GENERALISED INTERVAL-VALUED FUZZY IDEALS AND FILTERS IN ORDERED SEMIGROUPS

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TO MY BELOVED FATHER

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ABSTRACT

As a generalisation of fuzzy set, interval-valued fuzzy set has been formed by extending the grade of fuzzy membership function from a point to an interval. This concept has been applied to several algebraic structures, especially in semigroups and ordered semigroups. The importance of the ideas of "belongs to" ($\in$) and "quasi coincident with" (q) relations between a fuzzy point and fuzzy set is evident from the volume of researches conducted during the past two decades relating to these concepts. Some researchers employed these concepts in various algebraic structures like BCI-algebra, MV-algebra, semigroups and ordered semigroups by using interval-valued fuzzy set. However, it appears that ordered semigroups and their classification by the properties of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy concepts are not yet in the literature. In this research, for $\tilde{k} \in D[0,1)$, the concept of "$\tilde{k}$-quasi coincident with" ($q_{\tilde{k}}$) relation between interval-valued fuzzy point and interval-valued fuzzy subset is used to define the notions of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy ideal theory and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy filters in ordered semigroups. Interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy ideal theory refers to interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy ideals, generalised bi-ideal and bi-ideal. These generalised concepts are then linked with ordinary ideals, generalised bi-ideal, bi-ideal and filters by means of level subsets. Furthermore, characterisations of ordered semigroups are studied by the properties of generalised interval-valued fuzzy ideal theory and generalised interval-valued fuzzy filters. Finally, the notions of $\tilde{t}$-implication based interval-valued fuzzy ideals and filters are introduced. Thereafter, $[\frac{1-k^+}{2}, \frac{1-k^-}{2}]$-implication based interval-valued fuzzy ideal theory and filters are linked with interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy ideal theory and filters by using implication operators.

ABSTRAK

Sebagai pengitlakan bagi set kabur, set kabur bernilai-selang telah dibentukkan dengan memanjangkan gred bagi fungsi keahlian kabur dari satu titik kepada satu selang. Konsep ini telah diaplikasikan kepada beberapa struktur aljabar terutamanya dalam semikumpulan dan semikumpulan bertertib. Kepentingan ide hubungan "milik kepada" $(\in)$ dan "kuasi-kebetulan dengan" (q) antara titik kabur dan set kabur adalah jelas melalui penyelidikan berkaitan konsep-konsep ini yang telah dijalankan sepanjang dua dekad. Beberapa penyelidik menggunakan konsep ini dalam pelbagai struktur aljabar seperti aljabar-BCI, aljabar-MV, semikumpulan serta semikumpulan bertertib dengan menggunakan set kabur bernilai-selang. Walaubagaimanapun, kajian mengenai semikumpulan bertertib dan pengkelasannya melalui ciri-ciri konsep kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang masih belum dijumpai dalam kajian literatur. Dalam penyelidikan ini, untuk $\tilde{k} \in D[0,1)$, konsep hubungan "kuasi-kebetulan dengan $\tilde{k}$" $(q_{\tilde{k}})$ antara titik kabur bernilai-selang dan subset kabur bernilai-selang diguna untuk menentukan idea bagi teori unggulan kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang dan turas kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang dalam semikumpulan bertertib. Teori unggulan kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang merujuk kepada unggulan kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang, dwi-unggulan teritlak dan dwi-unggulan. Tambahan pula, konsep teritlak ini dihubungkan dengan unggulan biasa, dwi-unggulan teritlak, dwi-unggulan dan turas melalui kaedah peringkat subset. Seterusnya, pencirian bagi semikumpulan bertertib dikaji melalui ciri-ciri teori unggulan kabur bernilai-selang dan turas kabur bernilai-selang teritlak. Akhir sekali, ide bagi implikasi-$\tilde{t}$ berdasarkan unggulan dan turas kabur bernilai-selang diperkenalkan. Kemudiannya, implikasi-$[\frac{1-k^+}{2}, \frac{1-k^-}{2}]$ berpusatkan teori unggulan dan turas kabur bernilai-selang dihubungkan dengan teori unggulan dan turas kabur-$(\in, \in \vee q_{\tilde{k}})$ bernilai-selang dengan menggunakan pengendali implikasi.

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$\widetilde{t}$	-	arbitrary closed sub-interval $[t^{-}, t^{+}]$ of the unit closed interval
$\in$	-	belongs to relation
$\in \vee q$	-	belongs to or quasi-coincident with relation
$\in \wedge q$	-	belongs to and quasi-coincident with relation
$\in \vee q_{\widetilde{k}}$	-	belongs to or generalized quasi-coincident with relation
χ_A	-	characteristic function
$D[0,1]$	-	class of closed sub intervals of the unit closed interval
$\models$	-	double turnstile (used to indicate a tautology)
x_t	-	fuzzy point
f	-	fuzzy subset
$q_{\widetilde{k}}$	-	generalised quasi-coincident with relation
I_G	-	Gődel implication operator
$x_{\widetilde{t}}$	-	interval-valued fuzzy point
$A \cap B$	-	intersection of interval-valued fuzzy subsets A and B
$q_{\widetilde{k}}$	-	$\widetilde{k}$ -quasi-coincident with relation
$\widetilde{f}_A$	-	membership function of an interval-valued fuzzy subset A
$\overline{\in}$	-	negation of belongs to relation
$\overline{q}$	-	negation of quasi-coincident with relation
S	-	ordered semigroup
q	-	quasi-coincident with relation
$\widetilde{0}$	-	the interval $[0, 0]$
$\widetilde{1}$	-	the interval $[1,1]$
I_{cG}	-	the contraposition of Gődel implication operator

$\tilde{k}$	-	the arbitrary interval $[k^-, k^+]$ of $D[0,1)$
$[\Phi]$	-	the truth value of a fuzzy proposition Φ
$A \cup B$	-	union of interval-valued fuzzy sets A and B
$U(A;\tilde{t})$	-	$(\in)$ -level subset of an interval-valued fuzzy set A
$Q^{\tilde{k}}(A;\tilde{t})$	-	$(q_{\tilde{k}})$ -level subset of an interval-valued fuzzy set A
$[A]_{\tilde{t}}^{\tilde{k}}$	-	$(\in \vee q_{\tilde{k}})$ - level subset of an interval-valued fuzzy set A

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The concept of “belongs to” relation ($\in$) and “quasi-coincident with” relation (q) between a fuzzy point and a fuzzy set was introduced by Pu and Liu [1] and has boosted the significance of fuzzy set in different branches of algebra. A fuzzy set in X (where X is an ordinary non-empty set) is characterised by a function $f : X \to [0,1]$ [1]. A fuzzy set in X is called a fuzzy point if and only if its value is “0” at all $y \in X$ except at one point (say x) in X. If the value of the fuzzy point is $t \in (0,1]$ at $x \in X$, then it is denoted by x_t where the point x is its support [1]. This $f_A(x) \geq t$ means that a fuzzy point x_t is contained in a fuzzy set f and is denoted by $x_t \in f$. On the other hand, if $f(x) + t > 1$, then a fuzzy point x_t is said to be quasi-coincident with a fuzzy set f and is denoted by $x_t q f$. Moreover, Bhakat and Das [2, 3] used the notions of "belongs to" and "quasi-coincident with” relations and proposed the idea of (α, β)-fuzzy subgroup, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$. The idea of generalised fuzzy subgroups has increased the importance of algebraic structures by attracting the attention of many researchers and opened ways for further research in this field. In addition, Jun [4] generalised the concept of "quasi-coincident with" relation. According to him if $f(x) + t + k > 1$, $k \in [0,1)$, then x_t is said to be k-quasi-coincident with a fuzzy set f and is denoted by $x_t q_k f$. In [5] it is stated that, “As a rule, the membership functions of fuzzy sets representing particular verbal expressions cannot be defined appropriately on the basis of available information. Therefore it is not always possible for a membership function of type $f: X \to [0,1]$ to assign precisely one point from the interval $[0,1]$ to each element of x in X without loss of some information". Keeping this point in view, the concept of fuzzy set [6] is further generalised to interval-valued fuzzy set [7].

The main objective of the present study is to introduce the concepts of generalised interval-valued fuzzy ideal theory, generalised interval-valued fuzzy left filter and interval-valued fuzzy right filter in ordered semigroups by using the aforementioned concepts of "belongs to" relation and "quasi-coincident with" relation by Bhakat and Das [2, 3]. By generalised interval-valued fuzzy ideal theory we mean interval-valued fuzzy left ideal, interval-valued fuzzy right ideal, interval-valued fuzzy generalised bi-ideal and interval-valued fuzzy bi-ideal of type $(\in, \in \vee q_{\tilde{k}})$. Another objective is to characterise ordered semigroups by the properties of generalised interval-valued fuzzy ideal theory and interval-valued fuzzy filter of type $(\in, \in \vee q_{\tilde{k}})$.

1.2 Research Background

Objects in this real world, up to some extent have a degree of fuzziness. Therefore, classical set theory is not appropriate notion in dealing with real life problems. In order to overcome this problem, Zadeh [6] came up with an idea of fuzzy set. A fuzzy set is a set whose members may have a grade of membership between "0" and "1" as opposed to classical sets where each element must have either "0" or "1" as the membership grade. If the grade of the membership function is 0, then the element is completely outside the set and if its value is "1", then the element is completely inside the set. In the beginning, fuzzy set theory received vigorous negative replies from some renowned mathematicians and scientists. Regardless of the debate on the fuzzy theory, it drew the attention of many other researchers from different fields, especially mathematics. Fuzzy set theory discovered many applications in different areas like handwriting recognition and washing machines. These remarkable developments of fuzzy set theory attracted researchers to think about its applications in several applied disciplines such as theoretical physics, computer science, control engineering, information sciences, coding theory and fuzzy automata. Keeping this point in view, Rosenfeld [8] employed fuzzy set theory in algebra and initiated the concept of fuzzy subgroups. Since then, many researchers from different areas are working on thoughts like fuzzy semigroups [9-11], fuzzy ordered semigroups [12-18], fuzzy sub rings and near-rings [19, 20], fuzzy subgroups [2, 3, 8, 21, 22], BCK/BCI algebras [4, 23] and fuzzy topological spaces [1, 24].

In the contribution to Zadeh's idea [7] of interval-valued fuzzy sets, Gorzalczany [5] studied interval-valued fuzzy sets for approximate reasoning. Roy and Biswas [25] further investigated interval-valued fuzzy relations and applied these in Sanchez's approach for medical diagnosis. In addition, Biswas [22] used interval-valued fuzzy sets in algebra and introduced the notion of interval-valued fuzzy subgroups. Meanwhile, Narayanan and Manikantan [26, 27] used the concept of interval-valued fuzzy sets in semigroups and investigated various types of interval-valued fuzzy ideals in semigroups. Moreover, Yaqoob *et al.* [28] linked interval-valued fuzzy sets with left almost semigroups and introduced the concepts of interval-valued fuzzy interior ideal, interval-valued fuzzy left ideal, interval-valued fuzzy right ideal, interval-valued fuzzy two-sided ideal and interval-valued fuzzy bi-ideal. They provided several useful results on direct product of interval-valued fuzzy ideals in left almost semigroups.

Keeping the idea of the fuzzy point in view, Bhakat and Das [2, 3] defined $(\in, \in\vee q)$-fuzzy subgroups by using “belongs to” relation and “quasi coincident with” relation between fuzzy point and fuzzy set. Indeed, this is a useful generalization of Rosenfeld's fuzzy subgroup. Moreover, this concept of “belongs to” relation $(\in)$ and “quasi-coincident with” relation (q) has been applied in many other branches like R_0-algebras [29, 30], BL-algebra [31, 32], BCK/BCI-algebra [33], hemirings [34], ordered semigroups [35], semigroups [36]. Meanwhile, Jun [4] introduced a more general form of quasi-coincidence of a fuzzy point x_t with a fuzzy set f and defined $x_t q_k f$ where $k \in [0,1)$. Further, using Jun's idea of “generalised quasi-coincident with” relation (q_k), Shabir *et al.* [9] introduced the notion of $(\in, \in\vee q_k)$-fuzzy ideal of a semigroup as a generalization of (α,β)-fuzzy ideal. In addition, the concepts of interval-valued fuzzy left and right ideals, interval-valued fuzzy two-sided ideals, interval-valued fuzzy interior ideals and interval-valued fuzzy bi-ideals in semigroups are given in [27]. Further, Ma *et al.* [23] introduced the concepts of interval-valued $(\in,\in\vee q)$-fuzzy ideals of BCI-algebras. Moreover, ordered semigroups are characterised by the properties of interval-valued fuzzy left (resp. right) ideals, two-sided ideals, interior ideals and bi-ideals by Shabir and Khan [37], whereas in K-algebra, the concept of interval-valued fuzzy sets is applied by Akram *et al.* [38] and discussed K-algebra based on (α,β)-fuzzy subalgebra. Meanwhile, Khan *et al.* [39] initiated the notion of interval-valued $(\in,\in\vee q)$-fuzzy bi-ideals in ordered semigroups and provided a connection between ordinary bi-ideals and interval-valued $(\in,\in\vee q)$-fuzzy bi-ideals in the structure of ordered semigroups.

In this research work, by using the concept of belongs to relation ($\in$) and interval-valued generalised quasi-coincident with relation ($q_{\tilde{k}}$), a new type of fuzzy ideal theory of ordered semigroup called interval-valued $(\in,\in\vee q_{\tilde{k}})$-fuzzy ideal theory is introduced. Furthermore, the characterisation of ordered semigroups by the properties of interval-valued fuzzy ideal theory of type $(\in,\in\vee q_{\tilde{k}})$ is also provided.

1.3 Statements of the Problem

The theory of fuzzy ordered semigroups and fuzzy ideal theory in ordered semigroups has been recently developed [12-18, 37, 39-55], that played a significant role in the study of ordered semigroups. However, the available literature shows that a less attention has been given to the study of ordered semigroups in terms interval-valued fuzzy set theory (involving [37, 39]). Thus our problem statements are given as follows:

How to define more generalised form of interval-valued fuzzy ideal theory based on "$\tilde{k}$-quasi-coincident with" relation of an interval-valued fuzzy point with an interval-valued fuzzy subset. In other words, how to introduce interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left and right ideals and characterise ordered semigroups by the properties of these notions? In addition, how to link ordinary fuzzy ideals and generalised interval-valued fuzzy ideals? In what way can we initiate interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideals and classify ordered semigroups by the properties of this newly defined notion? How to give the concept of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal in ordered semigroup. Further, under what condition an interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal will be an ordinary interval-valued fuzzy bi-ideal? How to define the notions of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left filter and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy right filter. Moreover, how to establish relationship between ordinary interval-valued fuzzy filters and generalised interval-valued fuzzy filters?

1.4 Research Objectives

The objectives of this study are given as follows:

(i) To provide the concept of interval-valued fuzzy $(\in, \in \vee q_{\tilde{k}})$-fuzzy left and interval-valued fuzzy $(\in, \in \vee q_{\tilde{k}})$-fuzzy right ideals. Thereafter, to investigate several characterisations of ordered semigroups in terms of these concepts.

(ii) To consider ordered semigroups and define the notion of an interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideal. In addition, to characterise interval-valued fuzzy generalised bi-ideals and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideals by using implication operators and the notion of implication-based an interval-valued fuzzy generalised bi-ideals.

(iii) To give a more general form of interval-valued fuzzy bi-ideal than interval-valued $(\in, \in \vee q)$-fuzzy bi-ideal, called interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal. Further, to provide several characterisation results of ordered semigroups by the properties of this new concept.

(iv) To introduce the notions interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left filter and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy right filter in the structure of ordered semigroups and to link these notions with ordinary left and right filters, ordinary interval-valued fuzzy left and right filters and $(\in, \in \vee q_{\tilde{k}})$-implication based interval-valued fuzzy left filter and right filters by using implication operators and implication-based interval-valued fuzzy left and interval-valued fuzzy right filters.

1.5 Scope of the Study

This study focuses on ordered semigroups. An ordered semigroup is a set S with a partial ordered relation "$\leq$" and an associative multiplication "." which is compatible with the ordering. The present work is devoted to scrutinise in terms of interval-valued fuzzy left and right ideals, interval-valued fuzzy generalised bi-ideal, interval-valued fuzzy bi-ideal of type $(\in, \in \vee q_{\tilde{k}})$ and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left and right filters of ordered semigroups.

1.6 Significance of the Study

Ordered semigroups and semigroups personate a major role in the field of mathematics with a large number of applications in different areas like coding theory, automata theory and sequential machine [56-58], language theory and formal grammar [59, 60], and neural networks [61]. The significance of ordered semigroup theory is therefore evident from its vast applications in several disciplines as listed above. However, according to the available literature, ordered semigroups and their characterisations in terms of interval-valued fuzzy set theory have hardly been studied. Therefore, proper attention is required to fill this gap in a systematic way.

In this study, the notion of interval-valued fuzzy ideal theory is introduced and some characterisation results of ordered semigroups are provided in terms of interval-valued fuzzy ideal theory. These contributed results will bestow new theories and valuable contribution in mathematics in general and particular to the theory of ordered semigroups.

This research will provide motivation to the researchers in the fields of mathematics and other sciences as a long term benefit. The new results obtained from this research will establish a platform for further research of ordered semigroups and their applications in different branches of algebra.

1.7 Research Methodology

Based on the problem statements and research objectives, the methodology for the present study is given in detail in the following lines.

The idea of quasi-coincidence of a fuzzy point with a fuzzy set given by Bhakat and Das [2] motivated the researcher to conduct this study. The importance of the concept given in [2] is evident from the research conducted in the past two decades where different types of fuzzy subgroups were introduced by using this idea.

An interval-valued fuzzy subset $A = \{(x, \tilde{f}_A(x)) : x \in S\}$ of an ordered semigroup S of the form

$$\tilde{f}_A(y) := \begin{cases} \tilde{t}\,(\neq [0,0]), & \text{if} \quad y = x, \\ [0,0], & \text{if} \quad y \neq x, \end{cases}$$

is called an interval-valued fuzzy point with support x and the value $\tilde{t}$ (where $\tilde{t}$ is a closed sub-interval of $(0,1])$ and is denoted by $x_{\tilde{t}}$ [39]. In other words, an interval-valued fuzzy subset A of S taking value $[0,0]$ at every point of S except at single point is known as an interval-valued fuzzy point. It is clear that every interval-valued fuzzy set can be expressed as the union of all the interval-valued fuzzy points which belong to A. The interval-valued fuzzy point $x_{\tilde{t}}$ is said to "belongs to A", denoted by $x_{\tilde{t}} \in A$ if and only if $\tilde{f}_A(x) \geq \tilde{t}$. On the other hand if $\tilde{f}_A(x) < \tilde{t}$, then $x_{\tilde{t}}$ is said to "does not belong to A" and is denoted by $x_{\tilde{t}} \overline{\in} A$. Likewise, the interval-valued fuzzy point $x_{\tilde{t}}$ is said to be "quasi-coincident with A", denoted by $x_{\tilde{t}} qA$ if and only if $\tilde{f}_A(x) + \tilde{t} > [1,1]$. It is important to note that the interval-valued $(\in, \in \vee q)$-fuzzy theory completely depends on belongs to $(\in)$ or quasi-coincident with (q) relation between an interval-valued fuzzy point and an interval-valued fuzzy subset A. Here after, $\tilde{0}$ will be used to represent the interval $[0,0]$ and $\tilde{1}$ for the interval $[1,1]$. Some authors also used $\overline{0}$ and $\overline{1}$ to represent the intervals $[0,0]$ and $[1,1]$, respectively.

The following lines explain the generalisation of the concept of "quasi-coincident with" relation of an interval-valued fuzzy point $x_{\tilde{t}}$ with an interval-valued fuzzy set A.

Let us take an arbitrary element $\tilde{0} \leq \tilde{k} = [k^-, k^+] < \tilde{1}$. If $\tilde{f}_A(x) + \tilde{t} + \tilde{k} > \tilde{1}$, then we say that $x_{\tilde{t}}$ is $\tilde{k}$-quasi-coincident with A. As $\tilde{0} \leq \tilde{k} < \tilde{1}$ therefore, $\tilde{0} - \tilde{k} \leq \tilde{k} - \tilde{k} < \tilde{1} - \tilde{k}$ this implies $-\tilde{k} \leq \tilde{0} < \tilde{1} - \tilde{k}$, in which it follows that

$$-\tilde{k} \leq \tilde{0} \text{ and } \tilde{0} < \tilde{1} - \tilde{k},$$

therefore

$$\tilde{1} - \tilde{k} \leq \tilde{1} + \tilde{0} \text{ and } \tilde{0} < \tilde{1} - \tilde{k},$$

in which it follows that

$$\tilde{0} < \tilde{1} - \tilde{k} \leq \tilde{1}.$$

This implies

$$\frac{1}{2}[0,0] < \frac{1}{2}[1-k^{+}, 1-k^{-}] \leq \frac{1}{2}[1,1],$$

that is

$$[0,0] < \left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right] \leq \left[\frac{1}{2}, \frac{1}{2}\right],$$

in which it follows that $\left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right] \in D(0, \frac{1}{2}]$. Since, $\tilde{t} \in D(0,1]$, therefore $\tilde{t} > \left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right]$ or $\tilde{t} \leq \left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right]$.

Consequently, in this study it is emphasised that the interval-valued fuzzy sets throughout this thesis must satisfy the following:

(1) Any two elements of the set $D[0,1]$ are comparable.

(2) $\tilde{f}_A(x) > \left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right]$ or $\tilde{f}_A(x) \leq \left[\frac{1-k^{+}}{2}, \frac{1-k^{-}}{2}\right]$.

If the above conditions are omitted, then all of our main results may not be true.

The importance and usefulness of the ideas of Bhakat & Das [2, 3] and Jun [4] is evident from the research conducted on the basis of these concepts in the past two decades (see [19, 29-36, 38, 39, 62, 63]). Yet, while looking at the available literature it is observed that ordered semigroups by the properties of interval-valued fuzzy set theory have hardly been investigated. Eventually, the afore mentioned ideas of Bhakat & Das [2, 3] and Jun [4] are the bases of motivation to further investigate ordered semigroups in terms of interval-valued fuzzy concept. In this regards, these ideas are further extended to ordered semigroups to introduce more generalised forms fuzzy ideals, fuzzy generalised bi-ideal, fuzzy bi-ideal and fuzzy filters.

This study has been carried out in two main parts, the first part includes interval-valued fuzzy ideals, interval-valued fuzzy generalised bi-ideals and interval-valued fuzzy bi-ideals of type $(\in, \in \vee q_{\tilde{k}})$, while interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left filters are introduced in the second part. Further, ordered semigroups are classified by the properties of these newly defined notions.

This study starts with the introduction of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left and right ideals in ordered semigroups. Furthermore, defined notions are supported by constructing suitable examples. In addition, some characterisation results of ordered semigroups by the properties of interval-valued fuzzy ideals are established. Moreover, the relationships between interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left and right ideals and $[\frac{1-k^+}{2}, \frac{1-k^-}{2}]$-implication based interval-valued fuzzy ideals are established by using implication operators.

Furthermore, the notion of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideal in ordered semigroups is presented by using "$\tilde{k}$-quasi-coincident with" relation ($q_{\tilde{k}}$) between an interval-valued fuzzy point and an interval-valued fuzzy set. In addition, a condition for an interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideal to be an ordinary interval-valued fuzzy generalised bi-ideal is provided.

Next, based on the aforementioned idea of Jun [4], interval-valued fuzzy bi-ideal is further extended to interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal in the structure of ordered semigroup and some fundamental results are determined by the properties of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal. Also, ordinary bi-ideals are connected with $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideals using level subset.

In the second part of this research, the notion of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy filters in ordered semigroups is defined. Moreover, certain characterisations are discussed in terms of these notions. Moreover, $[\frac{1-k^+}{2}, \frac{1-k^-}{2}]$-implication based interval-valued fuzzy filters are linked with interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy filters by using implication operators.

Further, using implication operators and $[\frac{1-k^+}{2},\frac{1-k^-}{2}]$-implication based interval-valued fuzzy left ideal, interval-valued fuzzy right ideal, interval-valued fuzzy generalised bi-ideal and interval-valued fuzzy bi-ideal and $[\frac{1-k^+}{2},\frac{1-k^-}{2}]$-implication based interval-valued fuzzy left and $[\frac{1-k^+}{2},\frac{1-k^-}{2}]$-implication based interval-valued fuzzy right filters are linked with these defined notions. It is essential to highlight that this study gives another direction for further research in different algebraic structures and especially in the structure of ordered semigroup. In this regards, in Chapter 7, it is discussed in detail as suggestions for future work that how further research will take place. Figure 1.1 highlights the research framework of this study.

1.8 Thesis Outlines

This thesis is structured into seven chapters. Chapter 1 provides an introduction to the work on interval-valued fuzzy ideals theory in ordered semigroups. This chapter contains the research background which states a brief introduction followed by the statement of the problem, research objectives, scope and significance of this research and the methodology.

A detailed literature of several types of fuzzy ideals of ordered semigroups and their characterisations is illustrated in Chapter 2. In addition, some fundamental concepts and results that are used in this study are included in this chapter.

Chapter 3 illustrates the notions of interval-valued fuzzy ideals of type $(\in,\in\vee q_{\tilde{k}})$ of ordered semigroups. Some examples are constructed to support these notions. A condition is imposed on an interval-valued fuzzy subset that leads an interval-valued $(\in,\in\vee q_{\tilde{k}})$-fuzzy ideal to be an ordinary interval-valued fuzzy ideal. In addition, the notions of implication-based interval-valued fuzzy ideals are also given. Further, ordered semigroup is characterised by the properties of these concepts.

The notions of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideal and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal are defined in Chapter 4 and supported by suitable examples as well. Moreover, these concepts are linked with ordinary interval-valued fuzzy generalised bi-ideal and interval-valued fuzzy bi-ideal. In addition, by means of level subsets it is shown that these defined concepts of interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy generalised bi-ideal and interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal are equivalent to ordinary generalised bi-ideal and bi-ideal respectively. Finally, the notion of $\tilde{t}$-implication based interval-valued fuzzy bi-ideal is introduced and relationship between interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy bi-ideal and $\tilde{t}$-implication based interval-valued fuzzy bi-ideal is established.

The more generalised form of the idea of interval-valued fuzzy filters is discussed in Chapter 5 in detail. Moreover, several characterisations of ordered semigroups in terms of these notions are established. Also, it is shown by suitable examples that there are interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy filters which are not ordinary interval-valued fuzzy filters. In this respect, a condition for interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy filter to be an ordinary interval-valued fuzzy filter is imposed on an interval-valued fuzzy subset. Lastly, based on implication operators, $[\frac{1-k^-}{2}, \frac{1-k^+}{2}]$-implication based interval-valued fuzzy filters are linked with interval-valued $(\in, \in \vee q_{\tilde{k}})$-fuzzy left filters.

This thesis is concluded in Chapter 6 with the summary of the research as well as some recommendations for future research. Figure 1.2 shows the layout of this thesis.

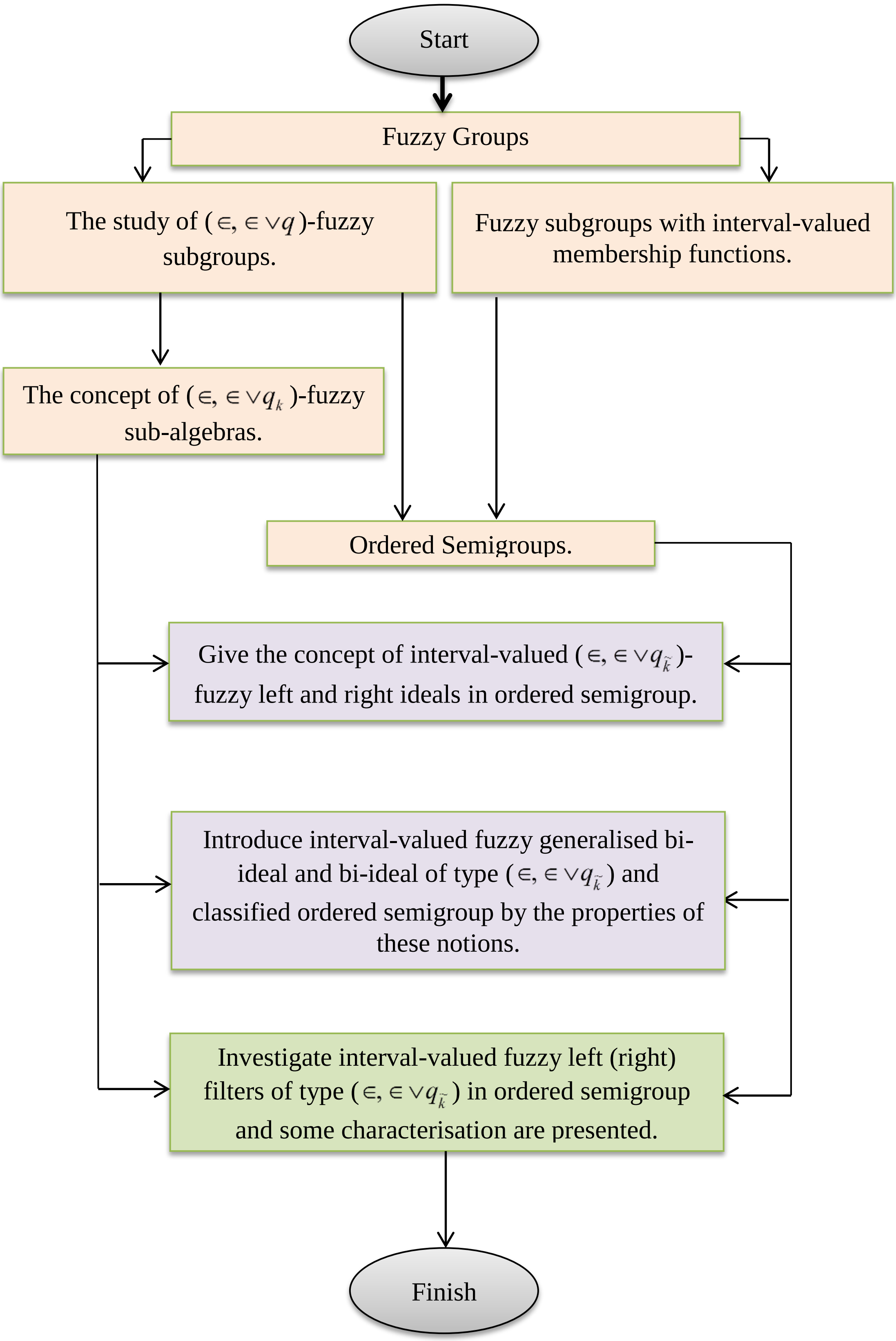

Figure 1.1: Research Framework

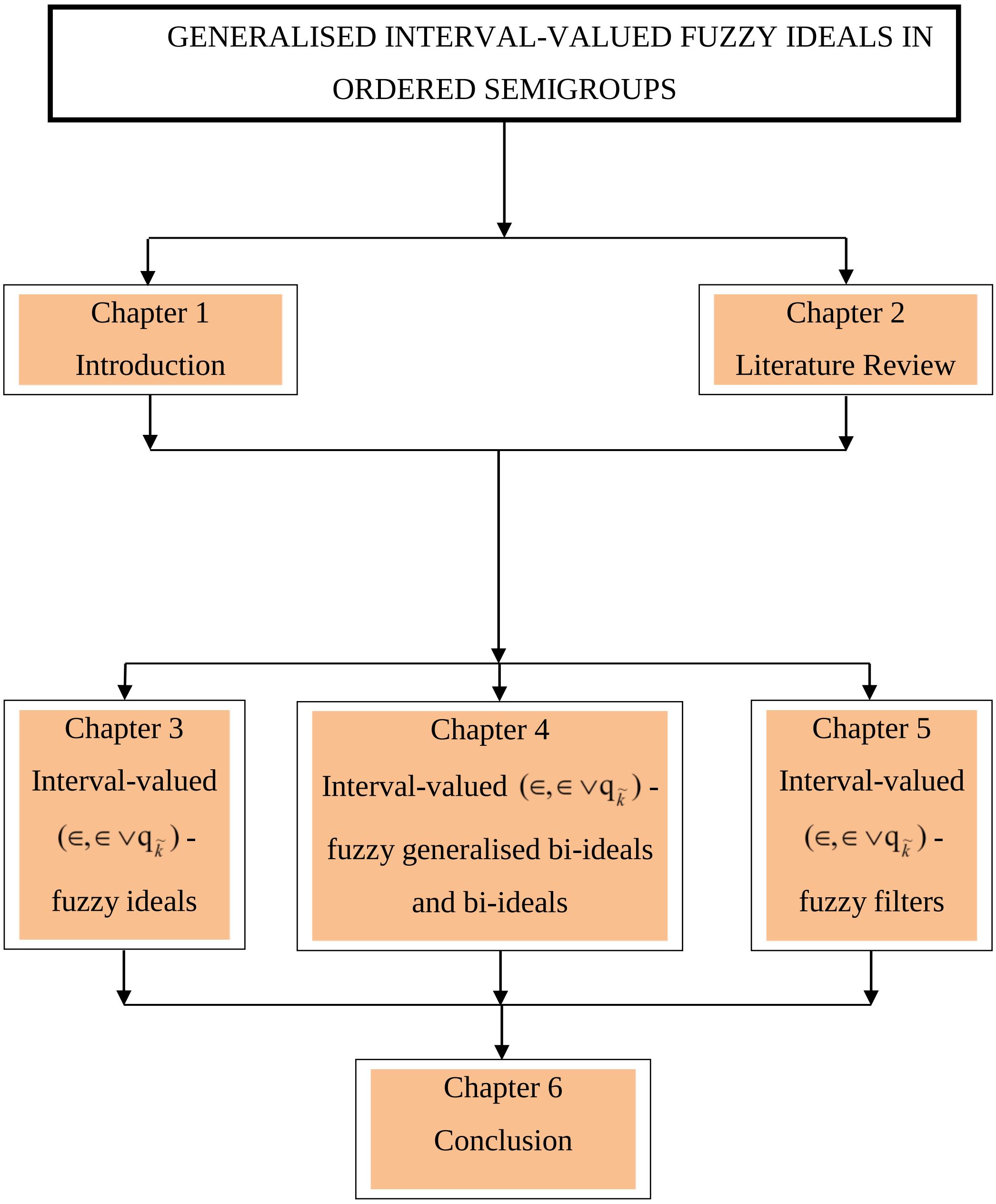

Figure 1.2: Thesis Outlines

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