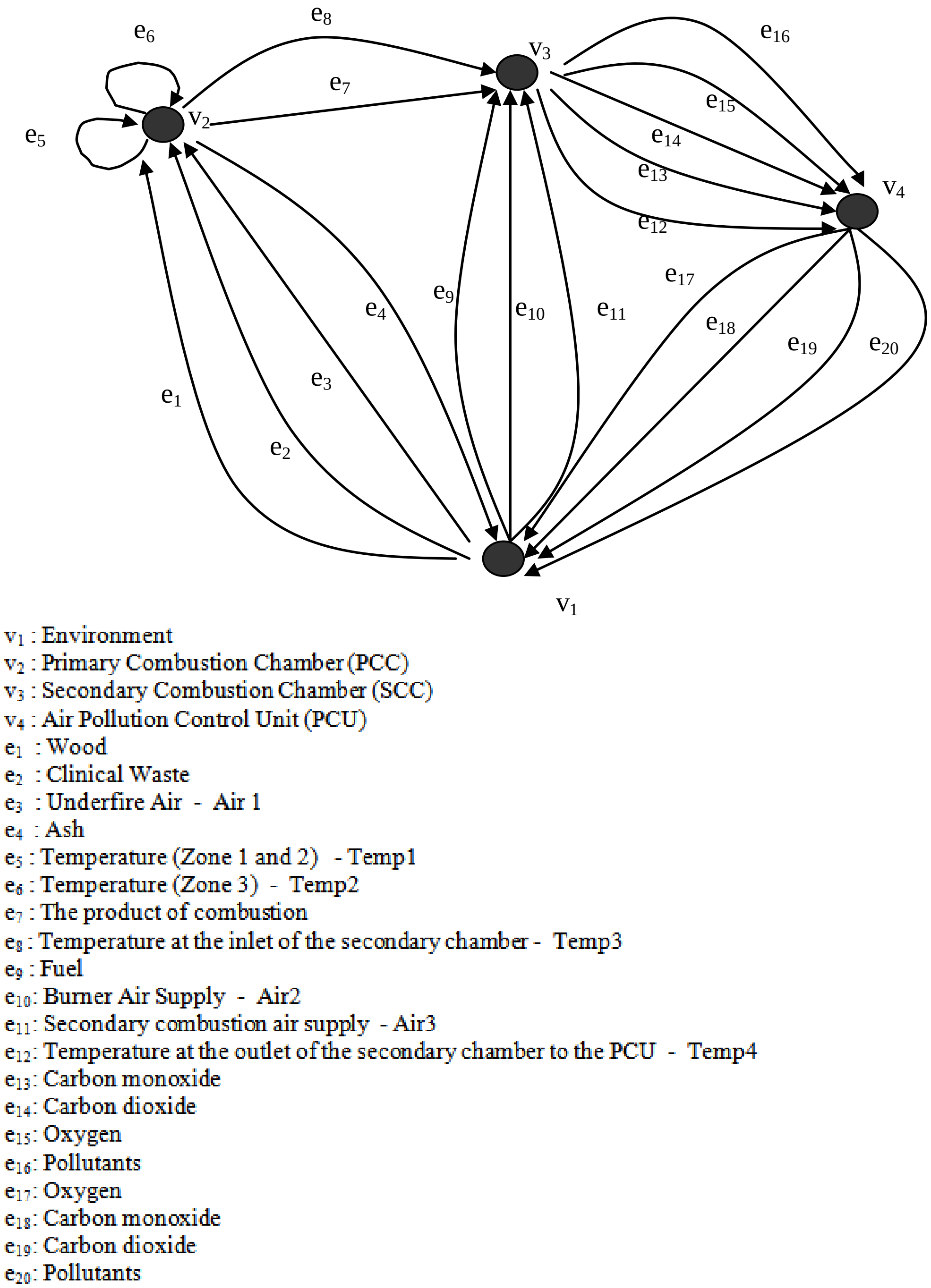

Figure 1.2 A Graph showing the association between input-output parameter and different components of the incineration process (Sabariah *et al*, 2002).

After some assumptions and refinements, a crisp model (Figure 1.3) was created (Sabariah, 2006) whereby $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the set of vertices representing six variables that play a crucial part in the clinical waste incineration process, namely waste, fuel, oxygen, carbon dioxide, carbon monoxide and other gases including water, respectively.

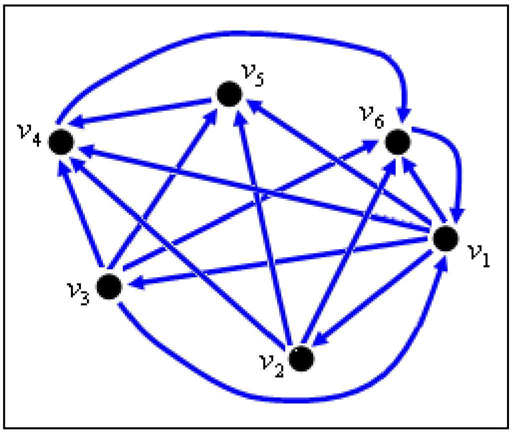

Figure 1.3 A Crisp Graph, G_d of the incineration process

The edges represent the connection between the variables in the process which indicate their catalytic relationship and are denoted by the set E, (v_i, v_j) which means there is a catalytic relation between v_i and v_j where v_i catalyzed the production of v_j. However, the crisp graph in Figure 1.3 was not able to illustrate the waste incineration process in detail.

Due to some discrepancies of this crisp model (Figure 1.3) in explaining the system has urged the study to look into the use of fuzzy theory, particularly the fuzzy graph. Sabariah (2006) then applied the concept of the fuzzy graph into the modeling of the clinical waste incineration process. The development and the results of the model have been presented in Figure 1.4 (Tahir *et al,* 2010).

On the other hand, Blue *et al*. (1997; 2002) generalized a catalogue of various possible types of fuzziness in a graph, which they named taxonomy of fuzzy graphs. There are 5 types of graph in total that are characterized below:

Type 1: Fuzzy Set of Graphs,

Type 2: Crisp Set of Vertices and Fuzzy Edges,

Type 3: Crisp Set of Vertices and Edges with Fuzzy Connectivity,

Type 4: Fuzzy Vertex set and Crisp Edge Set,

Type 5: Crisp Graph with Fuzzy Weight.

The fuzzy graph in Figure 1.4 is proved to be a fuzzy graph Type 3 (Tahir *et al*., 2010).

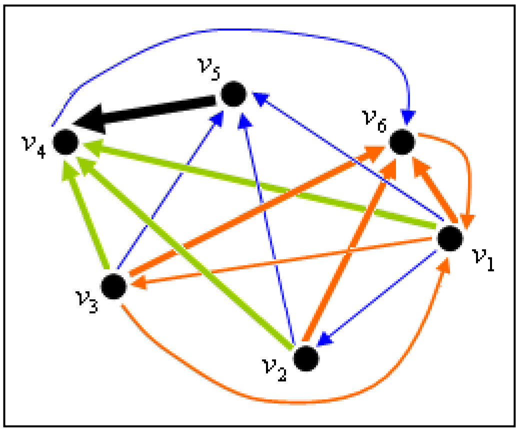

Figure 1.4 Fuzzy Graph of Type 3, G_F for the clinical incineration process.

A clinical waste incineration process modeled using this new concept (Figure 1.4) was more precise than when modeled with a crisp graph (Figure 1.3) (Tahir *et al*. 2010). As compared to the same color and thickness of each link in the crisp graph (Figure 1.3), the different color of edges in Figure 1.4 signifies the different range of membership values for the fuzzy edge connectivity. The changes in the concentration of the chemical elements are demonstrated more precisely with the changes in the thickness of the edges of each pair of elements in the process.

This graphical representation is found to be an Autocatalytic Set (ACS). A new concept known as the Fuzzy Autocatalytic Set (FACS) was then fashioned under the mishmash of fuzzy graph theory into ACS. Sabariah (2006) also showed that this representation enlightens the clinical waste incineration process more precisely.

The highlight of the investigation was the introduction of a fresh idea called the Fuzzy Autocatalytic Set (FACS). This relation between fuzzy graphs to autocatalytic sets has produced a few results in the forms of proven theorem (Sabariah, 2006).

1.1 Statement of the problem

The crisp graph (Figure 1.3) representation of the clinical waste incineration process gives the idea of connectivity between two elements of the process, while the fuzzy graph representation of the clinical waste incineration process (Figure 1.4) provides the changes of the concentration of the parameters during the process. However, these two graphs representations were not able to tell the details of the process as of yet. For example, the system cannot give information regarding the catalytic interaction and relation amongst each and every parameter in the system, not only among the pair.

In addition, the previous study was also unable to present all the possible catalytic reactions between variables of the system. The main interest in this research is the creation of the algebraic model to observe the system such that the interaction between parameters can be observed and built more comprehensively.

1.2 Objectives of the Study

The goal of this research is to extend the algebraic structure of the fuzzy autocatalytic set. In order to accomplish the goal, the following objectives are set as research tactics. The objectives are:

a) to transform the graphical representation of the fuzzy autocatalytic set to omega algebra.

b) to prove that the omega algebra of fuzzy autocatalytic set is a semigroup.

c) to model the omega algebra of the fuzzy autocatalytic set in terms of transformation semigroup.

d) to propose the manifold representation of the fuzzy autocatalytic set.

1.3 Scope of the Study

For this research, the emphasis is on the exploration of algebraic structures behind the clinical incineration reaction process that was modeled using FACS. The main interest is to model this model of FACS into few possible algebraic structures in order to study and algebraically explain the changes of the variable.

The omega algebra concept would be the foundation and the main aspect of the proposed algebraic model. The algebraic model assembled will then be analyzed in terms of validity in explaining the clinical waste incineration process.

1.4 Significance of Study

The main aim of this research is to extend the algebraic structure of the fuzzy autocatalytic set of the fuzzy graph. The omega algebra concept would be the foundation and the main aspect of the proposed algebraic model of the clinical waste incineration process. This new structure is called omega algebra of fuzzy autocatalytic set of the clinical waste incineration process. The new structure subsequently yields some physical interpretations in the form of proven lemmas and theorems for the clinical incineration process which has been modeled as a fuzzy autocatalytic set.

1.5 Summary and Outline of Thesis

The first chapter provides the general background and information regarding the research.

Chapter 2 gives the published literature of relevant topics to the research which includes graph theory, fuzzy theory, fuzzy graph, the autocatalytic set and Fuzzy Autocatalytic Set, the Fuzzy Autocatalytic Set of the clinical waste incineration process as well as omega algebra, the transformation semigroup and manifold.

The third chapter presents the construction of the omega algebra structure of the Fuzzy Autocatalytic Set. Then the omega algebra of the Fuzzy Autocatalytic Set is applied into clinical waste incineration as an example in explaining this algebraic structure. The same chapter also provides the definition of the membership value of fuzzy edge connectivity of the omega algebra of the Fuzzy Autocatalytic Set of the clinical waste incineration process.

Chapter 4 presents the extension algebraic model of omega algebra of the Fuzzy Autocatalytic Set which is constructed in the previous chapter. This includes the transformation semigroup of omega algebra of the Fuzzy Autocatalytic Set and manifold representation of transformation semigroup of omega algebra of the clinical waste incineration process.

Chapter 5 concludes the results and findings presented in the preceding chapters and suggest several ideas for further research on this topic.

REFERENCES

Abraham, R. and Marsden, J. E., (1978). *Foundation of Mechanics*. (2nd ed.). London: The Benjamin / Cummings Publishing Company, Inc.

Blue, M., Bush B. and Puckett, J. (1997). *Applications of Fuzzy Logic to Graph Theory*. Los Alamos National Laboratory. LA-UR-96-4792.

Blue, M., Bush B. and Puckett, J. (2002). Unified Approach to Fuzzy Graph Problems. *Fuzzy Sets and Systems*. 125: 355-368.

Chaplin, M. (2006). Do we underestimate the importance of water in cell biology? *Nature Reviews Molecular Cell Biology*. 7: 861-866.

Ding, C., He, X., Xiong, H., Peng, H. and Holbrook, S. R. (2006). Transitive Closure and Metric Inequality of Weighted Graphs: Detecting Protein Interaction Modules Using Cliques. *Int. J. Data Mining and Bioinformatics*. 1(2): 162-177.

Dubois, D. and Prade, H. (2005). Fuzzy elements in the fuzzy set. *Proceedings of the 10th International Fuzzy System Association (IFSA) Congress*. 28-31 July. China, Beijing. Pp. 55-60.

Eilenberg, S. (1976). *Automata, languages, and machines*. Vol B. New York: Academic Press.

Egry-Nagy, A., Nehaniv, C. L., Rhodes, J. L. and Schilstra M. J. (2008). Automatic Analysis of Computation in Biochemical Reactions. *Biosystem*. 94: 126-134.

Fortin, J., Dudois D. and Fargier, H. (2008). Gradual numbers and their Application to Fuzzy Interval Analysis. *IEEE Transactions on Fuzzy Systems.* 6(2): 388-402.

Gitman, I. and Levine, M.D. (1970). An Algorithm for Detecting Unimodal Fuzzy Sets and Its Application As A Clustering Technique. *IEEE Transaction on Computers*. 19(7): 583-593.

Halmos, P.R. (1960). *Naïve Set Theory*, New York: Van Nostrand.

Harary, F. (1969). *Graph Theory*. California, USA: Addison Wesley Publishing Company.

Howie, J. M., (1976). *An introduction To Semigroup Theory*. (1st ed.). London: Academic Press Inc. Ltd.

Jain, S. and Krishna, S. (1998). Autocatalytic Sets and the Growth of Complexity in an Evolutionary model. *Physical Review Letters*. 81: 5684-5687.

Jain, S. and Krishna, S. (1999). Emergence and the Growth of Complexity Networks in an Adaptive Systems. *Computer Physics Communications*. 121-122: 116-121.

Jain, S. and Krishna, S. (2003). *Graph Theory and the Evolution of Autocatalytic Networks*. In Bornholdt, S. & Schuster, H.G. eds. *Handbook of Graphs and Network.* (355-395). Berlin: John Wiley and VCH Publishers.

Kandel, A. and Yelowitz, L. (1974). Fuzzy Chains. *IEEE Transactions on Systems, Man and Cybernetics*. 4(5): 472-475.

Kauffman, S. A. (1971). Cellular Homeostasis, Epigenesist and Replication in Randomly Aggregated Macromolecular Systems. *Journal of Cybernetics*. 1: 71-97.

Kauffman, S. A. (1995). *At Home in the Universe- the Search for the Laws of Self-Organization and Complexity*. Oxford: Oxford University Press.

Keller, J.M. and Tahani, H. (1992). Implementation of Conjunctive and Disjunctive Fuzzy Logic Rules with Neural Networks. *International Journal of Approximate Reasoning*. 6: 221-240.

Krohn, K. and Rhodes, J. (1965). Algebraic Theory of Machine. I. Prime Decomposition Theorem for Finite Semigroup and Machine. *Trans. Am. Math. Soc*. 116: 450-464.

Moore, R. E., Kearfott, R. B. and Cloud, M. J. (2009). *Introduction to Interval Analysis*. Philadelphia: Society for Industrial and Applied Mathematics.

Nur Izura Udzir, Razali Yaakob and Mohamed Nordin Zakaria (2002). *C++: Pengenalan Kepada Pengaturcaraan*. Malaysia: Pearson Malaysia Sdn. Bhd.

Pathak, A. and Pal, S.K. (1986). Fuzzy Grammars in Syntactic Recognition of Skeletal Maturity from X-Rays. *IEEE Transactions on Systems, Man and Cybernetics*. 4(5): 657-667.

Plotkin, B.I., Greenglaz, L. Ja. and Gvaramija, A. A. (1992). *Algebraic Structures in Automata and Databases Theory*. Singapore: World Scientific.

Rabin, M. O. and Scott, D. (1959). Finite Automata and Their Decision Problems. *IBM J. Res Develop*. 3: 114-125.

Rhodes, J. L and Tilson B. R. (1968). *Semigroup: Elementary Definitions and Examples.* In Arbib, M. A.. *Algebraic Theory of Machines, Language, and Semigroup*. (1-13). New York: Academic Press Inc.

Riordan, D. and Hansen, B. K. (2002). A Fuzzy case-based System for Weather Preditcion. *Engineering Intelligent Systems.* 3: 139-146.

Rosenfeld, A. (1975). *Fuzzy Graphs*. In Zadeh, L.A., Fu, K.S. and Shimura, M. eds. *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*. (pp. 77-95). New York: Academic Press.

Rossler, O. E. (1971). A System Theoretic Model of Biogenesis. *Z. Naturforchung*. 26b: 741-746.

Sabariah Baharun, Tahir Ahmad and Khairil Anuar Arshad (2002). Graphical Presentation of a Clinical Waste Incineration Process. *Proceedings of the 10th National Symposium of Mathematical Sciences.* 23-24 December. Johor Bahru, Johor. Pp. 109-115.

Sabariah Baharun (2006). *Modeling of Clinical Waste Incinerator Process Using Novel Fuzzy Autocatalytic Set: Manifestation of Mathematical Thinking.* Doctor Philosophy, Universiti Teknologi Malaysia, Skudai.

Sabariah Baharun, Tahir Ahmad and Mohd Rashid Mohd Yusof (2009). Fuzzy Edge Connectivity Relates the Variable in Clinical Waste Incineration Process. *Matematica* 25(1): 31-38.

Tahir Ahmad (2000). Satu Kaitan Koszul Mendatar Untuk Berkas Tangen Unit Sfera $(\mathrm{T}S^2, \pi, S^2)$. *Matematika* 16(1): 41-46.

Tahir Ahmad, Sabariah Baharun and Khairil Anuar Arshad. (2010). Modeling A Clinical Incineration Process Using Fuzzy Autocatalytic Set. *Journal Math Chem*. 47:1263-1273.

Tamura, S., Higuchi, S. and Tanaka, K. (1971). Pattern Classification Based On Fuzzy Relation. *IEEE Transactions on Systems, Man and Cybernetics*. 1(1): 61-66.

Tutte, W. T. (1984). *Graph Theory*. California, U.S.A.: Addison-Wesley Publishing Company.

Watanabe, T. and Nakamura, A. (1983). On the Transformation Semigroup of Finite Automata. *J. Comput. System Sci*. 26:107-138.

Yeh, R. T. and Bang, S. Y. (1975). *Fuzzy relations, fuzzy graphs and their applications to clustering analysis*. In Zadeh, L.A., Fu, K.S. and Shimura, M. eds. *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*. (pp. 125-149). New York: Academic Press.

Zadeh, L.A. (1965). Fuzzy set. *Information and control*. 8(3): 338-353.