

FAILURE OF OHM'S LAW AND CIRCUIT DESIGN

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Dedication to my beloved parents, siblings and friends for their support and care

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ABSTRACT

Ohm's law is at the heart of circuit theory both for digital and analogue applications. Ohm's law depicts the linear current response I to the applied voltage V across a length of resistor. The resistance, as inverse slope of current-voltage (I-V) graph, is constant and is extensively used in the published literature. However, the linear response transforms to a sublinear response with current eventually saturating to a constant value I_{sat} . Nonohmic behaviour is distinctly visible when applied voltage V is larger than the critical voltage $V_c = V_t L / \ell_{\infty}$ ($V > V_c$), V_t is the thermal voltage with value 0.0259 V at room temperature and ℓ_{∞} (typically 100 nm) is the mean free path (mfp) in a nanoscale ($L < 1000\text{nm}$) device. The breakdown of Ohm's law affects heavily the flow of transporting carrier in a nanoscale device. The surge in direct resistance $R=V/I$ and incremental $r=dV/dI$ changes the time constants, power consumption, voltage and current division laws. The transient RC switching delay in micro/nano-scale circuit is strongly affected by the surge in the resistance arising out of sub-linear current-voltage (I-V) characteristics. The goal is to investigate the circuit laws when Ohm's law is not applicable. Factors affecting the critical voltage beyond which Ohm's law fails in scaled-down nanostructures have been studied in this project. The theory to 1D silicon nanowire, 2D AlGaAs nano-layer and 3D bulk resistor have been applied. The mechanism of current saturation is studied here. Numerical codes using MATLAB simulation software are developed. Each resistor in addition to its ohmic value R_o must also be described by either the critical voltage V_c or saturation current I_{sat} , connected by the relation $V_c = I_{sat} R_o$ whose default value is assumed to be infinite when Ohm's law is applicable. The research framework is based on Non-equilibrium Arora's Distribution Function (NEADF).

ABSTRAK

Hukum Ohm adalah jantung kepada kedua-dua teori litar untuk aplikasi digital dan analog. Hukum Ohm menggambarkan tindak balas arus linear I , dengan voltan V yang digunakan perintang. Rintangan, sebagai cerun songsang graf arus-voltan (IV), adalah tetap dan digunakan dengan meluas dalam kaedah yang diterbitkan. Walau bagaimanapun, sambutan linear untuk mengubah tindak balas sublinear dengan semasa akhirnya menepukan kepada nilai malar. Tingkah laku Non-ohmic adalah jelas apabila menggunakan voltan V lebih besar daripada voltan yang kritikal (V_c), iaitu voltan haba dengan nilai $0,0259$ V pada suhu bilik dan (Biasanya 100 nm) adalah mean free path (MFP) dalam skala nano yang (λ peranti. Pecahan Hukum Ohm menjejaskan banyak pengaliran dalam peranti skala nano. Peningkatan langsung rintangan $R = V / I$ dan r tambahan $= dV / dI$ mengubah hukum pemalar masa, penggunaan kuasa, voltan dan bahagian arus. Kewujudan RC pensuisan melengahkan masa mikro litar skala nano / yang dipengaruhi oleh kenaikan dalam rintangan yang timbul daripada sub-linear semasa ciri cir voltan (IV). Matlamatnya adalah untuk menyiasat hukum litar apabila hukum Ohm tidak digunakan. Faktor-faktor yang memberi kesan kepada voltan yang kritikal di luar hukum Ohm gagal dalam menurunkan nanostruktur telah dikaji dalam projek ini. Teori untuk 1D silikon nanowire, 2D AlGaAs nanowire dan 3D perintang pukal telah digunakan. Mekanisme arus tepu di kaji. Kod berangka menggunakan perisian simulasi MATLAB digunakan. Setiap perintang sebagai tambahan kepada nilai ohm yang mesti juga digambarkan oleh sama ada voltan yang kritikal atau arus tepu, Yang berkaitan dengan hubungan nilai lalai yang dianggap tidak terhingga dalam hukum Ohm berkenaan. Rangka kerja penyelidikan adalah berdasarkan NEADF, adalah fungsi taburan anisotropik yang merupakan hasil daripada fungsi taburan Fermi-Dirac yang isotropik.

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LIST OF SYMBOLS

Q3D	-	Quasi One Dimensional
Q2D	-	Quasi Two-Dimensional
Q1D	-	Quasi One-Dimensional
λ_D	-	De Broglie Wavelength
l_o	-	Mean Free Path
E_F	-	Fermi Energy
v_{Dn}	-	Intrinsic Velocity for Q3D
ε	-	Electric Field
ε_c	-	Critical Electric Field
v_{sat}	-	Drain Saturation velocity
V_c	-	Critical Voltage
V_T	-	Threshold Voltage
q	-	Electron Charge
ρ	-	Resistivity
ρ_s	-	Sheet resistivity
v_{th}	-	Thermal Velocity
m_l	-	Longitudinal Effective Mass
m_T	-	Transverse Effective Mass
V_T	-	Threshold Voltage
N_c	-	Density of States
μ	-	Electron Mobility
γ	-	Fitting Operator
n	-	Carrier Concentration

CHAPTER 1

OVERVIEW

1.1 Introduction

Science has given a perfect shape to the human civilization. Electricity was only the splendid invention in the twentieth century which greatly influenced the human life in the globe. Scientists invented, engineers implemented that is occurring in eras. This work is the integration of science and engineering. After discovery of preliminary elements of electricity like voltage, current and resistance Georg Ohm (1827) established a relation among them through a study of the conceptual framework in the famous book *Die galvanische Kette, mathematisch bearbeitet* (The Galvanic Circuit Investigated Mathematically) (1827) in which he gave his complete theory of electricity. According to him current through a resistor is directly proportional to the voltage applied on it. This is known to us as Ohm's law. Ohm's law is a basic law of circuit theory analysis in electronic engineering. It shows a linear voltage –current relationship for any conducting channel.

Primarily the value of the critical voltage V_c is assumed to be infinity and it is unreachable for the voltage V across a resistor in laboratory environment and hence the circuit behavior is linear. Greenberg and del Alamo [3] at Massachusetts Institute of Technology (MIT), Cambridge, carried out an experiment revealing that the critical

electrical field is 3.8 kV/cm for an InGaAs micro-channel with a derived mean free path of 68 nm. Considering a macro-resistor of InGaAs with $L= 1.0 \text{ cm}^2$, the critical voltage is $V_c=3.8 \text{ kV}$. This is quite high for any practical voltage in macro size devices. With $V_c = \infty$ the validity of Ohm's law is confirmed under normal circuit simulation, such as SPICE.

The length of conducting channel is decreasing day by day in the invention of new era of modern world which has entered the decananometer regime with a typical length of the order of 10-100 nm. Some features that are normally ignored in the long channel devices shows some special characteristics in nanoscale regime. One such effect is the swell in the resistance of a signal due to saturated current [1]. This increase in resistance affects all timing delays that are normally based on Ohm's law where resistance is constant due to current rising linearly with the applied voltage. Greenberg and del Alamo [3], at al Massachusetts Institute of Technology (MIT), Cambridge unveil the velocity and current saturation occurring in the extrinsic source and drain contacts resulting in resistance rise.

However, the linear relation between the current and voltage is not valid forever. As the critical voltage is finite, the current response to applied voltage is sublinear resulting in current saturation I_{sat} when $V \gg V_c = I_{sat} R_o$. In macro-dimensional circuits, both I_{sat} and V_c are very large, ideally infinity. However, for a micro- or nano-processor where the length is a few nanometers, both I_{sat} and V_c are very small. Critical voltage is typically fraction of a volt. If we reduce a length of resistive channel to $L=1.0 \text{ }\mu\text{m}$, for example, the critical voltage drops to 0.38 V. The applied logic voltage of 5 V—or even of 1.8 V in the VLSI circuit currently used in Intel technology—is now much larger than the critical voltage, and hence Ohm's law fails.

With 5 volt binary logic, the apparent rise in resistance (the inverse slope of current-voltage I-V graph) makes traditional circuit design invalid. As I-V curves become sublinear, the distinction between the direct resistance $R=V/I$ and incremental

resistance $r=dV/dI$ is necessary. The incremental resistance increase in r is much higher than in R over and above its ohmic value R_o . The surge in R and r changes the time constants, power consumption, voltage and current division laws. The transient RC switching delay in micro/nano-scale circuit is strongly affected by the surge in the resistance arising out of sub-linear current-voltage (I-V) characteristics.

The microcircuit resistors must be characterized by any two of the three parameters: Ohmic resistance R_o ; the critical voltage V_c , and the saturation (maximum) current in the resistor I_{sat} . These three parameters are connected through. Even though the operational devices in the VLSI domain are transistors, particularly the CMOS inverter, the concepts are easy to explain to the students using familiar resistive channels [8].

1.2 Background of Study

When beginning to explore the world of electricity and electronics, it is vital to start by understanding the basics of voltage, current, and resistance. These are the three basic building blocks required to manipulate and utilize electricity. Georg Ohm (1827) was a Bavarian scientist who studied electricity.

In January 1781, before Georg Ohm's work, Henry Cavendish experimented with Leyden jars and glass tubes of varying diameter and length filled with salt solution. He measured the current by noting how strong a shock he felt as he completed the circuit with his body. Cavendish wrote that the "velocity" (current) varied directly as the "degree of electrification" (voltage). He did not communicate his results to other scientists at the time [15] and his results were unknown until Maxwell published them in 1879 [16]. Ohm did his work on resistance in the years 1825 and 1826, and published his results in 1827 as the book *Die galvanische Kette, mathematisch*

bearbeitet ("The galvanic circuit investigated mathematically") [17] He drew considerable inspiration from Fourier's work on heat conduction in the theoretical explanation of his work. For experiments, he initially used voltaic piles, but later used a thermocouple as this provided a more stable voltage source in terms of internal resistance and constant potential difference. He used a galvanometer to measure current, and knew that the voltage between the thermocouple terminals was proportional to the junction temperature. He then added test wires of varying length, diameter, and material to complete the circuit.

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. Introducing the constant of proportionality, the resistance, one arrives at the usual mathematical equation that describes this relationship [18]

$$I = \frac{V}{R} \quad 1.1$$

Where I is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the R in this relation is constant, independent of the current [18]

Ohm's law states a linear relationship between voltage and current through a traditional resistive channel and it is vastly used for any circuit theory analysis in electrical engineering. It can evaluate any customary resistor and that is independent of the voltage applied across its length, L . We can assume the critical voltage V_c in any traditional resistor is infinity. This is in practice an unreachable value for a voltage applied across the length of the resistor in the laboratory environment and hence Ohm's law is valid under normal circuit simulation such as spice [2].

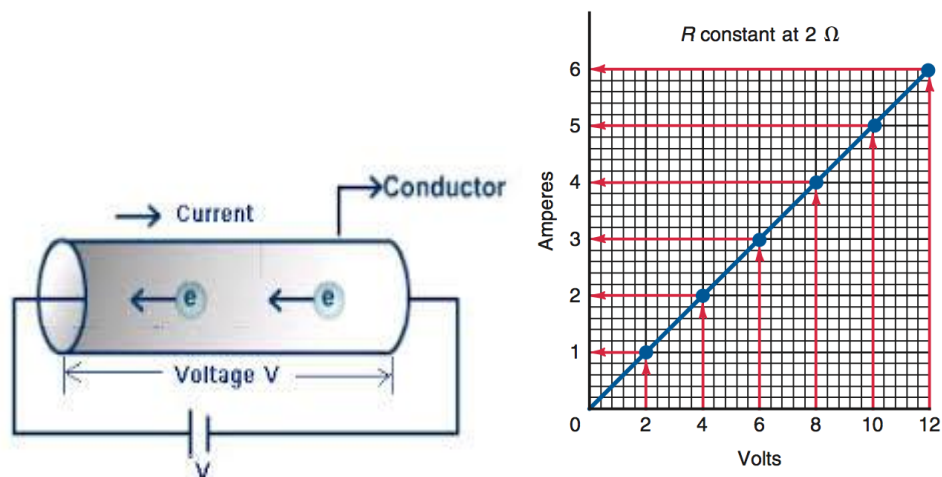


Fig. 1.1 Typical Resistor with Applied voltage V and Current-voltage linear relationship (19)

The number of transistors in successive generations of computer chips has risen exponentially or doubling every 1.5 years. This growth pattern had been predicted by Gordon Moore, the co-founder of Intel, Inc. in 1965 when the silicon chip only contained 30 transistors. The growth is largely due to continuing reduction in the size of the key elements in the devices, to about 10nm recently, with improvements in optical photolithography. Intense research is continuing to scale the silicon devices to smaller dimensions for the purposes of higher packing density, faster circuit speed and lower power dissipation. Devices such as metal oxide semiconductor field-effect transistor (MOSFET) are rapidly being scaled down. When MOS technology was developed in the 1960's, channel lengths were about 10 micrometers, but researchers are now building transistors with channel lengths of less than 10 nanometers.

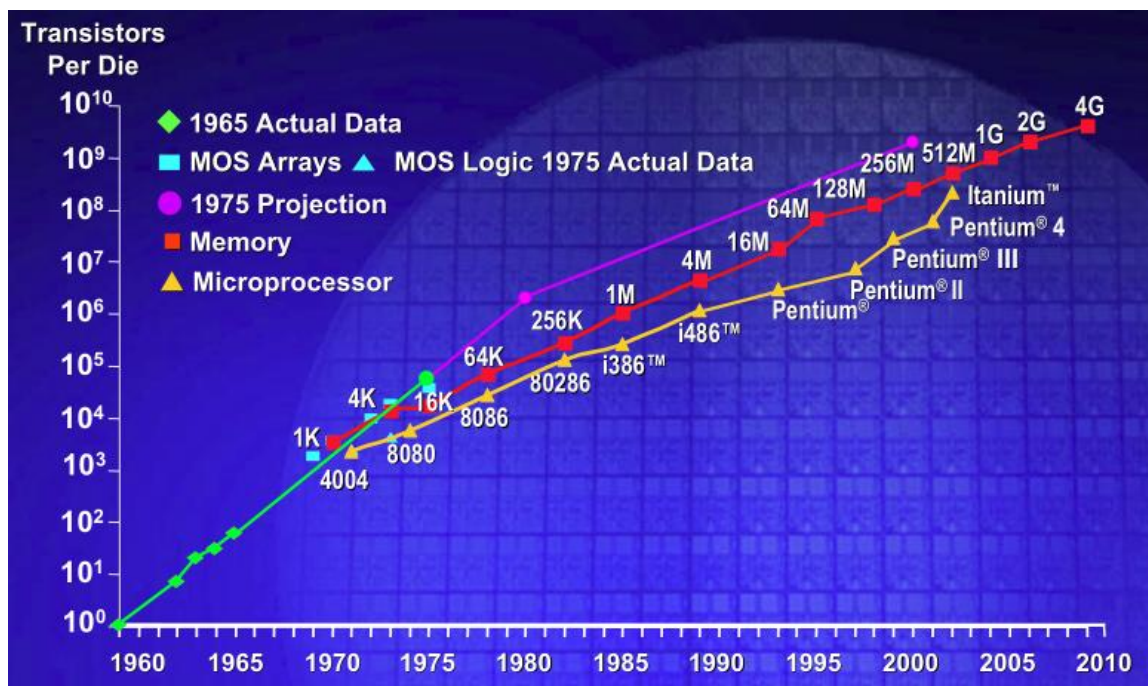


Fig. 1.2 Growth of the Number of Transistor as a Function of Years as Predicted By Gordon Moore. (21)

In this nanoscale regime linearity of conducting channel is not valid forever. In the short channel devices critical voltage is a fraction of a volt. However, the dimensions of most electronic devices in a VLSI configuration on a typical $1 \times 1 \text{ cm}^2$ chip are now on the micro- or nanoscale. When a resistive channel is reduced to $L=1 \text{ }\mu\text{m}$, for example, the critical voltage drops to 0.38 V. The applied logic voltage of 5 V—or even of 1.8 V in the VLSI circuit currently used in Intel technology—is now much larger than the critical voltage, and when the applied voltage V crosses the critical voltage V_c the device shows some special characteristics such as suddenly resistance rise and nonlinear behavior and hence Ohm's law fails [2]. Thus Ohm's law is broken down and affect device's properties.

1.3 Problem Statement

Ohm's law is the basic concept of circuit theory in both digital and analogue electronic which states that the current response to the applied voltage across a length of resistor is linear. In general, we are vastly experienced with linear relationship between the current and voltage due to the critical voltage very large, ideally infinity. However, the linear relation between the current and voltage is not valid forever and when critical voltage is finite its behavior is changed to nonohmic [2]. The properties of customary resistor is characterized by its Ohmic value that might be independent of the applied voltage across its length L for long channel devices. Nonohmic behavior becomes distinctly visible when applied voltage V is larger than the critical voltage $V_c = V_t L / \ell_{\infty}$ ($V > V_c$), V_t is the thermal voltage with value 0.0259 V at room temperature and ℓ_{∞} (typically 100 nm) is the mean free path (mfp) in a nanoscale ($L < 1000\text{nm}$) device. The breakdown of Ohm's law affects heavily the current of transporting carrier in a nanoscale device [1]. The surge in R and r changes the time constants, power consumption, voltage and current division laws. The transient RC switching delay in micro/nano-scale circuit is strongly affected by the surge in the resistance arising out of sub-linear current-voltage (I-V) characteristics.

1.4 Objective

The goal is to investigate and analyse the critical voltage of nano-scaled structure and observe the failure of Ohm's law. The direct and incremental resistance surge will be realised. The objective of this study are:

- To study factors affecting the critical voltage beyond which Ohm's law fails in scaled-down nanostructures.
- To investigate the cause of rise in resistance with the applied DC voltage.

- To apply the theory to 1D silicon nanowire, 2D AlGaAs nanowire and 3D bulk resistor
- To study the mechanism of current saturation

1.5 Scope

The research will focus mainly on the critical voltage beyond which Ohm's law fails in scaled-down nanostructures. The direct and incremental resistance rise with the applied dc voltage will be investigated. The implication of breakdown of Ohm's law on potential and current divider circuits will be analyzed in this study. In this research, the application of applied Mathematic-MATLAB simulation will be carried out to design an implemented circuit modeling. The failure of Ohm's law which is in the nanoscale regime is investigated.

- Analysis of the effects of breakdown of Ohm's Law to the fundamentals circuit such as voltage and current divider.
- Comprehension of the connections between velocity saturation and ohmic mobility.
- Development of capability for data processing using hardware and software.

1.6 Organization of the Project

This work will focus on the failure of Ohm's law in the nano-scaled regime for applied voltage beyond the critical voltage. Chapter one will discuss the overview of the project and then in the chapter two explains the related work and literature review.

In chapter three the strategy of whole work will be focused. Chapter four will discuss results and some other discussions. And lastly in chapter five a conclusion and future application will be brought out.

REFERENCES

- [1] V. K. Arora, "Nanoelectronics, Quantum Engineering of Low Dimensional nanoensemble" Wilks University, Universiti Teknologi Malaysia, 2014.
- [2] T. Saxena, D. C. Y. Chek, M. L. P. Tan, and V. K. Arora, "Microcircuit Modeling and Simulation Beyond Ohm's Law," IEEE Transactions on Education, 2008.
- [3] D. R. Greenberg and J. A. del Alamo, "Velocity saturation in the extrinsic device: A fundamental limit in HFET's," IEEE Trans. Electron Devices, vol. 41, no. 8, pp. 1334–1339, Aug. 1994.
- [4] M. L. P. Tan, T. Saxena, and V. K. Arora, "Resistance Blow-Up Effect in Micro-Circuit Engineering," Solid State Electronics, vol. In Press, 2010.
- [5] B. L. Anderson and R. L. Anderson, Fundamentals of Semiconductor Devices. New York: McGraw-Hill, 2005.
- [6] V. K. Arora, et al., "Transition of equilibrium stochastic to unidirectional velocity vectors in a nanowire subjected to a towering electric field," Journal of Applied Physics, vol. 108, pp. 114314-8, 2010.
- [7] V. K. Arora and M. B. Das, "Effect of electric-field-induced mobility degradation on the velocity distribution in a submicron-length channel of InGaAs/AlGaAs heterojunction MODFET," Semicond. Sci. Technol., vol. 5, p. 967, 1990.
- [8] V. K. Arora, "Quantum engineering of nanoelectronic devices: The role of quantum emission in limiting drift velocity and diffusion coefficient," Microelectron. J., vol. 31, no. 11–12, pp. 853–859, 200

- [9] Z. Yao, C. L. Kane, and C. Dekker, "High-Field Electrical Transport in Single-Wall Carbon Nanotubes," *Physical Review Letters*, vol. 84, pp. 2941-2944, 2000.
- [10] M. T. Ahmadi, R. Ismail, and V. K. Arora, "The ultimate ballistic drift velocity in a carbon nanotubes," *J. Nanomater.*, p. 8, 2008.
- [11] M. L. P. Tan, V. K. Arora, I. Saad, M. T. Ahmadi, and R. Ismail, "The drain velocity overshoot in an 80-nm metal-oxide-semiconductor field-effect-transistor," *J. Appl. Phys.*, vol. 105, p. 074503, 2009.
- [12] I. Saad, M. L. P. Tan, A. C. E. Lee, R. Ismail, and V. K. Arora, "Scattering-limited and ballistic transport in a nano-CMOS circuit," *Microelectron. J.*, vol. 40, pp. 581–583, 2009.
- [13] I. Saad, R. Ismail, and V. K. Arora, "Investigation on the effects of oblique rotating ion implantation (ORI) method for nanoscale vertical double gate NMOSFET," *Solid State Sci. Technol. Lett.*, vol. 15, no. 2, pp. 69–76, 2008
- [14] <https://learn.sparkfun.com/tutorials/voltage-current-resistance-and-ohms-law>
- [15] "Electricity". *Encyclopædia Britannica*. 1911.
- [16] Sanford P. Bordeau (1982) *Volts to Hertz...the Rise of Electricity*. Burgess Publishing
- [17] G. S. Ohm (1827). *Die galvanische Kette, mathematisch bearbeitet*. Berlin: T. H. Riemann.
- [18] Robert A. Millikan and E. S. Bishop (1917). *Elements of Electricity* American Technical Society. p. 54.
- [19] https://www.google.com.my/search?q=ohm%27s+law&biw=1252&bih=557&tbm=ch&source=lnms&sa=X&ei=Zl2SVMKzGM3IuATl5ICIBA&ved=0CAgQ_AUoAQ&dpr=1.09#tbm=isch&q=ohm%27s+law+graph&revid=459693739
- [20] V. K. Arora, "High-field distribution and mobility in semiconductors," *Japanese Journal of Applied Physics, Part 1: Regular Papers & Short Notes*, vol. 24, pp. 537-545, 1985.
- [21] <http://hightechforum.org/100-years-of-moores-law/>

- [22] V. K. Arora, "High-field distribution and mobility in semiconductors," Japanese Journal of Applied Physics, Part 1: Regular Papers & Short Notes, vol. 24, pp. 537-545, 1985.