FOURIER TRANSFORM TECHNIQUE FOR ANALYTICAL SOLUTION OF DIFFUSION EQUATION OF CONCENTRATION SPHERICAL DROPS IN ROTATING DISC CONTACTOR COLUMN

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ABSTRACT

A Fourier transform technique has been utilized to obtain analytical solution of diffusion equation of concentration of spherical drops in the Rotating Disc Contactor (RDC) column. The result obtained by Fourier transforms technique is compared with the result that obtained using separation variables. This study shows that the concentrations of spherical shape drops obtained by using both methods are the same. We find that the time for the drops to attains equilibrium depend on the size of drops. The smaller size of drops cause quicker time for the drops to attains the equilibrium.

ABSTRAK

Suatu teknik jelmaan Fourier telah digunakan untuk mendapat penyelesaian analisis bagi persamaan resapan kepekatan titisan di dalam Turus Pengekstrakan Cakera Berputar (RDC). Keputusan yang diperolehi dengan menggunakan teknik jelmaan Fourier ini dibandingkan dengan keputusan yang diperolehi dengan menggunakan teknik pemisahan pembolehubah. Didapati kepekatan titisan yang didapati dengan menggunakan kedua-dua teknik ini adalah sama. Seterusnya profile bagi jumlah kepekatan bagi titisan sfera dalam RDC dapat dikira. Didapati masa untuk titisan mencapai kesaimbangan dipengaruhi oleh saiz titisan. Saiz titisan yang kecil menyebabkan masa yang diambil untuk mencapai kesaimbangan adalah lebih cepat.

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LIST OF SYMBOLS/NOTATIONS

a	-	radius of a sphere
RDC	-	rotating disc column
C_0	-	concentration of one drop on surface
C_1	-	initial uniform concentration of drop
$C(r,t)$	-	concentration at distance r from the center at time t
C_s	-	total concentration of one drop
D	-	diffusion equation constant
F	-	Fourier transform
IBVP	-	Initial Boundary Value Problem

CHAPTER 1

INTRODUCTION

1.0 Preface

In recent years, the study of liquid – liquid extraction has attracted widespread interest from all major chemical engineering, petroleum, and pharmaceutical company including mathematician. Liquid – liquid extraction is a process for separating components in solution by their distribution between two immiscible liquid phases. This extraction is used primarily when distillation is impractical or too costly to use. It may be more practical than distillation when the relative volatility for two components falls between 1.0 and 1.2 [1].

There are a lot of instruments used for the processes of liquid-liquid extraction. One of the instruments that concern in this research is Rotating Disc Contactor Column. This rotating disc liquid-liquid extraction unit has been designated to demonstrate the basic principles of a liquid-liquid extraction process. When separation by distillation is ineffective or very difficult, liquid-liquid extraction is one of the main alternatives to consider [2]. Close boiling mixtures or substances extraction, which utilizes chemical differences instead of vapor pressure differences.

The mass transfer is defined as the movement of mass or molecules from an area of high concentration to that of low concentration until a homogenous or equilibrium concentration in the system is achieved. Basically, there are two modes of mass transfer in a single liquid phase. The model of mass transfer process in Rotating Disc Contactor (RDC) Column has been developed by some researchers. The past studies have discussed on how to solve the diffusion equation by using separation of variables. In this study, we want to focus solving the diffusion equation by using Fourier transform.

1.1 Problem Statement

Rotating Disc Contactor Column is one of the agitated mechanical devices that is being widely used in the study of liquid-liquid extraction. One of the main important in liquid-liquid extraction is the concentration of drops of liquid mixtures. In industry, the scientist will measure the concentration of drops through experiment. However, this study is more interested to find the solution of diffusion equation of concentration of spherical drops in RDC. Previous researchers used separation of variables technique to obtain the solution of the diffusion equation. In this study, there are three related questions need to be answered:

1. What is the analytical solution for diffusion model of spherical shape drops in RDC ?
2. Is the concentration of drops that obtained by using Fourier transform technique the same with the separation of variables technique ?
3. What are the factors for spherical drops in RDC influence the time of equilibrium?

1.2 Objective of Study

The objectives of this research are:

1. To study on the application of Fourier transform technique on Initial Value Boundary Problem.
2. To solve diffusion equation of concentration of spherical drops using Fourier transform technique.
3. To solve diffusion equation of concentration of spherical drops using Separation of variables technique.
4. To verify the Fourier transform method with separation variable method.
5. To determine the profile of total concentration of drops by using MATLAB

1.3 Scope of Study

This study focuses on the investigation of the diffusion equation for spherical shape drop in order to find the total concentration in Rotating Disc Contactor Column (RDC) of drops using Fourier transform. This study simulates the drops that have spherical shape with ten different volumes. The volume of drops in this study is based on the volume of ten spherical shape drops with diameter in the range of 0.705 mm to 7.05 mm.

1.4 Significant of Study

This study wants to prove that we have another method to get the diffusion model for spherical drops. The liquid-liquid extraction can be determined by using the simulation value of the profile concentration for spherical shape drops with solving the diffusion equation by separation variables and Fourier transform. Hence, this method can be used in finding the total concentration of drops in RDC column. We can show that the total of concentration of drops at time *t* are same for both method.

1.5 Summary

In this introductory chapter, a short introduction on the liquid – liquid extraction process particularly on the RDC column has been presented. The current chapter gives direction and purpose to the research and the discussions presented here are the basis for the work done in the subsequent chapter.

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