

**HYDRODYNAMIC COEFFICIENTS OF MANOEUVRING FOR
SMALL VESSELS**

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ABSTRACT

Manoeuvring analysis of a small vessel is more complex as compared to the conventional merchant ship. For small vessels such as fishing vessels, those have a relatively small prismatic coefficient as compared to large ships, would have less restoring force. On the other hand, the hydrodynamic heeling moment caused by yaw and sway at higher operational speed will become significantly high. As a consequence, the roll motion should not be neglected in the analysis of manoeuvring for small vessel, as it is in the case of big ships. The analysis for small vessels would involve at least 4-degrees of freedom and non-linearity of the forces and motions would have to be considered. During turning (yawing), the centrifugal force acting and also the developed hydrodynamic pressures surrounding the hull will induce a rolling moment. This moment will force the vessel to list either inward or outward with the turning path depending on the height of the centre of the mass. Conjunction with the development of the ship simulator in marine technology laboratory, a study on the hydrodynamic coefficient of manoeuvring has been conducted on fishing vessels. The mathematical model of manoeuvring has been reviewed in order to incorporate the heel angle effect. By conducting the experiment in the towing tank, the coefficients in the mathematic model are obtained. In this study, the effects of heel angle on the hydrodynamic coefficients have been studied. The coefficients that obtained from model testing have been used in the time domain simulation and the manoeuvring criteria of the vessel have been simulated.

ABSTRAK

Analisis bagi olah-alih untuk kapal kecil adalah amat kompleks berbanding dengan kapal dagang besar. Kapal kecil, contohnya kapal perikanan, mempunyai pekali prismatic yang kecil berbanding dengan kapal besar, maka daya menegaknya bagi kapal kecil adalah relatifnya kurang. Namun begitu, daya hidrodinamik yang menyebabkan kapal sendeng ketika kapal teranjak atau berolah-alih pada kelajuan yang tinggi. Akibatnya, gerakan olengan bagi kapal kecil tidak boleh dikecuai bagi gerakan analisis kapal kecil. Analisis bagi gerakan kecil akan melibatkan sekurang-kurangnya 4-darjah kebebasan dan juga berbentuk tidak liner bagi daya dan momen. Ketika kapal berolah-alih, tindakan daya centrifugal dan juga tekanan hidrodinamik di sekitar kapal akan menyebabkan momen olengan. Momen ini akan menyebabkan kapal sendeng ke sebelah dalam atau sebelah luar bagi lokus gerakan pusingan kapal. Darjah kesendengan ini amat bergantung kepada lokasi pusat graviti. Bersempena dengan pembentukan sistem simulasi bagi pergerakan kapal di Makmal Teknologi Marin, kajian terhadap pekali hidrodinamik bagi olah-alih kapal nelayan telah dijalankan. Model matematik bagi olah-alih kapal telah dikaji untuk menyertakan komponen yang dapat menghuraikan kesan kesendengan bagi kapal dan program simulasi. Pekali-pekali dalam model matematik telah didapati dengan menjalankan ujikaji dalam makmal. Bagi kajian ini, kesan sudut sendang terhadap pekali-pekali hidrodinamik telah diberi penumpuan. Pekali-pekali yang didapati daripada ujian makmal telah disertakan kedalam program simulasi dalam bentuk domain masa. Ciri-ciri olah-alih bagi kapal telah simulasikan .

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LIST OF ABBREVIATION

Latin symbols

a_1, a_2, a_3	Constants for propeller open characteristics
a_H	Rudder to hull interaction coefficients
AR	Aspect Ratio
B	Breadth
C_b	Block Coefficient
C_D	Drag coefficient
C_L	Lift coefficient
C_p	Prismatic Coefficient
D_P	Propeller Diameter
F	Force
F_N	Rudder normal forces
GZ	Restoring Arm
I_x	Inertia about x axis.
I_z	Inertia about z axis.
J	Propeller Advance ratio
K	Moment about x - axis
KG	Height of the centre of gravity from the keel line
KM	Height of the metacentric from the Keel line
K_r	Roll moment due to yaw rate
K_v	Roll moment due to sway velocity
K_δ	Roll moment due to rudder angle
$K_{\dot{\theta}}$	Added mass coefficient for roll motion
$K_{\dot{\theta}}$	Roll damping coefficient

K_ϕ	Roll restoring moment
L	Length of vessel
LCG	Longitudinal centre of gravity
m	Mass
m'	Added mass
M	Moment
n	Propeller revolution
N	Moment about z - axis
N'	Non dimensional value of force N

$$N' = \frac{N}{\frac{1}{2} L^2 \rho U^2}$$

N'_v	Acceleration coefficient of yaw moment due to sway acceleration
N'_r	Acceleration coefficient of yaw moment due to yaw angular acceleration
N'_v	Yaw moment due to sway motion
N'_{vvv}	Non-linear component of yaw moment due to sway motion
N'_r	Yaw moment due to yaw motion
N'_{rrr}	Non-linear component of yaw moment due to yaw motion
N'_ϕ	Yaw Moment due to heel angle
$N'_{\phi\phi}$	Non-linear component of yaw moment due to heel angle
$N'_{v\phi}$	Coupling coefficient of sway motion and heel angle for yaw moment
$N'_{vv\phi}$	Non-linear coupling coefficient of sway motion and heel angle for yaw moment
$N'_{v\phi\phi}$	Non-linear coupling coefficient of sway motion and heel angle for yaw moment
$N'_{r\phi}$	Coupling coefficient of yaw motion and heel angle for yaw moment
$N'_{rr\phi}$	Non-linear coupling coefficient of yaw motion and heel angle for yaw moment
$N'_{r\phi\phi}$	Non-linear coupling coefficient of yaw motion and heel angle for yaw moment

r	Yaw velocity
\dot{r}	Yaw acceleration
S_f	Forward actuator amplitude
S_a	Aft actuator amplitude
T	Draught of vessel
t	Thrust deduction factor
t_R	Rudder to hull interaction coefficients
u	Surge velocity
U	Forward speed
\dot{u}	Surge acceleration
v	Drift velocity
\dot{v}	Drift acceleration
W	The width of the towing tank
x	Right handed axes fixed in ships; x positive forward.
X	Force in x direction
X_f	Forward Longitudinal force
X_a	Aft longitudinal Force
x_f	Longitudinal distance from CG of the model to forward transducer
x_a	Longitudinal distance from CG of the model to aft transducer
x_H	Rudder to hull interaction coefficients
x_R	Distance of centre of rudder to KG
x_s	Distance along the longitudinal axis of the vessel
y	Right handed axes fixed in ships; y positive Starboard.
Y	Force in y direction
Y_f	Forward lateral force
Y_a	Aft lateral Force
Y'	Non dimensional value of forces Y

$$Y' = \frac{Y}{\frac{1}{2}L\rho U^2}$$

Y'_v	Acceleration coefficient of sway force due to sway acceleration
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Y'_r	Acceleration coefficient of sway force due to yaw angular acceleration
Y'_v	Sway force due to sway motion
Y'_{vv}	Non-linear component of sway force due to sway motion
Y'_r	Sway force due to yaw motion
Y'_{rr}	Non-linear component of sway force due to yaw motion
Y'_ϕ	Sway force due to heel angle
$Y'_{\phi\phi}$	Non-linear component of sway force due to heel angle
$Y'_{v\phi}$	Coupling coefficient of sway motion and heel angle for sway force
$Y'_{vv\phi}$	Non-linear coupling coefficient of sway motion and heel angle for sway force
$Y'_{v\phi\phi}$	Non-linear coupling coefficient of sway motion and heel angle for sway force
$Y'_{r\phi}$	Coupling coefficient of yaw motion and heel angle for sway force
$Y'_{rr\phi}$	Non-linear coupling coefficient of yaw motion and heel angle for sway force
$Y'_{r\phi\phi}$	Non-linear coupling coefficient of yaw motion and heel angle for sway force
z	Right handed axes fixed in ships; z positive downwards.
z_a	Rudder area
z_{rud}	Height from lateral force and rudder centre

Greek Symbols

α	Phase angle for Sinusoidal motion
δ	Rudder angle
ϕ	Heel angle
ψ	Heading angle
ω	Actuator frequency
Δ	Weight of the vessel

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CHAPTER I

INTRODUCTION

1.1 Research Background

For ship motion analysis, the use of simulator has become more significant. Somehow in this country, ship simulation is still in a very preliminary stage, where ship motion is analysed by model testing method. Even though, model testing is very time consuming, need a lot of man power and very costly and it involves various types of high technology equipment.

For this reason, a real-time Ship Simulator has been developed to investigate and to predict the ship motions in variable seas environment. The simulator program available in Universiti Teknologi Malaysia is developed based on standalone personal computer (PC) system platform and network PC system platform.

This simulation program successfully demonstrated most of the critical seakeeping motions, which included various capsizing sequences (Yeak, 1997). This program also included strong non-linear coupling from heave and pitch onto roll and yaw (Maimun, 1993). On the extensions of the simulation program, the effect of diffraction on large amplitude of vessel motion has been described very thoroughly by Yeak (1997). However, for the simulation of surfriding and broaching conditions, the seakeeping approach itself is not sufficient for these applications. It is found that the forces and moments for a manoeuvring vessel are not only

predominated by seakeeping forces, but also the manoeuvring forces (Renilson, 1982).

The hydrodynamic equation incorporated in existing simulation program involved is only linear manoeuvring equation. This is not only unsuitable to simulate the conditions described above, but also insufficient to simulate the normal turning or zigzag manoeuvre in calm water. As a consequence, the hydrodynamic of manoeuvring should be focused in order to have a more realistic simulation.

The linear equation using perturbation method adopted by SNAME (Crane, 1989) is a simplified in linear form for easier analysis. However, it is limited for small motion. As compared to big vessel, generally, the Froude numbers for small vessel are much bigger than the big vessel; this is similar for non-dimensioned yaw and sway motion of small vessel. The linear equation used previous study was limited to predict the real manoeuvring path for small vessel. In order to study the manoeuvring characteristics in 6 degrees of freedom (6 DOF), the motion of small vessel is focused in this study.

Beside the non-linearity, small vessel motion will involve a least equations in 4 degree of freedom, which are surge, sway, yaw and roll in additional. The small vessel with smaller prismatic coefficient will have less restoring moment due to the smaller water plane area. Hence, the centrifugal force during a turning will cause a significant heel angle. The under water hull form would become asymmetry during a heel. As understood, the hydrodynamic coefficients are highly depending on the hull form geometry, as a result, the hydrodynamic coefficient will be affected by the heel angle. Different from big vessel, which is usually in 3 degree of freedom, the additional roll motion for small vessel become very interesting for this research.

1.2 Aim Of The Thesis

The Aim of this research is the searching of hydrodynamic coefficients related to manoeuvring of small vessels. In this research, the small vessel manoeuvring criteria will be focus. The non-linear equation of motion of small vessels will be studied. In additional, the heel angle effect on the manoeuvring derivatives and their coupling effect will be analysed.

1.2.1 Objective

- (a) To conduct literature survey on the important hydrodynamic coefficient that affect manoeuvring of small vessels.
- (b) To derive suitable experimental techniques to obtain the hydrodynamic coefficients.
- (c) To incorporate the hydrodynamic coefficient in existing simulation program and gauge the sensitivity of coefficients with respect to capsizing sequences.

1.3 Frame of Approach

For the study of hydrodynamic coefficients of manoeuvring of small vessels, firstly the mathematical model on the manoeuvring equation will be studied. A suitable mathematical model will be used as the reference to incorporate the heel angle effect. Base on the mathematical model, towing tank testing will be conducted in order to obtain the hydrodynamic derivatives. Besides that, the heel angle effect on hydrodynamic coefficient also had been studied.

From the experimental, each parameter is tested separately. The effect of each particular parameter could be studied mean while others parameter are controlled. By this method, a function of certain parameter could be obtained easily. However, for a real motion of a vessel during manoeuvring, the dynamic of motion will involved all parameters and will be complex. Each single parameter effect obtained form experimental should be combined into one system of motion in order

to study the real motion behaviour. However, this could not been done in the captive model testing method. As the alternative, by the simulation approach, the parameters could be combined virtually. By using the coefficient values obtained from experimental, an in house time domain simulation will be develop by utilising MATLAB-Simulink software for simulation purpose.

REFERENCES

- Abkowitz, M.A. : 'Lectures on ship hydrodynamics-steering and manoeuvrability'.
HyA report No Hy-5, May 1964
- Ankudinov, V.K. "Simulation analysis of ship motion in waves', Inetrnational
Workshop on Ship and Platform Motions, University of California at
Berkeley, 1983
- Ankudinov, V.K. et al., "Assessment and principal structure of modular
mathematical model for ship manoeuvrability prediction and real-time
manoeuvring simulations', Int. Conference on Marine Simulation and Ship
Manoeuvrability MARSIM '93, St. John's. Newfoundland, 1993.
- Barr, R.A., "A review and comparison of ship maneuvering simulations methods",
Transactions of SNAME, Vol. 101, 1993.
- Bhattacharyya, R. : " Dynamics Of Marine Vehicles", Anapolis, Maryland, John
Wiley & Sons, 1979
- Bishop, R.E.D., Burcher, R.K. and Price, W.G. : ' The Determination of Ship
Manoeuvring Characteristics from Model Tests'. Trans. RINA.1974
- Clarke , "The application of Manoeuvring Criteria in Hull Design Using Linear
Theory" Transactions of RINA, 1982

- Crane, C.L., Eda, H. & Landsburg, A. 'Controllability' Chapter IX, Principles of Naval Architecture Second Revision, Vol III, 1989
- Davison, K.S.M. and Schiff, L.J., "Turning and course-keeping qualities", SNAME Proceeding, New York NY, 1946.
- Eda, H., "Maneuvering performance of high speed ship with effect of roll motion", Ocean Engineering, Vol.7, pp. 379-397, 1980.
- Gill, A.D. : 'The Analysis and Synthesis of Ship Manoeuvring'. Trans. RINA, Vol.136 1979, pp 209-225
- Gui, Q.Y., Chuang, J. M., Hsiung, C.C., 1990, "Computation of Ship Intraction Forces and Moments in Restricted Waterways using the Numerical Conformal Mapping Method", International Shipbuilding Progress, Vol. 31, No. 412.
- Hamamoto, M., Tsukasa, Y., 1992, "An Analysis of Side Force and Yaw Moment on ship in quatering Waves", Journal of Society of Naval Architects of Japan, No 171.
- Hirano, M. and Takashina, J. : 'A Calculation of Ship Turning Motion Taking Coupling Effect Due to Heel into Consideration' Technical Committee of the West-Japan Society of Naval Architects, 1980
- Hooft, J.P. and Pieffers, J.B.M., "Maneuverability of Frigates in waves", Marine Technology, Vol. 25, No. 4, 1988
- Hooft, J.P.: 'Standard Procedure for Performing Captive Manoeuvring tests'. MARIN Draft Report No 57351-1-WR, May 1997
- Inoue, S. et al., 1981 "A practical calculation method of ship manoeuvring motion", International Shipbuilding Progress, Vol. 28.

- Inoue, S., Hirano, M. and Mukai, K. , 1980 : ‘The non-linear Terms of Lateral Force and Moment Acting on Ship Hull in the Case of Manoeuvring, Technical Committee of the West-Japan Society of Naval Architects
- Kan, M. and Hanaoka, T., 1964, “Analysis for the effect of shallow water upon turning”, Japan Society of Naval Architects, Vol 49, Pp 115
- Karasuno, K., Yoneta, K., Jyanuma, S., 1990, “Physical-Mathematical Moels of Hydro- or Aero-Dynamic Forces acting on Ships Moving in an Oblique Direction’, Proceedings of MARSIM and ICSM ’90, Tokyo, Japan, pp393-400.
- Kijima, K., Furukawa, Y., He Qing, 1991, “The Interaction Effects between Two Ships in the Proximity of Bank Wall”, Transaction of the West-Japan Society of Naval Architects, No. 81, pp. 101-112.
- Kijima, K. and Furukawa, Y. : “ Effect of Roll Motion on Manoeuvrability of Ship” Man’98, Val de Reuil, Sept 1998
- Kose, K.: “ On A New Mathematical Model Of Maneuvering Motions Of A Ship And Its Applications’, International Shipbuilding Progress, Rotterdam, Netherlands, Vol 29, No. 336, August 1982.
- Leeuwen, G. Van, 1964, “The lateral damping and added mass of an oscillating shipmodel”, Shipbuilding Laboratory, Technological University, Deft, Pub. No 23.
- Maimun, A. , “ Stability of Fishing Vessel in an Astern Sea- Shallow water Environment”. PhD Thesis, Dept. of Ship and Marine Technology, University of Strathclyde, 1993
- McCallum. Ian., “ An Introduction to Ship Mathematical Models and Manoeuvring Behaviour”, Institute of Science and Technology, University of Wales, 1967.

- Newman, J. N., "Some theories for ship manoeuvring", International Symposium on Directional Stability and Control of Bodies Moving in Water, The Institute of Mechanical Engineers, 1972.
- Nomoto, K. et al., "On the steering qualities of ships", International Shipbuilding Progress, Vol. 4, No. 35 1957.
- Nonaka, K., 1990, "Ship Handling Standards – Capabilities and Requirements" Proceedings of MARSIM and ICSM '90, Tokyo, P 37.
- Norrbin, N.H., "Theory and observations on the use of a mathematical model for ship manoeuvring in deep and confined waters", SSPA Report No.68, Gothenborg, 1971.
- Otlmann, P., "Roll – An often neglected elements of manoeuvring", int. Conference on Marine Simulation and Ship Manoeuvrability MARSIM '93, St. John's, Newfoundland, 1993.
- Price, W. G., Tan, M. Y., 1992,I "Applications of Boundary Element Methods in Hydrodynamic Problems Relating to Manoeuvring bodies", 19th Symposium on Naval Hydrodynamics.
- Price, W. G., Tan, M. Y., 1992,II, "A Viscous Boundary Element Approach to Evaluate slow Motion Derivatives", Proceedings of MCMC, pp. 437-449.
- Renilson, M.R. : "Broaching-An Investigation into the Loss of Directional Control in Severe Following Seas" Trans. RINA 1982, Vol 124
- Renilson, M. R., Ch'ng, P. W., 1990, "The Effects of Bank Slope and water depth on the Forces on a Ship in Restricted Water", Proceedings of MARSIM and ICSM '90, pp. 485-491.
- Renilson, M.R. and Manwarring, T. : 'An Investigation into Roll/Yaw Coupling and Its Effect on Vessel Motions in Following and Quartering Seas'. 7th

International Conference on Stability of Ship and Ocean Vehicles,
Launceston, Tasmania, Australia, Feb 2000

Shampine, L. F. and M. K. Gordon, Computer Solution of Ordinary Differential Equations: the Initial Value Problem, W. H. Freeman, San Francisco, 1975.

Smitt, L. W., Chislett, M. S., 1974, "Large amplitude PMM test and maneuvering prediction for a Mariner class vessel", Proceedings of the 10th Symposium on Naval Hydrodynamics, pp. 131-157.

Strøm-Tejsen, J., Chislett, M. S., 1966, "A model testing technique and method of analysis for the prediction of steering and manoeuvring qualities of surface vessels", Rep. No. Hy-7. HyA, Lyngby.

Vassalos, D., Devenakiotis, C., 1992, "The Effect of Environmental on the Stability and Turning of ships", Proceedings of Manoeuvring and Control of Marine Craft. MCMC '92, Southampton, UK, pp. 147-165.

Yeak, S.H.: "The Effect of Diffraction on Large Amplitude Simulation Off Fishing Vessel", M.Eng Thesis, Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, 1997.