MODELLING OF REINFORCED CONCRETE BEAM BY USING PLASTICITY APPROACH

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DEDICATION

For my beloved

Dad: Mansor Bin Hambali, Mom: Norliza Binti Hassan

Siblings: Mohd Nizar , Nur Hidayah, Mohamad Hafiz,

Wan Salfarini, and Ahmad Jais

Nephew: Adam Rizq and Adam Zahin

Fiancé: Nik Mohd Haniff

Thanks for your support, understanding and concern.

I will love you all

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Even thousand words could not express my gratitude.

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ABSTRACT

Reinforced concrete structure is a composite material made of fresh concrete and steel bars most commonly used in construction industry due to economic, efficiency, and strong in sustained stress. Even though reinforced concrete is widely used with many modification applied to increase the strength and durability, structure are still exposed to failure due to limitation of plastic behaviour knowledge on concrete. The objectives of this study are to determine formula of moment-curvature and load-displacement; ultimate loads to achieve yield state, the moment-curvature and load-displacement relationships of reinforced concrete and verify the applicability of the proposed model by comparing the predicted behaviours with those observed in experimental results. This study is carried out created a model of simply-supported beam with two concentrated load in symmetrically way. MATLAB was chosen in this study to determine the plastic behaviour of concrete beam under bending load. The results on moment-curvature and load-displacement formula, ultimate loads, moment-curvature and load-displacement relationships and comparison of results are presented in this report. Based on the analysis, it is found that the moment and displacement of reinforced concrete beam is affected by load applied and plasticity behaviour of a reinforced concrete beam. Moreover, the formulation of moment-curvature and load-displacement of reinforced concrete beam is successfully developed.

ABSTRAK

Struktur Konkrit bertetulang adalah bahan gabungan yang diperbuat daripada konkrit dan keluli yang penggunaannya amat luas dalam industri pembinaan kerana faktor ekonomi, kecekapan, dan kekuatan dalam tekanan yang berterusan. Walaupun struktur konkrit bertetulang digunakan secara meluas dengan banyak pengubahsuaian untuk meningkatkan kekuatan dan ketahanan, struktur konkrit masih terdedah kepada kegagalan kerana pengetahuan perihal tingkah laku plastik pada konkrit adalah terhad. Objektif kajian ini adalah untuk menentukan formula momen-kelengkungan dan beban-anjakan; beban maksimum untuk struktur gagal dalam keadaan plastik, hubungan momen-kelengkungan dan beban-anjakan konkrit bertetulang dan mengesahkan kesesuaian model yang dicadangkan dengan membandingkan tingkah laku yang diramalkan dengan yang diperhatikan dalam keputusan eksperimen. Kajian ini dijalankan dengan menggunakan satu model rasuk yang disokong secara mudah dengan dua beban tumpu dalam keadaan simetri. MATLAB telah dipilih untuk menentukan tingkah laku plastik rasuk konkrit di bawah beban lenturan. Hasil analisis berkenaan formula momen-kelengkungan dan beban-anjakan, beban maksima, hubungan momen-kelengkungan dan beban-anjakan dan perbandingan keputusan telah dibentangkan di dalam laporan ini. Berdasarkan analisis, didapati bahawa momen dan anjakan rasuk konkrit bertetulang dipengaruhi oleh beban dan kelakuan plastik dalam rasuk konkrit bertetulang. Malah, formula momen-kelengkungan dan beban-anjakan telah berjaya dihasilkan.

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LIST OF SYMBOLS AND ABBREVIATION

As	=	Reinforcement Steel Area
b	=	Width of Beam
c	=	Material Reinforcement Ratio
df	=	Internal Forces of beam
E	=	Modulus of Elasticity
fc	=	Ultimate Compression Stress of Concrete
f'c	=	Yield Compression Stress of Concrete
h	=	Height of Beam
I	=	Moment of Inertia
K	=	Curvature
kN	=	kiloNewton
L	=	Length of Beam
M	=	Moment
mm	=	Millimetres
P	=	Concentrated Load
RC	=	Reinforced Concrete
u	=	Deflection of Beam
ν	=	Displacement of Beam
ω	=	Geometrical Reinforcement Ratio
σ	=	stress of material
ε	=	strain of material
σ_p	=	Yield Tensile Stress of Concrete
σ_y	=	Yield Stress of Steel
ρ	=	Radius of Curvature

CHAPTER 1

INTRODUCTION

1.1 Introduction

Reinforced concrete is one of the most commonly used building materials nowadays. It is a composite material made of fresh concrete and steel bars surrounded in the concrete. Concrete is very strong in compression but weak in tension. Therefore, steel bar known as reinforcement is needed to resist the tensile stresses resulting from the applied loads. The economy, efficiency, strength and stiffness of reinforced concrete make it an attractive material for a wide range of structural engineering applications.

Many tragedy that involved with engineered structure such as building, had given the human and economy a real problem. Engineer is in charge to reduce the impact came from all the major problems related to engineer as failures of structures difficult to be avoided. Behaviours of reinforced concrete should be properly defined so that the precise reinforcement for concrete can be positioned at one structure. The laboratory analysis and analytical or theory analysis should be done to the structure.

Concrete is a heterogeneous, cohesive-frictional material exhibiting a complex nonlinear inelastic behaviour under multi-axial stress states. Detailed understanding of material response under variety of loads that determine the presence of bending, shear, and axial force enables the structural designer to forecast the material behaviour under conditions of plastic (Chandrasekaran *et al*.,2009).

There are relatively a few ways in order to do a research regarding concrete beams such as by experimental test. Beams will be casted manually and cured in several in order to achieve the targeted strength. In order to increase the understanding of reinforced concrete beams under load, stiffness method and analytical method also another popular choice. However, the reliability of the finite element model developed must be verified by comparison with the previous laboratory testing results.

In addition, MATLAB was used as a tool for analyse and execute complicated calculations. MATLAB tolerates matrix manipulations and convenient for writing simple finite element programs. It provides the typical constructs, such as loops and conditionals and plotting results.

MATLAB was first implemented by practitioners and researchers in control engineering that spread quickly to various other fields. It is now widely used in education, in particular the teaching of linear algebra and numerical analysis and is popular among scientists involved in image processing.

1.2 Statement of problem

Analytical procedures which may accurately determine stress and deformation states in reinforced concrete members are complicated due to many factors. Among them are the non-linear load-deformation response of concrete and difficulty in forming suitable constitutive relationships under combined stresses, progressive cracking of concrete under increasing load and the complexity in formulating the failure behaviour for various stress states.

Generally in design, the application of stress and strain relationship in limit state is adapt but considerations of the behaviour of plastic in reinforced concrete beam are limited. It is found that the plasticity formulation of moment and displacement for reinforced concrete beam is very limited.

Furthermore, widely used stiffness method has its limitation to analyse reinforced concrete structure using plasticity approach. Stiffness method analysis shows no transition of structure behaviour where structure behave as fully plastic yield after elastic. This study is focus on the properties and behaviour of reinforced concrete beam from elastic state until it reaches plastic state.

1.3 Aims and Objectives of Study

The research study is conducted in order to acquire understanding on the reinforced concrete beam behaviour. The aims of this study are to assemble the information about the reinforced concrete beam moments and displacements under loadings. The objectives of the research are to study the plastic behaviour of reinforced concrete under two concentrated point pure bending loads as:

i. To develop formula of moment-curvature and load-displacement of reinforced concrete beam.
ii. To identify the ultimate loads for reinforced concrete beam to achieve yield state.
iii. To compare the moment-curvature and load-displacement relationships of reinforced concrete.
iv. To verify the applicability of the proposed model by comparing the predicted behaviours with experimental results obtained by other researcher.

1.4 Scope of Study

The scope of this study is to develop modelling and analysis of rectangular reinforced concrete beam with dimension of 120x250mm with 2600mm length under two concentrated pure bending load involves considerations of plastic behaviour. The scope of study is based on:

- Static analysis
- Elastic
- Plastic
- Rectangular reinforced concrete beam will be based on mathematical approach and stiffness method
- MATLAB software

1.5 Significance of Study

Currently, engineers are required to use limit state design in their structures without knowing the development of failures in beam's section. This study focused in understanding the behaviour of beam under concentrated loads at two points by formulated the stresses using partial differential equations. The formulation will be based on stress-strain block of beam section and with the help of MATLAB software for complicated calculations. With this study, the awareness of plastic behaviour in beams can be increase hence avoid the failure of structures.

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