

# The Strengths and Limitations of Risk Measures in Real Estate: A Review

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## Abstract

The purpose of this paper is to provide a brief overview of the concept of risk measures that used in real estate with a particular emphasis on the merits and drawbacks of these risk measures. The finding reveals that each risk measure has its distinctions and limitations. The finding has a far-reaching implication to investors, particularly, institutional investors, which they should be conscious with the advantages and disadvantages of these risk measures in order to determine the most apt risk measure in formulating desirable risk investment strategy.

*Keywords:* Risk Measures, Variance and Investment Risk

## 1. Introduction

Nowadays, variance is the most popular risk measure, which has been employed widely in many real estate and finance studies; and it is also the most popular risk measure for investors (Evans, 2004). Interestingly, variance is not the only risk measure, while there are several fruitful alternatives that can be used as risk measure in current real estate literature.

These alternative risk measures are Lower Partial Moment (downside risk) (LPM), Mean Absolute Deviation (MAD), Minimax and Maximum Drawdown (MaxDD). However, these risk measures have not received overwhelming response as variance in finance and real estate literature. Does variance is the best risk measure? If not, which risk measure is the 'best' risk measure?

Unfortunately, the determination of the best risk measure that offer the best return and risk trade-off might be a vain exercise unless a common risk measure is identified (Cheng and Wolverton, 2001; Byrne and Lee, 2004). The difficulty for determining it is also demonstrated by the Cheng and Wolverton (2001) and Cheng (2001) in which they reveal that downside risk and variance are

not directly comparable. As such, it is implausible to conclude which risk measure is superior than others.

Additionally, Biglova et al., (2004) and Byrne and Lee (2004) also reveal empirical evidences that different risk measures provide different portfolio allocation and different performance result for an asset. These findings have a practical implication to investors that the choice of the risk measure depends on the investors' risk attitudes and investment objectives.

Therefore, understanding the concept, distinctions and drawbacks of each different risk measure is inevitable for investors. In this way, they can select the most appropriate risk measure according to their investment objectives and attitudes toward risk.

The purpose of this paper is to provide a brief overview on the concept of risk measures that employed in real estate and determine the advantages and disadvantages of these measures. In Section 2, the concept of available risk measures, the strengths and weaknesses of these risk measures are discussed. The choice of risk measures is discussed in Section 3 in which it provides a review for

investors on the appropriate risk measures in different circumstances. Section 4 concludes and provides the future research direction.

## 2. Risk Measures

Many studies have proposed alternative risk measures in line with the motivation for overcoming the limitations of variance. At least four alternative risk measures, namely Lower Partial Moment (LPM), Mean Absolute Deviation (MAD), Minimax, and Maximum Drawdown (MaxDD) are found in real estate literature.

### 2.1 Variance

Since the introduction of Mean Variance (MV) Analysis by Markowitz (1952), Variance (or Standard deviation) is the most common risk measure that is used by practitioners and researchers (Evans, 2004). Variance is measured by the dispersion of a return distribution around the mean or average. While, standard deviation is the square root of the variance. It is defined as follow:

$$\sigma^2 = \frac{1}{T} \sum_{i=1}^T [R_i - \bar{R}]^2 \quad (1)$$

where  $R_i$  is the return for asset  $i$ ,  $\bar{R}$  is the average (mean) of the returns and  $T$  is number of returns. Notably, most of the studies on finance and real estate are dealing with the sample rather than population, hence, the sum of the square deviations should be divided by  $T - 1$  rather than (Strong, 2003).

It should be noted that MV model has several strict assumptions such as asset return distribution must be normally distributed and all investors have a constant quadratic utility function<sup>2</sup>. However, substantial studies have demonstrated that returns from real estate are not necessarily normally distributed (Myer and Webb, 1993, 1994 for U.S. commercial real estate and Real Estate Investment Trusts; Graff et al., 1997, Lee, 2006 and Peng, 2005 for Australian commercial real estate and Listed Property Trusts; Maitland-Smith and Brooks, 1999 for commercial real estate in U.K.; Lee et al., 2006 for Malaysian property shares). Besides, Fishburn (1977), Harrington (1987), Nawrocki (1999) and Sharpe and Alexander (1990) also argue that no compelling reason for assuming all investors have a static quadratic utility function.

<sup>1</sup> A normal/symmetrical distribution assumption allows MV model can be completely described by mean and variance (the first two central moments) (Brown and Matysiak, 2000).

<sup>2</sup> Quadratic utility function assumes all investors are risk averse.

### 2.2 Lower Partial Moment

Lower Partial Moment (LPM) is also known as downside risk. It relies upon safety first rule, which is developed by Roy (1952). It measures only the likelihood of bad outcomes that is the likelihood of return below the target return for an investment.<sup>4</sup> It must be noted that semi-variance is a special case of the LPM in which the  $\alpha$  value is equal to 2 (Harlow, 1991). According to Bawa (1975) and Fishburn (1977), n-degree Lower Partial Moment can be written as:

$$LPM_{\alpha}(\tau, R_i) = \int_{-\infty}^{\tau} (\tau - R)^{\alpha} dF(R)$$

$$= \frac{1}{T-1} \sum_{i=1}^T [Max(0, (\tau - R_{it}))]^{\alpha} \quad (2)$$

where  $dF(R)$  is the cumulative distribution function of the investment return  $R$ ,  $\tau$  is the target return,  $\alpha$  is the degree of the LPM,  $R_{it}$  is the return for asset at time  $t$  and  $T$  is number of returns.

While, there is another LPM measure that is generalised or asymmetric co-LPM (CLPM). It is proposed by Hogan and Warren (1974) and Bawa and Linderberg (1977). The measure for CLPM is defined as:

$$CLPM_{\alpha}(\tau, R_i, R_j) = \frac{1}{T-1} \sum_{i=1}^T [Max(0, (\tau - R_{it}))]^{\alpha-1} (\tau - R_{jt}) \quad (3)$$

where  $\tau$  is the target rate of return,  $R_{it}$  and  $R_{jt}$  are the rate of return of the asset  $i$  and  $j$  at time  $t$ ,  $T$  is the total number of returns, and  $\alpha$  is the degree of CLPM.<sup>5</sup>

Symmetric co-LPM (SLPM), a simple algorithm for calculating downside risk is proposed by Nawrocki (1991). The SLPM is written as follows:

$$CLPM_{\alpha}(\tau, R_i, R_j) = CLPM_{\alpha}(\tau, R_j, R_i)$$

$$SLPM = [LPM_{\alpha}(\tau, R_i)]^{1/\alpha} [LPM_{\alpha}(\tau, R_j)]^{1/\alpha} (\rho_{i,j}) \quad (4)$$

<sup>3</sup> See also Pratt (1964); Arrow (1971); Wipern (1971) and Sarnat (1974) for the details on the limitations of quadratic utility function that is used to describe the actual behaviour of investors.

<sup>4</sup> Markowitz (1959) also recognises the importance of this argument and suggests using semi-variance.

<sup>5</sup> CLPM is an asymmetric measure and  $CLPM_{\alpha}(\tau, R_i, R_j)$  is not equal to  $CLPM_{\alpha}(\tau, R_j, R_i)$ . See also Nawrocki (1999) and Lee et al. (2006) for details.

where  $\rho_{ij}$  is the correlation coefficient between the return of asset  $i$  and  $j$ ,  $\tau$  is the target rate of return,  $R_i$  and  $R_j$  are the return of the asset  $i$  and  $j$ ,  $T$  is the total number of returns, and  $\alpha$  is the degree of SLPM.

The advantages of the LPM have been demonstrated in many studies. In general, LPM appears some theoretical superiorities such as no assumption on asset return distribution and liberates investors from the assumption of quadratic utility function (Lee et al., 2005). Byrne and Lee (2004), however, argue that LPM sensitive to observations that are distant from their target. Besides, Konno et al. (2002) show that it involves the prolonged computation time for large scale portfolio optimisation. Hamelink and Hosli (2004) also argue that it does not consider the issue of serial correlation of returns<sup>6</sup>.

### 2.3 Mean Absolute Deviation

Mean Absolute Deviation (MAD) is proposed by Konno (1989), which is used to overcome the limitations of Mean Variance (MV) model. The MAD can be computed from the following:

$$MAD = \frac{1}{T} \sum_{t=1}^T |R_i - E(R_i)| \quad (5)$$

Where  $R_i$  is the return asset  $i$ ,  $E(R_i)$  is the expected return or mean and  $T$  is the total number of returns.

The MAD measure has a number of attractive features such as bypass the covariance matrix computation and easier solving algorithm (portfolio optimisation). So it requires a shorter computation time and improves the computation of optimal portfolios. Moreover, MAD is more stable over time than variance which it is less sensitive to outliers and it does not require any assumption on the shape of a distribution. Interestingly, it retains all the positive features of the MV model. MAD is also apt to be used in situations when the number of assets ( $N$ ) is greater than the number of time periods ( $T$ ) (Konno & Yamazaki, 1991; Byrne and Lee, 1997, 2004; Brown and Matysiak, 2000; Konno, 2003).

However, the computation time is less significant nowadays due to the advancement of computer. Additionally, the use of MAD is precluded in line with the findings of Simaan (1997) in which the ignorance of the covariance matrix leads greater estimation risk that outweighs the benefits.

<sup>6</sup> Serial correlation (smoothing) is a flaw with real estate, particularly when estimating the returns from indices. See Blundell and Ward (1987); MacGregor and Nanthakumaran (1992); Geltner (1993); Newell and MacFarlane (1995, 1996) and Brown and Matysiak (1996, 2000) for the smoothing effect and the desmoothing techniques.

### 2.4 Minimax

Young (1998) proposes a new principle for portfolio selection rule using a simple linear programming solution, which is Minimax. The rule uses minimum return as a measure of risk rather than variance.

A Minimax portfolio is defined “as minimising the maximum loss, where loss is defined as negative gain, or, alternatively, maximizes the minimum gain.” (Young, 1998: 674). In other words, Minimax can be viewed as an extreme and special case of the Conditional Value at Risk as it represents the maximum loss over all past historical returns (Biglova et al., 2004). The solution for Minimax portfolio is as follows:

$$\max M_p \quad (6)$$

Subject to:

$$\sum_{i=1}^N w_i r_{it} - M_p \geq 0, t = 1, \dots, T.$$

$$\sum_{i=1}^N w_i \bar{R}_i \geq G$$

$$\sum_{i=1}^N w_i \leq W,$$

$$w_i \geq 0, i = 1, \dots, N.$$

where  $r_{it}$  is return of asset  $i$  in time period  $t$ ,  $\bar{R}_i$  is average return on asset  $i$ ,  $w_i$  is portfolio allocation to security  $i$ ,  $M_p$  is minimum return on portfolio, subject to the constraint that average return on portfolio exceeds some minimum level, say  $G$ , and that the sum of the portfolio allocations less than the total allocation, say  $W$ .

Such model has a number of superiorities by comparing with MV model such as it retains the positive features of MV model if the return distribution is normally distributed. Furthermore, the complex decision variables can be accommodated in the model. It also liberates investors from the assumption of asset return distribution is normally distributed and it more consistent with investors' utility functions, particularly, investors who have a strong aversion to downside risk (Young, 1998).

However, Minimax rule is subject to the limitation of requiring sufficient historical data on the past returns and a predictive probability model for future returns. If not, Minimax rule is not appropriate to be used (Young, 1998). Moreover, Byrne and Lee (2004) also argue that Minimax is sensitive to outliers of the data because it

specifically minimises the maximum loss (negative return).

## 2.5 Maximum Drawdown

The Maximum Drawdown (MaxDD) is defined as “the loss suffered when an asset is bought at a local maximum, and sold at the next local minimum.” (Hamelink and Hosli, 2004: 6). The MaxDD evaluated at time T is defined as:

$$MaxDD(T) = \max_{t \leq T} \{P(t) / P_{\max}(t) - 1\} \quad (10)$$

where  $P(t)$  is the price at time  $t$  and  $P_{\max}(t)$  is the maximum of all prices in overall to this point in time:  $P_{\max}(t) = \max_{\tau \leq t} P(\tau)$

The MaxDD appears as the more sensible by incorporating of the time-dependence of financial series and the serial correlation problem is taken into account in which the correlation effect will be shown in price variations and no assumption on the distribution of returns (Johansen and Sornette, 2001; Hamelink and Hosli, 2004). Besides, it is stable over time and less influenced by the added observations, which it will remind constant as long as there is no a new drawdown occurs (Hamelink and Hosli, 2004). It also can be used as a constraint in portfolio optimisation (Pereira Câmara Leal and Vaz de Melo Mendes, 2005).

The shortcoming for MaxDD, whereas, is influenced considerably by the data interval. Hamelink and Hosli (2004) highlight that the higher the frequency, the larger the MaxDD. This is consistent with the findings of Acar and James (1997), which the MaxDD from intra-day data is higher than monthly MaxDD.

## 3. Choice of Risk Measures

Table 1 reveals the characteristics of different risk measures. Obviously, each risk measure has its advantages and disadvantages. In other words, in certain circumstance, certain risk measure is more suitable to be used than other risk measures.

Variance is the most popular risk measure and it emerges as a risk measure that is more suitable for individual investors who normally have some basic background on risk management and the computation of variance is not as complex as other risk measures. While, variance is the only risk measure requires assumption on the asset return distributions.

On the other hand, LPM and Minimax could be a better choice for investors (risk seekers or strong risk aversion investors) who require a risk measure that is consistent with investor’s degree of risk aversion. While, MaxDD and MAD are apt for those desire a stable risk measure that less sensitive to outliers over the period of study. However, they should avoid from using variance, LPM and Minimax since these risk measures are sensitive to the outliers.

Institutional investors who have a large scale of portfolio (more than 1,000 assets), which MAD is more favourable risk measures. But, empirical evidences demonstrate that MAD suffers from the significant estimation error. This undermines the use of MAD. Notably, the large scale portfolio factor might be not very significant for real estate portfolios since real estate investors or funds rarely have more than 1,000 numbers of properties in their real estate portfolios.

Table 1: The Characteristics of Different Risk Measures.

Characteristics	Variance	LPM	MAD	Minimax	MaxDD
Popular and simple	√				
Consistent with investors’ degree of risk aversion		√		√	
No assumption on return distribution		√	√	√	√
Less sensitive to outliers			√		√
Suitable for large scale portfolio optimisation			√		
Significant estimation error			√		
Time-dependence of financial series					√
Serial correlation Consideration					√

While, MaxDD emerges as an ideal risk measure for those need a time-dependence risk measure and it is the only risk measure that takes into the account the serial correlations are found in the return series.

#### 4. Conclusion

In this paper, risk measures that have been employed for measuring the riskiness of real estate are reviewed. The advantages and disadvantages of each risk measure are also determined.

In general, there are several alternative risk measures that have been used in real estate literature and each risk measure has its strengths and limitations. As such, understanding the merits and drawbacks of these risk measures will assist investors to select the most appropriate risk measure in formulating a desirable risky investment strategy accordingly their investment criteria and objectives.

However, researchers should continue to make contributions in the area of the way in which surveying the acceptance level of these risk measures in practice. Additionally, the endeavours should be expended to employ other alternative risk measures such as Value at Risk and Conditional Value at Risk to real estate market.

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