APPLICATION OF SIMILARITY SOLUTION ON GASEOUS POLLUTANT PROBLEMS

NG SIN YI

UNIVERSITI TEKNOLOGI MALAYSIA

APPLICATION OF SIMILARITY SOLUTION ON GASEOUS POLLUTANT PROBLEMS

NG SIN YI

A report submitted in partial fulfillment of the

requirements for the award of the degree of

Master of Science (Engineering Mathematics)

FACULTY OF SCIENCE

UNIVERSITI TEKNOLOGI MALAYSIA

FEBRUARY 2015

To my beloved father, Mr. Ng Kheng Keong, mother, Mrs. Lim Poh Peng, brother, sister and all of my dear friends for their love and support.

ACKNOWLEDGEMENT

In preparing this thesis, I was exposed to many journals and articles of the researchers. They have contributed towards my understanding and thoughts. In particular, I wish to express my gratefulness and sincere appreciation to my supervisor, Professor Dr. Mohd Nor Mohamad, for encouragement, guidance, critics and friendship throughout the duration of this study. Without his interest and willingness to allocate his precious time to advise me when I am facing problems in completing this report, this thesis would not have been the same as presented here.

I am also indebted to Universiti Teknologi Malaysia (UTM) for supplying the sources for relevant literatures. My sincere appreciation also extends to all my colleagues and others who have provided assistance at various occasions. Their views and tips are useful indeed. Unfortunately, it is not possible to list all of them in this limited space. I am grateful to all my family members for giving financial and spiritual support to the contribution of this thesis.

ABSTRACT

This study concentrates on dispersion of the gaseous pollutants. The diffusion equation is used to model the concentration from a various distance by the temperature invariants. This study features the group-theoretical method form the similarity techniques. This method is applicable and appropriate to nonlinear partial differential equations. From the similarity solutions, a one-parameter group of transformation provides the equation to be conformally invariance. By introducing of similarity variables, the nonlinear partial differential equation can be reduced to a second order differential equation. Applying boundary conditions and assuming the arbitrary constant for some unknowns, a particular solution of the diffusion equation is obtained. The result is shown for two cases where no pollutant is absorbed by the ground and where all pollutants are absorbed by the ground.

ABSTRAK

Kajian in tertumpu kepada penyerakan oleh pencemaran gas. Persamaan penyebaran digunakan untuk permodelan kepekatan daripada pelbagai jarak dengan suhu yang tidak berubah. Kajian ini bercirikan kaedah teori kumpulan membentuk teknik seiras. Kaedah ini adalah sesuai dan diapliksikan pada persamaan pembezaan separa yang tidak linear. Daripada penyelesaian similariti, transformasi kumpulan dengan satu-parameter membekalkan persamaan tersebut sebagai konformal tetap. Dengan pengenalan pemboleh ubah seiras, persamaan pembezaan separa yang tidak linear boleh dikurangkan kepada satu persamaan pembezaan susunan kedua. Pengaplikasian keadaan sempadan dan jangkaan untuk pemalar yang rambang bagi beberapa anu, suatu persamaan tertentu bagi penyelesaian untuk persamaan penyebaran telah diperolehi. Hasil kajian ditunjukkan untuk dua kes yang berbeza iaitu tiada pencemaran diserap oleh tanah dan semua pencemaran diserap oleh tanah.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

C	-	Concentration
K_1, K_2	-	Diffusion coefficient
x, y	-	Distance
u	-	Mean velocity
u_0	-	Reference velocity
h	-	Height
η	-	Similarity variable
ε	-	phase shift constant

CHAPTER 1

INTRODUCTION

1.1 Introduction

Engineering problems involving a continuum, such as the elastic behaviour of solids, the motion of fluids and the movement and transport of heat and contaminants, are most often modelled with partial differential equations. In actual fact, all of these models are an approximation to reality, but the correspondence between solutions of these equations and reality is often so close that these models can be used both to study the physical behaviour of reality and to also predict future behaviour for engineering design purposes. Therefore, even students who may never actually calculate a new and original solution to a set of equations will find exposure to this topic helpful for two reasons: firstly, it provides an efficient way to study and understand the physical behaviour of engineering problems, and, secondly, analytical and numerical solutions of partial differential equations provide the basis for a large number of computer software programs that are used for engineering designs.

A number of methods for obtaining the numerical solutions can be used to solve the problems in science and engineering fields. Thus, we have to choose a suitable method to work out the problem but at the same time we need to try to reduce the size of the system of the equations and also the computation time while maintaining its accuracy. With the assist of some computer system or symbolic computation software such as MATLAB, C programming, FORTRAN, MAPLE and so on, it is helpful in compiling the numerical result and to get highly accurate result within a short period of time.

1.2 Background of the Study

A differential equation is an equation that involves specified derivatives of an unknown function, its values, and known quantities and functions. Many physical laws are most simply and naturally formulated as differential equations. For this reason, differential equations have been studied by many great mathematicians and mathematical physicists since the time of Newton. Ordinary differential equations are differential equations whose unknowns are functions of a single variable; they arise most commonly in the study of dynamical systems and electrical networks. They are much easier to treat than partial differential equations, whose unknown functions depend on two or more independent variables. Ordinary differential equations are classified according to their orders. The order of a differential equation is defined as the largest positive integer, n, in which the highest derivative, the nth order, occurs in the equation. Thus, an equation of the form $\emptyset(x, y, y') = 0$ is said to be of the first order. A differential equation of the form $t^2y'' + aty' + by = 0, \mathrm{t} > 0$ where a, b are real constants, is called Euler's equation, and it is a second order equation.

Differential equations have wide range of applications in various engineering and science disciplines. In general, modeling of the variation of a physical quantity, such as temperature, pressure, displacement, velocity, stress, strain, current, voltage, or concentration of a pollutant, with the change of time or location, or both would result in differential equations. Similarly, studying the variation of some physical quantities on other physical quantities would also lead to differential equations. In fact, many engineering subjects, such as mechanical vibration or structural dynamics, heat transfer, or theory of electric circuits, are founded on the theory of differential equations. It is practically important for engineers to be able to model physical problems using mathematical equations, and then solve these equations so that the behavior of the systems concerned can be studied.

Linear and nonlinear partial differential equations arise in various fields of science and numerous applications, e.g., heat and mass transfer theory, wave theory, hydrodynamics, aerodynamics, elasticity, acoustics, electrostatics, electrodynamics, electrical engineering, diffraction theory, quantum mechanics, control theory, chemical engineering sciences, and biomechanics.

Differential equations, both ordinary and partial, are sometimes invariant to groups of algebraic transformations, and these algebraic invariances are also symmetries. About a hundred years ago, a Norwegian mathematician, Sophus Lie, hit upon the idea of using the algebraic symmetry of ordinary differential equations to aid him in their solutions. In the course of his work, he achieved two profoundly important results: he showed how to use the knowledge of the transformation group to construct an integrating factor for first-order ordinary differential equations and to reduce second-order differential equations to first order by a change of variables. These two results are all the more important because they do not depend on the equation’s being linear. (Helgason (1994))

1.3 Statement of the Problem

In engineering and science world, there are many problems that are difficult to solve. If these problems are modelled as ordinary differential equations, then the problem may become easier to solve. We have been accustomed to solving ordinary differential equations by obtaining the sum of a general solution and a particular solution and then using either initial or boundary conditions to determine unknown constants in the general solution. The general solution of a partial differential equation, however, contains unknown functions with specified arguments, and the determination of these unknown functions is not always easy. Furthermore, general solutions of second-order partial differential equations can be found only for a small number of the very simple equations. In problems involving partial differential equations, a solution method that work for one problem may not work for slightly different problems.

There are two problems need to be solved in this study.
Problem 1: there is no pollutant absorbed by the ground.
Problem 2: all pollutants are absorbed by the ground.

Group-Theoretical Method forms the similarity techniques has been discovered in order to reduce the nonlinear second order partial differential equation into second order ordinary differential equation. In short, group theoretical method is a powerful method applicable in solving nonlinear differential models.

1.4 Objectives of the Study

The objectives of the study are as follow:

1. To apply a parameter group of transformation on diffusion equation.
2. To predict the pollutant concentration from a source by application of similarity techniques.
3. To analyze the numerical results for two problems using MAPLE software.

1.5 Scope of the Study

This study will focus on the dispersion of the gaseous pollutants. The governing equation describes on the phenomena in the partial differential equations. By using similarity solution techniques, the equation can be reduced to the second order ordinary differential equation. The later equation is then solved by MAPLE.

1.6 Significance of the Study

The result of this research will give benefits to mathematics and engineering fields. Many engineering problems involve nonlinear problems and as we know nonlinear equations are difficult to be solved analytically compared with linear equations. Therefore, the mathematical concept, analytic method or mathematic tools such as group-theoretical method form the similarity techniques and MAPLE program are used to simplify the complicated problems. Sedov (2009) had proved that many physical applications were successfully solved by applying similarity solution into the equations.

1.7 Outline of Report

The aim of this report is to apply the similarity solutions for solving some nonlinear problems. This report consists of six chapters and it is organised as follows. In Chapter 1, the introduction, describing background of the problem, statement of the problem, objectives of the study, scope and significance are demonstrated. In Chapter 2, the concept, theory and review of diffusion equation and group-theoretical method are discussed. Similarity concepts, transformation group and concept of invariance are introduced in Chapter 2 as well. Chapter 3 discussed about the mathematical model and solution steps of similarity solution. Chapter 4 reviewed on numerical results that obtained from software or tools. Some graphs may use to present the numerical result. In the last chapter which is Chapter 5, the study is concluded and summarized. A number of useful recommendations are also suggested for the purpose of further research.

REFERENCE

Abd-el-Malek M.B. (1997) Group Method Analysis of Nonlinear Temperature Variation Across the Lake Depth. Proc. XXI International Colloquium on Group Theoretical Methods in Physics Group 21 (Goslar, Germany. 15-20 July, 1996), World Scientific, Singapore, 255–260.

Abd-el-Malek M. B. (1998) Application of the Group-Theoretical Method to Physical Problems. Journal of Nonlinear Mathematical Physics, V.5, N 3, 314–330

Altshuller A.P. and Taft R. A. (1958) Natural Sources of Gaseous Pollutants in The Atmosphere. Tellus 10(4): 479-492

Ames W. F. (1965) Similarity for the Nonlinear Diffusion Equation. Department of Mechanical Engineering, University of Delaware, Newark, Del. Vol. 4 No. 1

Ayres J.G. (1998) Health effects of gaseous air pollutants. In: Hester RE, Harrison RM, editors. Air Pollution and Health. Cambridge: The Royal Society of Chemistry: 1–20.

Badran N.A. and Abd-el-Malek M.B., (1993) Group Method Analysis of the Dispersion of Gaseous Pollutants in the Presence of a Temperature Inversion. Proc. Modern group analysis: Advanced Analytical and Computational Methods in Mathematical Physics (Acireale, Catania, Italy 1992), 35–41, Kluwer Acad. Publ., Dordrecht.

Birkhoff G. (1948) Mathematics for Engineers III: Dimensional Analysis of Partial Differential Equations. Harvard University, Cambridge, Mass: 1185-1188

Bluman G. W. and Cole J. D. (1969) The General Similarity Solution of Heat Equation. Journal of Mathematics and Mechanics, Vol. 18, No. 11

Boutros Y.Z. et al. (1998) Group Method for Temperature Analysis of Thermal Stagnant Lakes, Acta Mech.

Debnath L. (2012) Nonlinear Partial Differential Equations for Scientists and Engineers Third Edition DOI 10.1007/978-0-8176-8265-1_1, © Springer Science+Business Media, LLC

Deconinck B. (2006) Introduction to Differential Equations and Applications. Department of Applied Mathematics University of Washington Campus Box 352420 Seattle, WA, 98195, USA

Djordjevic V.D., Atanackovic T.M. (2008) Similarity solutions to nonlinear heat conduction and Burgers/Korteweg–deVries fractional equations. Journal of Computational and Applied Mathematics 222: 701–714

Everstine G. C. (2012) Analytical Solution of Partial Differential Equations. Gaithersburg, Maryland

Helgason S. (1994). Sophus Lie, the mathematician. In Contenido en Proceedings of The Sophus Lie Memorial Conference, Oslo (pp. 3-21).

Gittins M. J. (1999) Air pollution. In: Bassett WH. Clay's Handbook of Environmental Health, 18th ed. London: E & FN Spon: 729–76

Higgins G. (2012) Similariry solutions to Partial differential equations: IntroSimilarityMethod2.nb

Megainey C. (1999) Introduction to environmental protection. In: Bassett WH. Clay's Handbook of Environmental Health. London: E & FN Spon: 684–703

Mehrer H. (2009) Heroes and Highlights in the History of Diffusion. Diffusion-fundamentals.org 11, pp 1-32

Moran M. J. and Gaggioli R. A. (1968) Reduction of The Number of Variables in Systems of Partial Differential Equations, with Auxiliary Conditions. Siam J. Appl. Math Vol. 16, No. 1

Na T. Y. and Hansen A. G. (1971) Similarity Analysis of Differential Equations by Lie Group. Journal of the Franklin Institute Vol. 292, No. 6

Neta B. (2009) Partial Differential Equations Ma 3132 Lecture Notes. Department of Mathematics Naval Postgraduate School Code MA/Nd Monterey, California 93943

Olver P. J. and Rosenau. P (1987) Group-Invariant Solutions Of Differential Equations. Siam J. Appl. Math. @ 1987 Society for Industrial and Applied Mathematics Vol. 47, No. 2

Patel M. et al. (2013) Group Invariance for Non-Linear Partial Differential Equations. Int. J. of Appl. Math and Mech. 9 (13): 83-102

Polianin A. D. (2002) Handbook of Linear Partial Differential Equations for Engineers and Scientists. Chapman & Hall/CRC QA377 .P568

Sachev, P. L. (2000) Self-similarity and Beyond: Exact Solutions of Nonlinear Problems. Chapman & Hall/ CRC Monographs and Surveys in Pure and Applied Mathematics; 113

Schwela D. (2000) Air pollution and health in urban areas. Rev Environ Health. 12(1–2):13–42

Seshadri R. and Na T.Y. (1985) Group Invariance in Engineering Boundary Value Problems. Springer-Verlag, New York

Sheth S. S. (2013) Similarity Solution of Dispersion of Gaseous Pollutants in the Presence of Temperature Inversion. Int. J. of Appl. Math and Mech. 9 (15): 53-61

Socolofsky S. A. & Jirka G. H. (2002) Environmental Fluid Mechanics Part I: Mass Transfer and Diffusion Engineering- Lectures 2nd Edition University Karlsruhe 76128-Karlsruhe, Germany

Tracy C. A. (2012) Lectures on Differential Equations Department of Mathematics University of California Davis, CA 95616

Vallero D. A. (2008) Fundamentals of Air Pollution. 4th ed. London: Elsevier Inc