MATROID STRUCTURE OF DYNAMIC GRAPH MODEL OF EVAPORATION PROCESS IN A BOILER SYSTEM

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To my beloved family and friends
especially
my dear abah..Khamis Bin Sam
and
my lovely mak..Maria Binti Mislani.

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ABSTRACT

Graph and matroid are strongly bonded to each other. In fact, a graph can be transformed to a matroid structure. In this study, we are going to to discuss on matroid and its examples and to show that the dynamic graph model of an evaporation process in a boiler system can be viewed as a matroid. The definition of matroid based on the independence axiom is used in this study to achive the objectives that mentioned. The evaporation process model that denoted as G_s (V, E) in this study is developed using the integration of the concept of autocatalytic set (ACS) and graph theory. An Autocatalytic Set (ACS) is a set of reactions whose product catalyzes one another. In term of graph theoretic approach, ACS is a subgraph each of the nodes has one incoming link from a node belonging to the same subgraph. The model had listed about seventeen variables to represent the nodes and thirty six links which are based on the catalytic relationship among the nodes to represent the edges.

ABSTRAK

Graf dan matroid sangat berkait antara satu sama lain dan graf juga boleh dilihat sebagai struktur matroid tertentu. Kajian ini membincangkan mengenai matroid dan contoh-contohnya serta membuktikan bahawa model graf dinamik bagi proses penyejatan dalam sistem dandang boleh dilihat sebagai satu struktur matroid. Definisi matroid berdasarkan axiom ketidakbergantungan telah digunakan di dalam kajian ini untuk mencapai objektif yang telah dinyatakan. Model graf dinamik bagi proses penyejatan dalam sistem dandang yang dinyatakan sebagai G_s (V, E) telah dibina menggunakan pengintegrasian konsep set pengautomangkinan (ACS) dan teori graf. Set pengautomangkinan (ACS) adalah satu set tindak balas yang mana sesuatu produk menjadi pemangkin antara satu sama lain. Dari segi pendekatan teori graf, ACS ialah subgraf yang mana setiap nod mempunyai satu pautan masuk dari nod subgraf yang sama. Model penyejatan ini telah menyenaraikan sebanyak tujuh belas pembolehubah yang mewakili nod dan tiga puluh enam pautan yang merujuk kepada hubungan pemangkinan antara nod yang diwakili oleh pinggir.

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LIST OF SYMBOLS AND ABBREVIATION

e_i	-	The i^{th} edge/link
v_i	-	The i^{th} vertex/node
V / $V(G)$	-	Set of vertices
E / $E(G)$	-	Set of edges
A	-	Adjacency Matrix
a_{ij}	-	Entries of the adjacency matrix A representing the link between j^{th} node and the i^{th} node
$U_{k,n}$	-	Uniform matroid
$\rho(A)$	-	rank of A
$\|A\|$	-	Cardinality of A
$\in$	-	element of/member of
$\subset$ / $\subseteq$	-	subset of
ACS	-	Autocatalytic set
CO_2	-	carbon dioxide
CuO	-	copper oxide
HCl	-	hydrochloric acid
H_2	-	hydrogen
H_2O	-	water
H_2S	-	hydrogen sulfide
H_2SO_4	-	sulphuric acid
$NaCl$	-	sodium chloride
$NaOH$	-	sodium hydroxide
Na_2SO_4	-	sodium sulfite

N_2	-	nitrogen
O_2	-	oxygen
SiO	-	silicon dioxide
SO_2	-	sulfur dioxide

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Matroids were introduced by Whitney in 1935 to try to capture abstractly the essence of dependence (Oxley, 2003). Whitney's definition embraces a surprising diversity of combinatorial structures. Moreover, matroids arise naturally in combinatorial optimization since they are precisely the structures for which the greedy algorithm works. Since then, it has been recognized that matroids can be used as a framework for approaching a diverse variety of combinatorial problems. Indeed, the introduction of matroids by Whitney has provided a unifying abstract treatment of dependence in linear algebra and graph theory.

The name "*matroid*" suggests a structure that related to matrix. In the early introduction to the name "*matroid*", it has not always been universally admired. Rota (1970), which is one of the most important contributors to matroid theory include the coauthorship of the first book on the subject had mounted a campaign to try to change the name *"matroid"* to "*geometry*" which is an abbreviation of "*combinatorial geometry*". At the height of this campaign in 1973, several other terms have been used to replace the name "*geometry*" but the name "*matroid*" is the only one that survived. Today, both "*geometry*" and "*matroid*" are still in use although "*matroid*" is certainly predominates.

Apart from that, graph theory is a branch of mathematics that has real-world application in many areas, including industrial, electrical and transportation engineering, computer science, environmental conservation, management, marketing, scheduling, education, psychology, biology and also chemistry (Chartrand, 1985; Evans and Minieka, 1992). In fact, it is rich in theoretical results, especially to study the interconnection among elements in natural and man-made systems. Traditionally, research on graph theory only focused on studying the properties of static graphs. However, almost all real networks are dynamic and large in size. The study on the evolution of variables or processes in a network has led to the integration of the concept of autocatalytic set (ACS) and graph theory.

The concept of autocatalysis which comes from chemistry is a fundamental concept that has been used in a wide range of domains. The notion of "autocatalysis" was first introduced by Ostwald in 1890 to describe reactions that showing rate acceleration as a function of time for example, the case of esters hydrolysis that is at the same time acid catalyzed and producing an organic acid (Raphael *et al*., 2010). On the other hand, an autocatalytic set (ACS) is a set of reactions whose product catalyzes one another. In term of graph theoretic approach, ACS is a subgraph each of the nodes has one incoming link from a node belonging to the same subgraph (Jain and Krishna, 1998).

Applications of this concept are explored in modeling the clinical waste incineration process and combustion process in a circulating fluidized bed boiler (Sumarni *et al.*, 2013; Tahir *et al.*, 2010). In a typical oxy-fuel combustion boiler system, the two main processes that involved are the combustion and evaporation process. In Noor Ainy *et al.* (2014), the concept of ACS was applied in developing a dynamic graph model of an evaporation process in a boiler system. As for the process, seventeen variables that give significant contributions to the evaporation process are identified to represent the nodes and thirty six links which are based on the catalytic relationship among the nodes are represented by edges in the graph.

Since the introduction of matroids by Whitney, they are closely related to graph. Even the study by Oxley (2003) highlights several broad areas in which interesting interactions occur between graphs and matroids. Oxley proved that the graph theorems that “can be expressed in terms of edges and circuits only” are always going to suggest new matroid results. One of the aims of his study was to show that, by taking a matroid perspective on graphs, one can frequently produce new results not only for matroids but also for graphs.

1.2 Statement of the Problem

The study on evolution of processes or variables in a network has led to the integration of the concept of autocatalytic set (ACS) and graph theory which then has been applied by Noor Ainy *et al.* (2014) to develop a dynamic graph model of evaporation process in a boiler system.

In this study, we are interested to proof that the graph model that has been developed is a matroid, so as to discover another mathematical structure of the graph and to come out with another example of matroid in real network of a man-made system that is in this study the evaporation process in an oxy-fuel combustion boiler system. However, this study also attempts to provide enough understanding towards the concept of matroid and the evaporation process itself before proceeding to the proof.

1.3 Research Objectives

The objectives of this study are as follows:

i. to discuss on matroid and its examples.
ii. to prove that the dynamic graph model of an evaporation process in a boiler system is a matroid.

1.4 Scope of Study

The scope of the study is to show that the dynamic graph model of evaporation process in a boiler system developed by Noor Ainy *et al.* (2014) is a matroid. However, since this study is the case study, the proof is only limited to the graph that has been mentioned.

1.5 Significance of Study

The study of matroid is an attempt to capture the mathematical structure. This study aims to further discover another matroid structure in graph theory which provides a mathematical modelling for studying interconnection among elements in natural and man-made systems (in this study, the graph model of evaporation process in a boiler system).

1.6 Research Layout

In this study, we are going to show a matroid structure of an evaporation graph model developed by Noor Ainy *et al.* (2014). Therefore, the study is designated to achieve that with the objectives that have been mentioned in section 1.3 which served as guidance throughout the study as well as to write this dissertation. The research layout is given as follows.

Chapter 1 introduces the framework of the study. Chapter 2 gives a review on the previous literature work and some basic knowledge of related topics. In details, chapter 2 reviews about the concept that related to matroid. This chapter presents about the definition of matroid, theorems, and examples of matroid and how matroid arise from graph which expected to provide an insight towards the concept of matroid.

In chapter 3, the concept of graph in a network study which covers on the definition of graph and some other definitions that related to it will be reviewed. The dynamic graph model of evaporation process which will be used in this study will be presented in this chapter as well as the concept of autocatalytic set (ACS) that has been used to develop the model.

Chapter 4 shows the proof that the dynamic graph model of evaporation process in a boiler system developed by Noor Ainy *et al.* (2014) is a matroid based on the Oxley's definition.

The last chapter which is chapter 5 presents some conclusions made for the study and also the recommendations for future works of the research.

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