A RELATION BETWEEN *TUDUNG SAJI* WEAVING PATTERNS AND GROUP THEORY

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To my dearest,

Papa (Zamri bin Abdul Rahman)

Mama (Sakinah bt Harun)

My Late Mama (Siti Maimunah bt Abdul Halim)

Nenek (Zainab bt Ismail)

My Late Atuk (Abdul Rahman bin Jusoh)

Lovely supervisor (Prof Dr Nor Haniza Sarmin)

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Examiners

Families

Friends

Supporters

Sponsors

You Know Who

ALLAH SWT KNOWS BETTER

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ABSTRACT

Tudung saji is a traditional utensil used by the Malays to cover their food to be served. *Tudung saji* is woven with strands of dried leaves using a specific technique called triaxial weave, where the strands are plaited in three directions. Previously, a tool known as triaxial template had been created to represent the patterns of *tudung saji* weaving in a planar pattern. Based on this template, many beautiful and symmetrical patterns were successfully generated, creating some of the original patterns of *tudung saji*. These patterns are categorized according to the number of colours of the strands, from the basic 2-strand up to 6-strand template. The purpose of study is to find several finite groups to represent the triaxial weaving patterns on two dimensional templates, focussing only on the 2 and 3-strand templates. It is found that the symmetric group of two letters, S_2 and the cyclic group of order six, C_6 are isomorphic to the triaxial template of Flock of Pigeons and Sailboats patterns, respectively. These isomorphisms are determined by mapping the elements of the Flock of Pigeons and Sailboats onto the elements of the two groups. Using a software iMac Grapher, several graphs are generated based on the elements of the triaxial template patterns. Next, graph theory is used to analyze the properties of these graphs. The graphs are sorted by the numbers of strands, namely the graphs of block two, graphs of block three up to the graphs of block six. All such graphs are found to feature the characteristics of three types of graphs, namely a complete graph with three vertices, K_3, a simple graph with six vertices, and an acyclic graph. Lastly, this research reports on modifications to the template by adding extra colours to the framework strands and the insertion strands. This is done by using new colour ordering in addition to the same colour ordering for the 2-strand template. As a result, a new characteristic on the modified template has been found, namely the existence of different triaxial patterns in one template.

ABSTRAK

Tudung saji adalah sejenis alat tradisi yang digunakan oleh masyarakat Melayu untuk menutup makanan hidangan mereka. Tudung saji dianyam dengan helaian daun kering menggunakan satu teknik anyaman tiga paksi, dengan untaiannya dijalin dalam tiga arah. Sebelum ini, suatu alat yang dipanggil templat tiga paksi telah dihasilkan untuk membentuk corak anyaman tudung saji berpola satah. Berdasarkan templat tiga paksi ini, terdapat banyak corak cantik dan mempunyai simetri berjaya dihasilkan termasuk corak asal tudung saji. Corak-corak ini dikategorikan mengikut jumlah warna yang dimasukkan ke dalam setiap untaian, iaitu templat 2-untai sehingga templat 6-untai. Kajian ini bertujuan mencari beberapa kumpulan terhingga untuk mewakili corak anyaman tiga paksi dalam templat dua dimensi, dengan tumpuan kepada templat 2-untai dan 3-untai. Hasil kajian mendapati bahawa kumpulan simetri dua huruf, S_2 dan kumpulan kitaran peringkat enam, C_6 adalah isomorfik kepada templat tiga paksi yang bercorak Pati Sekawan dan Kapal Layar. Isomorfisma ini telah ditentukan melalui pemetaan di antara unsur Pati Sekawan dan Kapal Layar kepada unsur-unsur dua kumpulan tersebut. Dengan menggunakan perisian *iMac Grapher*, beberapa graf telah dihasilkan berdasarkan unsur templat tiga paksi ini. Seterusnya, teori graf telah digunakan untuk menghuraikan ciri-ciri yang terdapat pada graf yang terhasil daripada unsur corak templat tiga paksi. Graf-graf ini disusun mengikut jumlah untaian, yang dinamakan sebagai graf blok dua, graf blok tiga sehinggalah graf blok enam. Kajian menunjukkan semua graf ini mempunyai ciri tiga jenis graf, iaitu graf lengkap tiga bucu, K_3, graf ringkas enam bucu, dan graf *acyclic*. Akhir sekali, kajian ini melaporkan beberapa pengubahsuaian yang dilakukan terhadap templat dengan memasukkan warna tambahan untuk menggambarkan untaian rangka dan juga untaian sisipan. Kaedah ini dilaksanakan dengan menggunakan susunan warna baru dan juga susunan warna yang sama dengan templat 2-untai. Hasilnya, satu ciri baru telah dijumpai pada templat yang diubahsuai iaitu kewujudan corak-corak yang berbeza pada satu templat yang sama.

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LIST OF SYMBOLS

$\mathbb{R}^n$	-	n-dimensional space
ϕ	-	phi
G	-	group
$\cong$	-	isomorphic
D_4	-	dihedral group of order eight
S_4	-	symmetric group on four symbols
$V(G)$	-	set of vertices
$E(G)$	-	set of edges
ψ_G	-	incidence function
$\subseteq$	-	subset of
θ	-	theta

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CHAPTER 1

INTRODUCTION

1.1 Introduction

All things that happen in everyday life can be connected with mathematics. In other words, mathematics is a wide field of knowledge that tests human logic through problem solving skills. It can also be expressed as a basis for many other areas of knowledge in the world. In addition, a mathematical study is a continuous process where new theorems and definitions can be found along with research. This research reveals the connection of mathematics with the practice of *tudung saji* weaving, an ancient artwork of the Malay people.

Art is an expression. It is beautifully expressed through various mediums to attract people to it. Hence, people’s life are surrounded by many forms of art without them realizing it. There are some examples of art that are considered attractives, namely wall paintings, space decorations, murals and framed paintings. Besides, artistic expressions can be found in crafts, paintings, weavings and ornaments, where their beauty is exhibited through their geometrical elements.

A field of study that seeks the connection between mathematics and the arts is known as ethnomathematics. Alternatively, ethnomathematics is a research domain that highlights the relationship between mathematics and cultural arts. This field is relevant because people have long been interested in the mathematical aspects of the arts, and many studies have been conducted to investigate the connection.

This research analyzes the artistic properties of a craft named *tudung saji* which can be represented through mathematical elements in group theory. Hence, the symmetrical operations of *tudung saji* weaving are observed where the properties of its triaxial template are analyzed and connected to group theory and graph theory. This triaxial template is created by Adam [1], which is used to present some of the patterns on the surface of *tudung saji* onto a planar template. *Tudung saji* is originally a traditional food cover which is mostly used by the Malays to cover their meals. Nowadays, people use food covers as home decoration. These food covers are produced in certain states in Malaysia such as Kelantan, Terengganu, Melaka, Sabah, Sarawak and Negeri Sembilan. However, this research focuses only on the food covers that are created in Melaka and Terengganu.

A food cover is woven by using a specific technique called triaxial or hexagonal weave, where the strands are plaited in three directions [1]. Some examples of patterns that can be seen in the *tudung saji* weaving are Flock of Pigeons (Pati Sekawan), Cape Flower (Bunga Tanjung), Sailboats (Kapal Layar), Five States (Lima Buah Negeri) and Cookie Flower (Bunga Biskut). Figure 1.1 shows several patterns of tudung saji weaving created in Melaka and Terengganu [1].

(a) (b) (c) (d)

Figure 1.1 Patterns of *tudung saji* weaving a) Cape Flower, b) Bald Head, (c) Standing, d) Five States

1.2 Research Background

This research starts with a detail review on the ethnomathematical study of *tudung saji* weaving, where the combination of mathematics and arts is beautifully expressed throughout the study [1]. Adam has also analyzed the techniques and patterns of *tudung saji* weaving in Melaka and Terengganu. From her research, she has created a planar template, namely triaxial template based on the patterns that appeared at the surface of *tudung saji* weaving. In addition, Adam has also created a labelling system to define the elements for each template, according to their colours inserted and also the number of strands. Therefore, in this study, the properties of group theory are used to map the elements of the template to the elements of groups.

Furthermore, Adam has also developed several graphs based on the elements of the trixial templates by using a software named iMac Graph. In this research, the characteristics of these graphs are also analyzed by using the properties of graph theory. In addition, this research also extends the study on the current triaxial template, where some modifications are done based on the techniques of *tudung saji* weaving. However, these modifications do not lead into the production of three dimensional triaxial template.

There are several studies by previous researchers that highlight the connection between mathematics and the arts. One of the popular examples is the discovery of the Golden Section (ratio or proportion) called phi, φ [2]. This ratio represents a fundamental structural element of the physical universe. Other examples that feature mathematics in arts are the study of modular design system [3], the discovery of patterns in mathematics and poetry [4], the calculus of gothic architecture [5] and analysis of a multi-layered geometric pattern from a mosque in Yazd [6].

Since this research focuses mainly on group theory and graph theory, there are several examples of their applications in other study. One of the crucial applications of group theory is in the study of crystallography. Other than that, group theory can also be applied in the study of symmetrical and geometrical patterns of tiles and ornaments. Meanwhile, graph theory can be applied into various real life problems such as networking.

1.3 Problem Statement

Adam [1] conducted a research on *tudung saji* weaving and created a triaxial template to represent the planar patterns of *tudung saji*. However, the author did not know how to relate the elements of the templates with groups. Therefore, this research is done to answer the question on how to model the patterns of triaxial template of *tudung saji* weaving by using group theory.

Besides that, Adam [1] has also generated several graphs based on the elements of the triaxial template by using an iMac Grapher software. However, the author did not explain the meaning of the graphs created. Therefore, this research is done to find out how these existing graphs can be related to graph theory.

1.4 Research Objectives

The objectives of this research are:

i. to determine suitable finite groups that can model triaxial weaving patterns on a two dimensional template,
ii. to analyze the connections between graphs of triaxial template with graph theory,
iii. to modify the current triaxial template based on the techniques of *tudung saji* weaving.

1.5 Scope of the Study

This research focuses on four patterns of *tudung saji* weaving created commonly in Terengganu and Melaka, namely Flock of Pigeons, Sailboats, Buttons and Cape Flower. This research also focuses only on finite groups. In addition, the analysis is done only on two dimensional triaxial template comprising of 2-strand and 3-strand templates.

1.6 Significance of the Study

By relating the *tudung saji* weaving patterns and group theory, this research is aimed to provide a new representation of *tudung saji* weaving by using the elements of groups. Hence, this research has leads to a new application of group theory in real life, which is the *tudung saji*. This research has also contributes to several modified templates of *tudung saji* which is useful to create new patterns of *tudung saji*.

1.7 Research Framework

This research starts with the study on the basic concepts of three main topics which are *tudung saji* weaving, group theory and also graph theory. Based on the basic concepts of *tudung saji* weaving, the basic materials, the techniques, process and the patterns are easily understood.

Next, the triaxial template which is created by Adam [1] is studied. There are two things that became the concern for this research. The first one is the patterns and the elements that are existed from this template. The research is conducted by first analyzing the patterns, and then finding suitable finite groups that hold the properties of symmetrical operations. After that, the elements of finite groups are mapped with the elements of triaxial template.

The second part concerned the graphs that are developed based on the elements of triaxial template. These graphs are illustrated by using some of the properties in graph theory.

Lastly, this research suggested a few modifications for the current triaxial template. These modifications are completed by referring on the techniques and process of *tudung saji* weaving, where some framework strands and insertion strands are added on this template in order to see the difference. However, these modifications do not lead into the production of three dimensional triaxial template.

The flowchart of research framework is illustrated in Figure 1.2.

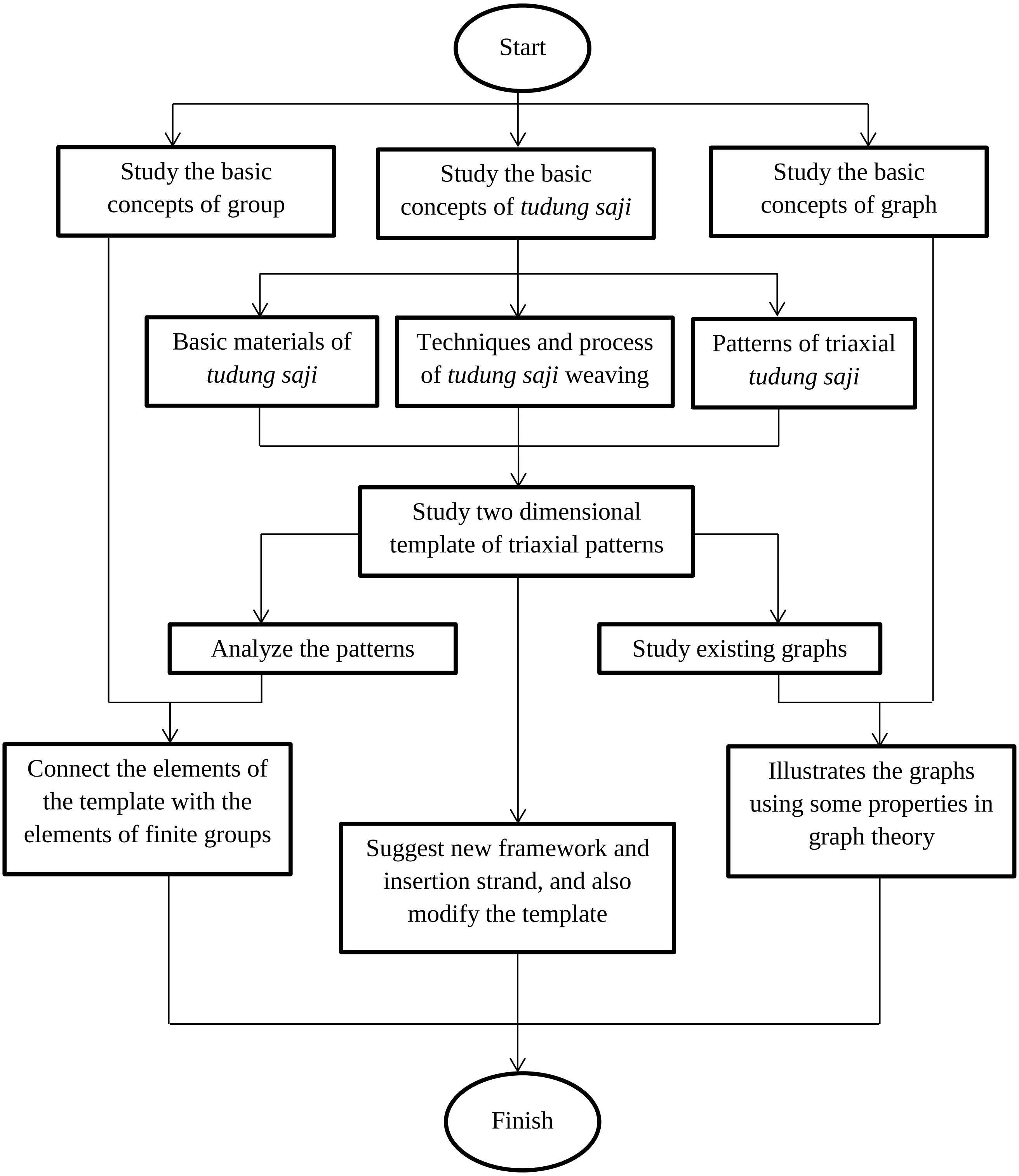

Figure 1.2 Research framework

1.8 Thesis Organization

The first chapter begins with an introduction which describes the whole thesis. This chapter defines the basic concepts in the study of mathematics and arts which are shown in the practice of *tudung saji* weaving. Chapter 1 also includes the research background, problem statement, research objectives, scope, significance of the study and research methodology.

Chapter 2 elucidates the literature review of this research. It starts with the basic concepts of *tudung saji* weaving including the material used, the weaving techniques and the patterns created. Other than that, there are also some definitions and terminologies in group theory and graph theory which are used in this research. In addition, previous studies that are related to groups and graphs are also stated in this chapter.

Chapter 3 reveals the relations between triaxial template patterns with some elements in group theory. This chapter also clarifies the properties of two dimensional triaxial template patterns. Also, this chapter highlights some embedment of elements in groups into the symmetrical elements of triaxial patterns.

Chapter 4 introduces some graphs that have been produced based on the triaxial template patterns. These graphs are produced by using a software called iMac Grapher, where the number of strands used is between two and six. The graphs are analyzed by using the properties and characteristics in graph theory. The results from the analysis are given in this chapter.

Chapter 5 presents some approaches for the modifications of the triaxial template. In this chapter, the properties of the current template and its weaknesses are discussed. Some attempts are made to match the template with the original *tudung saji* weaving. However, the modifications are done only by referring on the techniques and process of *tudung saji* weaving, which means that the properties of groups and graphs are not being used during the modification process.

Finally, Chapter 6 gives a summary of the whole research. Some suggestions for future research which are related with the triaxial patterns of *tudung saji* weaving are also included in this chapter.

The flowchart of the thesis organization is illustrated in Figure 1.3.

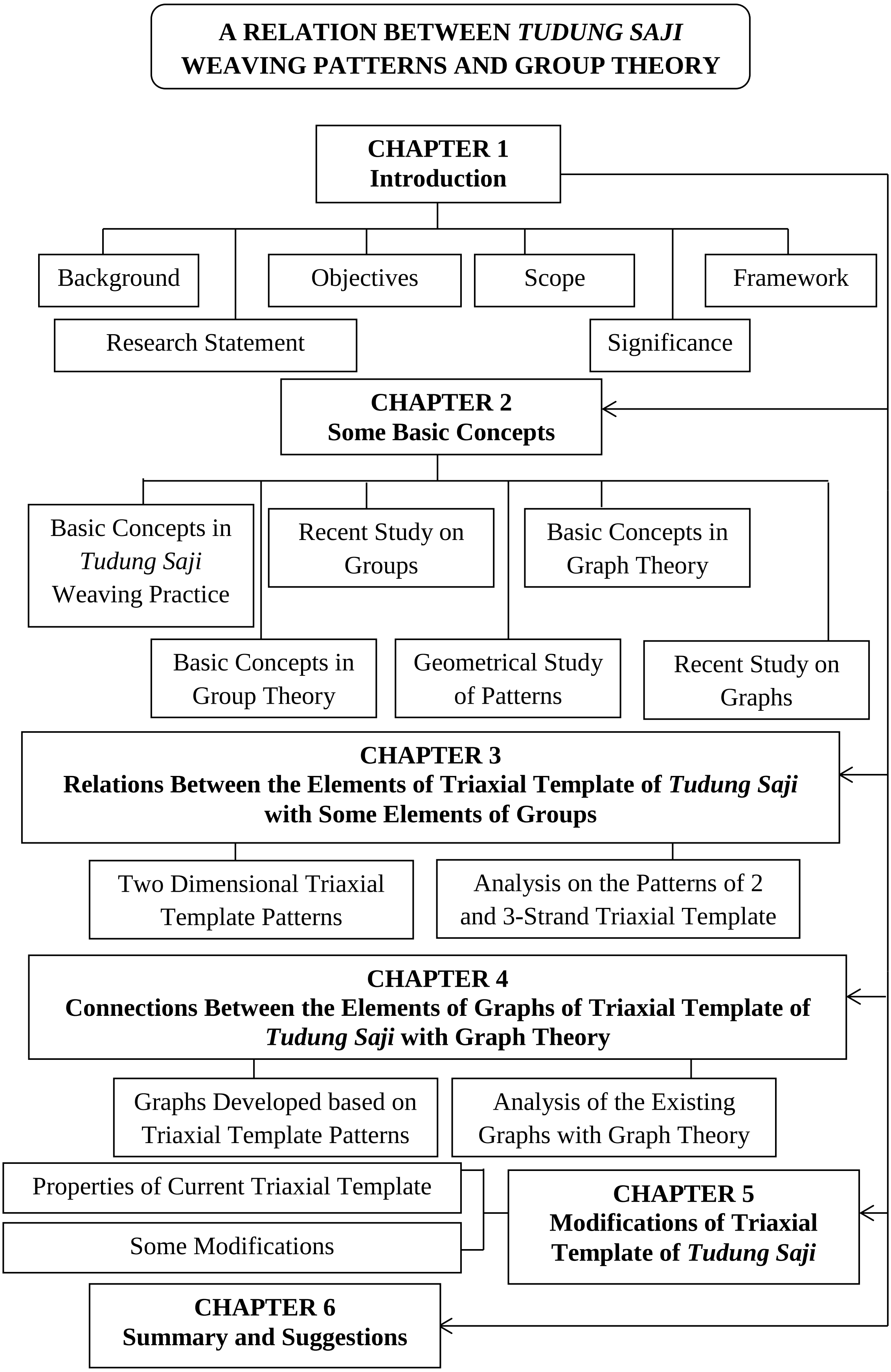

Figure 1.3 Thesis organization

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