A NOVEL MATHEMATICAL MODEL OF BLOOD FLOW

AHMAD KHUDZAIRI BIN KHALID

A dissertation submitted in partial fulfillment of the

requirements for the award of the degree of

Master of Science (Mathematics)

Faculty of Science

Universiti Teknologi Malaysia

JUNE 2014

Specially dedicated to my beloved father and mother,

Khalid Bin Jusoh and Kalthom Binti Atan,

to my siblings

and

those people who have guided and inspired me throughout my journey of education

ACKNOWLEDGEMENTS

All praise to Allah, The Most Gracious The Most Merciful. I hereby placing all my faith and thankfulness of His greatness all that he piled on me, as I could afford to complete my task timely.

In order to complete my final year project, I have a lot of assistance from many people especially my supervisor. In the first place I wish to express my sincere appreciation to my beloved supervisor, PM Dr Mukheta Isa, for his willingness to provide me with guidance, knowledge suggestions, valuable comments, constructive criticisms and encouragement. Without his continued support and interest, this project would not have been completed in the stated time.

I am also thankful to all my family members especially my loving parents, Khalid Bin Jusoh and Kalthom Binti Atan, for their patience and support in my journey of education. They have given me so much comfort, care and love, either financially or spiritually, of which word could not express and will forever be remembered in my heart.

Besides, i would like to thank to the Librarian at Faculty of Science also Librarian at Universiti Teknologi Malaysia who supplies guidance to me in process of finding relevant books. Last but not least, I would like to thank all my fellow friends especially Rachel Aswin Bin Jominis and Azmirul Bin Ashaari who had given me their support and assistance to enable the completion of this research.

Thank You

ABSTRACT

The interface between mathematics and biology has initiated and fostered new mathematical areas, where the ideas from mathematics and biology are synergistically applied. Study of fluid dynamics plays a significant role in fluid flow inside the human body, and modeling of blood flow is an important field in cardiovascular physics. However, models have been developed so far are very complex with three dimensional analysis. This project presents a novel and simple mathematical model of blood flow. The main fluid component of the cardiovascular system is the body. Assuming blood is a Newtonian fluid which is governed by the Navier-Stokes equations and continuity equation and with making use of the Navier-Stokes equation, a simple differential equation called as the Cardiovascular System equation is derived. Then by applying the logical assumptions on this model, the general mathematical model of the normal blood flow rate is developed. Using Poisuelli's equation, the Cardiovascular System equation is also used to develope a model for blood pressure. These two models are then analyzed against surface, pressure gradient and the vessel's length using MAPLE 13. Our results are in agreement those obtained by Sanjeev, Chandel and Harjeet (2011) in A Mathematical Model for Blood Flow and Cross Sectional Area of an Artery.

ABSTRAK

Antara muka antara matematik dan biologi telah memulakan dan membentuk kawasan matematik baru, di mana idea-idea daripada matematik dan biologi secara sinergi digunakan. Kajian dinamik bendalir memainkan peranan penting dalam aliran bendalir di dalam tubuh manusia, dan pemodelan aliran darah adalah tetapan yang penting dalam fizik kardiovaskular. Walau bagaimanapun, model telah dibangunkan setakat ini adalah sangat kompleks dengan tiga analisis dimensi. Projek ini membentangkan model matematik novel dan mudah aliran darah. Komponen utama cecair sistem kardiovaskular adalah badan. Menganggap darah adalah cecair Newtonian yang ditadbir oleh persamaan Navier Stoke dan persamaan kesinambungan dan dengan menggunakan persamaan Navier Stoke, persamaan perbezaan mudah dipanggil sebagai persamaan sistem kardiovaskular telah dihasilkan. Kemudian dengan menggunakan andaian logik pada model ini, model matematik umum kadar aliran darah yang normal dibangunkan. Dengan menggunakan persamaan Poisuelli ini, persamaan Sistem Kardiovaskular juga digunakan untuk membangunkan model untuk tekanan darah. Kedua-dua model kemudiannya dianalisis terhadap permukaan, kecerunan tekanan dan panjang saluran menggunakan MAPLE 13. Keputusan kami adalah dalam perjanjian yang diperolehi oleh Sanjeev, Chandel dan Harjeet (2011) di dalam “A Mathematical Model for Blood Flow and Cross-Sectional Area of an Artery”.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

μ	-	Kinematic viscosity of blood
$\frac{dP}{dz}$	-	Pressure gradient
Q	-	Volumetric flow rate
P	-	Pressure
t	-	Time
F	-	Force
M	-	Mass
V	-	Volume
$\boldsymbol{v}$	-	Velocity vector
d**S**	-	Element of the surface
ρ	-	Density of blood
R(z,t)	-	Radius of blood vessel
L	-	Length of blood vessel

w(γ,z,t)	-	The axial velocity component
f(γ,z,t)	-	The radial velocity component
$\frac{D}{Dt}$	-	Substantive derivative
∇^2	-	Laplacian operator
F, F_s, F_i	-	A vector body force, its components
π	-	Pi
τ	-	Shear stress
S	-	Cross sectional area of the blood vessel

LIST OF TERMINOLOGYS

Blood	-	the fluid and its suspended formed elements that are circulated through the heart, arteries, capillaries, and veins; blood is the mean by which 1) oxygen and nutritive materials are transported to the tissues, and 2) carbon dioxide and various metabolic products are removed for excretion. The blood consists of a pale yellow or gray-yellow fluid, plasma, in which are suspended red blood cells (erythrocytes), white blood cells (leukocytes), and platelets.
Cardiac cycle	-	the complete round of cardiac systole and diastole with the intervals between, commencing with any event in heart's action and ending when same event is repeated.
Cardiovascular	-	relating to the heart and the blood vessels or the circulation.
Diastole	-	normal postsystolic dilation of the heart cavities, during which they fill with blood; diastole of the atria precedes that of the ventricles; diastole of either chamber alternates rhythmically with systole or contraction of that chamber.
Diastolic	-	relating to diastole.

Laminar flow	-	the relative motion of elements of a fluid along smooth parallel paths, which occurs at lower values of Reynolds number.
Systole	-	contraction of the heart, especially of the ventricles, by which the blood is driven through the aorta and pulmonary artery to tranverse the systemic and pulmonary circulations, respectively; its occurrence is indicated physically by the first sound of the heart heard on auscultation, by the palpable apex beat, and by the arterial pulse.
Systolic	-	relating to, or occuring during cardiac systole.
Vessel	-	a structure conveying or containing a fluid, especially a liquid.
Viscous	-	sticky; marked by high viscosity.

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

According to Bender (2000), a mathematical model is a description of a system using mathematical concept and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models is used not only in the natural science and engineering disciplines, but also in the social sciences. A model may help to explain a system and to study the effects of different components and to make predictions about behaviour.

Blood flow is the continuous circulation of blood in the cardiovascular system. The cardiovascular system in the body consists of three component which is blood, heart, and blood vessel. This process blood flow in the cardiovasuclar system ensures the transportation of nutrients, hormones, metabolic wastes, oxygen, and carbon dioxide through the body to maintain cell, level metabolism, the regulation of the pH osmotic pressure and temperature of the whole body, and the protection from microbial and mechanical harms (Gerard and Bryan, 2012). The science dedicated to describe the physics of blood flow is called hemodynamics.

For the basic understanding it is important to be familiar with anatomy of the cardiovascular system and hydrodynamics. However it is crucial to mention that blood is the non-Newtonian fluid and blood vessels are not rigid tubes (Fieldman, Phong, Aubin, and Vinet, 2007). In this study, we considered that the blood is behaves like a Newtonian fuid which is governed by the Navier-Stokes equations and the continuity equation (Rahman and Haque, 2012).

In many previous analytical studies in concept with blood flow have been carried out in the recent past, the non linear velocity profile of pulsatile flow of blood through the descending aorta have been satisfactorily investigated by Ling and Atabek (1972) and Imaeda and Goodman (1980) who treated the artery as an uniformly tapered thick cylindrical tube of isotropic and incompressible material. Moreover, some of studies have been performed on the measurement of electrical activity (Faris, Evans and etc, 2003), deformation (Masood, Yang, Pennell and Firmin, 2000), flow (Kilner, Yang, Wilkes and Mohiaddin, 2000), modeling of the heart (Shoaib, Haque, and Asaduzzaman, 2010), and computational design of cardiac activity in the body (Rahman, 2011).

In this research, our aim is to models blood flow of the cardiovascular system, blood flow rate, and blood pressure. In whole literature, some assumptions have been considered, which include that although the blood vessels are different in size, they are all considered being cylindrical shaped and deformable components with circular cross-sections. They expand as the blood enters into and contract as the blood leaves it. Although the blood needs the help of the lungs for the supply of oxygen, its properties remain unchanged by the addition of that oxygen. Another assumption is required, and that is, the blood has both the radial and axial flow in only one direction z-direction in a three dimensional system. So, the other two components x and y-direction are vanished.

1.2 Basic Concepts

This project is concerned on how to develop solutions to the governing equations used to describe incompressible flows. It begins by applying the continuity and Navier-Stokes equations to analyze steady, laminar flow problems involving a constant density and constant viscosity fluid. Before discussing the problem to be discussed in this project, it is necessary to understand a few basic concepts that are related to blood flow. In the next section will discuss briefly about the blood flow in cardiovascular system.

1.3 Blood Flow in Cardiovascular System

According to Campbell and Reace (2005), beginning with the pulmonary (lung) circuit, the right ventricle pumps blood to the lung via the pulmonary arteries. As the blood flows through capillary beds in the left and right lungs, it loads oxygen and unloads carbon dioxide. Oxygen rich blood returns from the lung via the pulmonary veins to left atrium of the heart. Next, the oxygen rich blood flows into the left ventricle as the ventricle opens and the atrium contracts. The left ventricle pumps the oxygen rich blood out to body tissues through the systemic circuit.

Blood leaves the left ventricle via the aorta, which conveys blood to arteries leading throughout the body. The first branches from the aorta are the coronary arteries, which supply blood to the heart muscle itself. Then, flow branches leading to capillary beds in the head and arms. The aorta continues in a posterior direction, supplying oxygen rich blood to arteries leading to arterioles and capillary beds in the abdominal organs and legs. Within the capillaries, oxygen and carbon dioxide diffuse along their concentration gradients, with oxygen moving from the blood to the tissues and carbon dioxide produced by cellular respiration diffusing into the bloodstream.

Capillaries rejoin, forming venules, which convey blood to veins. Oxygen poor blood from the head, neck and forelimbs is channeled into a large vein called the anterior or superior vena cava. Another large vein called the posterior or inferior vena cava drains blood from the trunk and hind limbs. The two venae cavae empty their blood into the right atrium, from which the oxygen poor blood flows into the right ventricle.

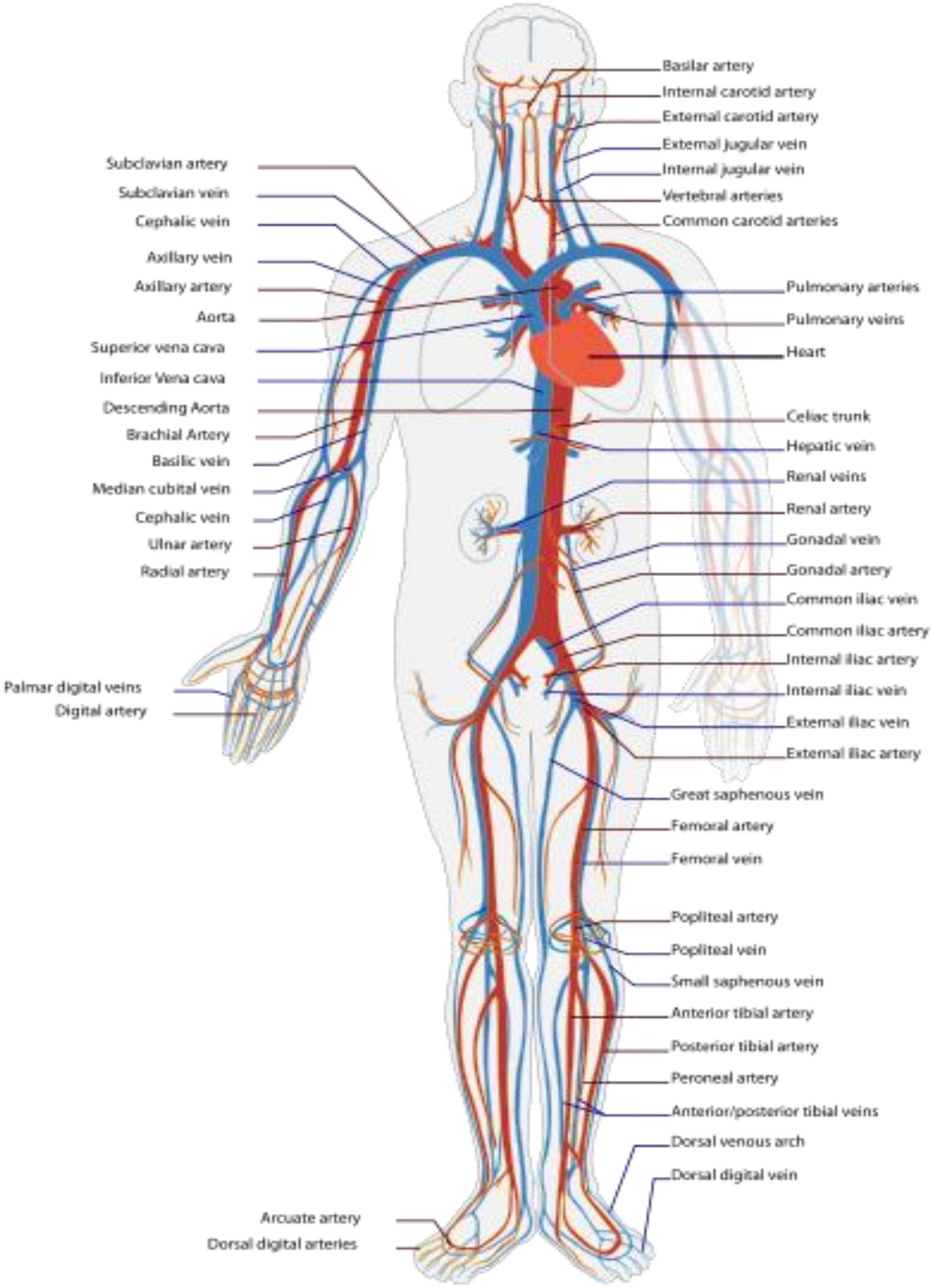

Figure 1.1: The human cardiovascular system. Red indicates oxygenated blood while blue indicated deoxygenated blood (Wikipedia, 2010).

1.4 Arteries of the Cardiovascular System

Internally, the heart is divided into four hollow chambers, two on the left and two on the right. The upper chambers, called atria, have relatively thin walls and receive blood returning through the veins. The lower chambers, the ventricles, force blood out the heart into the arteries to be carried back to the various sites throughout the body. Arteries are strong, elastic vessels that are adapted for carrying blood away from heart under relatively high pressure. Arteries divide into progressively thinner and thinner tubes and eventually become fine branches called arterioles and capillaries. Arteries parallel the courses taken by veins, which carry the blood back to the heart, and usually have the same names as their companion veins. For example, the renal artery parallels the renal veins; the common iliac artery parallels the common iliac vein, and so forth (Innerbody.com, 1999-2011).

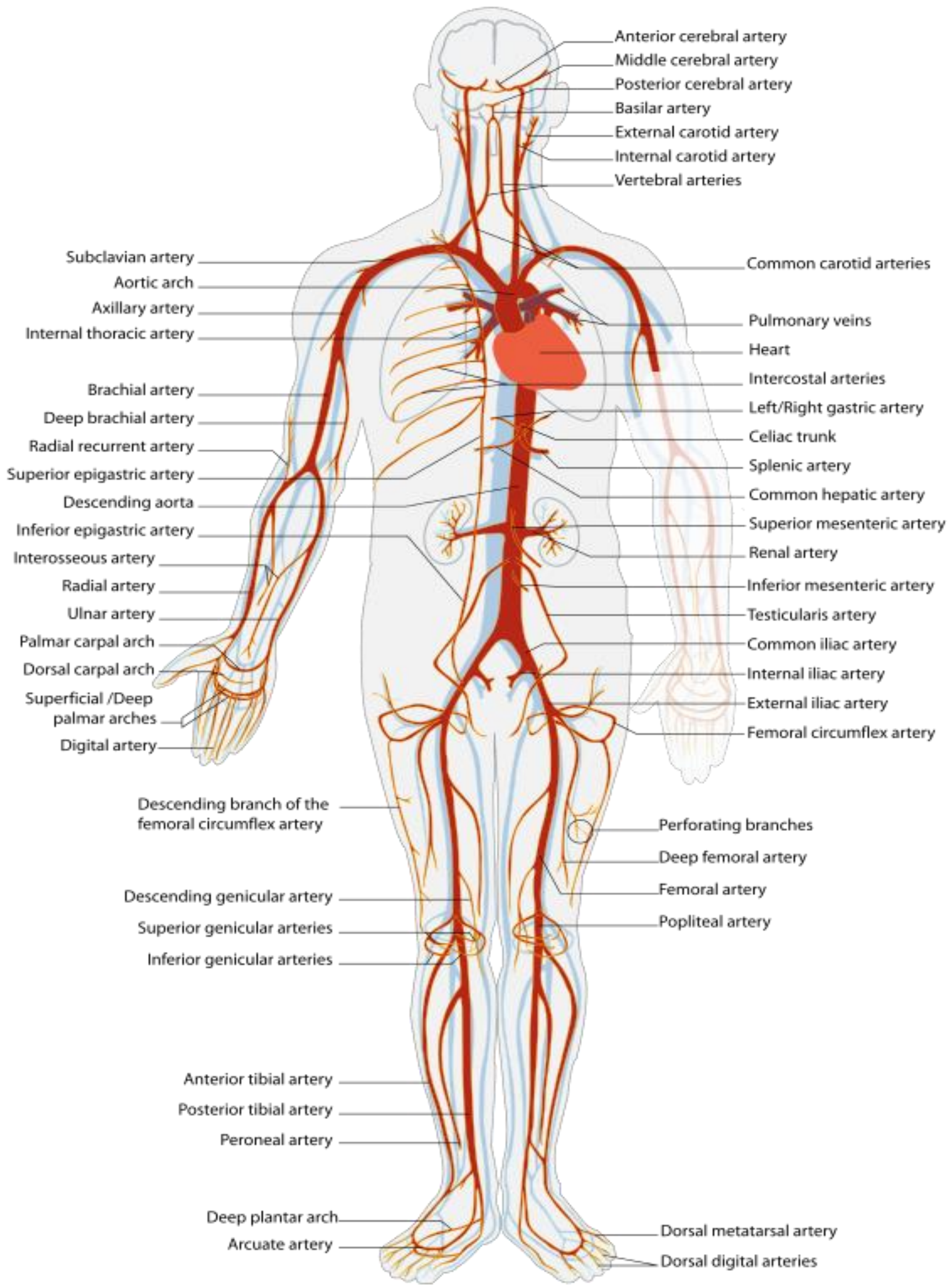

Figure 1.2: Arteries of the cardiovascular system. Red indicates oxygenated blood (Wikipedia, 2010).

1.5 Vein of the Cardiovascular System

Figure 1.3 shown the human circulatory system since the blue indicates deoxygenated blood or we call it as a vein. Veins are responsible for returning blood to the heart after exchanges of gases, nutrients, and wastes have been made between the blood and the body cells. Vein begin when capillaries merge into venules, the venules into small veins, and the small veins merge into larger ones. They are harder to follow than the arteries, because these vessels are interconnected with irregular networks, so that many small unnamed venules may join to form a larger vein. On the other hand, larger veins typically parallel the courses taken by named arteries, and the veins are often given the same name as the companion arteries. The veins from all parts of the body (except from the lungs back to heart) converge into two major paths that lead to the right atrium of the heart. These veins are the superior vena cava and the inferior vena cava (Innerbody.com, 1999-2011).

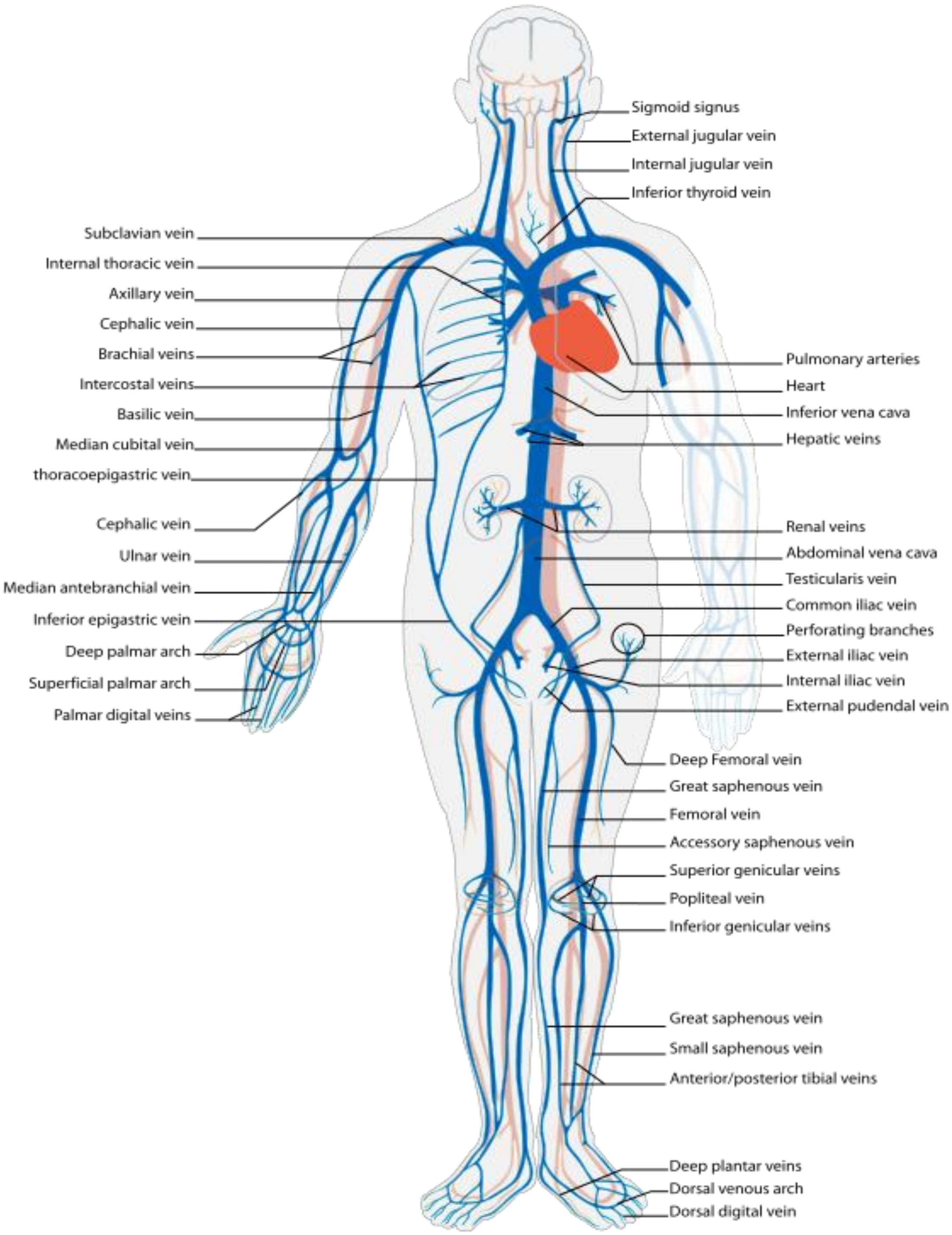

Figure 1.3: Vein of the cardiovascular system. Blue indicates deoxygenated blood (Wikipedia, 2010).

1.6 Blood

Blood is made of four component which are plasma, red blood cells, white blood cells, and platelets. Plasma is a mixture of water, sugar, fat, protein, potassium, and calcium salts. It also contains many chemicals that help form blood the clots necessary to stop bleeding and more than 92% of plasma is water. Red blood cells contain a special protein called hemoglobin, which carries the oxygen we inhale with our lungs to all of the parts of our bodies. It then returns carbon dioxides from our body to our lungs so we can exhale it.

Hemoglobin is also responsible for making red blood cells red. We have so many red blood cells that our blood itself appears red, even though it contains more than red blood cells. White blood cells are clear round cells that are bigger than red blood cells. White blood cells produce proteins called antibodies that help our bodies fight infections caused by bacteria, viruses, and foreign proteins. Platelets are not cells, they are just fragments of cells. When we are injured, platelets gather at the site of the injury and stick to the edges of the wound. They release chemicals that help start the process of blood clotting so that bleeding will stop.

Blood carries the oxygen that needed around the body for use then returns the oxygen poor blood back to the lungs to replace the missing oxygen. This process known as blood flow or blood circulation. Due to complexity of blood rheology, a mathematical description of blood itself has not yet been completely formulated. In the systematic circulation, the large vessels are approximated by tubes with thin, elastic walls, while the blood filling the vessels is considered as a continuum, and incompressible fluids. According to Rudinger (1970). Blood is known to be an incompressible viscous fluid in a number of applications such as flow in large blood vessels. Demiray (2000) studied the contribution of higher order terms to solitary waves in fluid-filled elastic tubes by employing the modified multiple scale expansion. As far as the biological applications are concerned, in a real life, the blood is an incompressible and viscous blood.

Blood is also known to be an incompressible non-Newtonian fluid. However, in the course of flow in arteries, the red cells in the vicinity of arteries move to the central region of the artery so that the hematocrit ratio becomes quite low near the arterial wall and the blood viscosity does not change very much with shear rate. Due to high shear rate near the arterial wall, the viscosity of blood is further reduced. Therefore, for flow problems in large blood vessels, the blood may be treated as a Newtonian fluid.

As is well known, it is not easy to deal with the exact equations of motion of the viscous fluid or also known as Navier-Stokes equation. So, we will make some simplifying assumptions called hydraulic approximations. By doing this approximation, it is assumed that the axial velocity is much larger than the radial and permissible for an averaging procedure with respect to cross-sectional area.

1.7 Cardiac Cycle

According to Guyton and Hall (2006), the cardiac cycle is a term referring to all or any of the events related to the flow or blood pressure that occurs from the beginning of one heartbeat to the beginning of the next. The frequency of the cardiac cycle is described by the heart rate. Each beat of the heart involves five major stages.

The first two stages, often considered together as the “ventricular filling” stage, involve the movement of blood from the atria into the ventricles. The next stages involve the movement of blood from the ventricles to the pulmonary artery which is in the case of the right ventricles and the aorta which is in the case of the left ventricles.

1.8 Blood Pressure

Blood moves through our circulation system because it is under pressure, caused by the contraction of the heart and by the muscles that surround our blood vessels. The measure of this force is blood pressure. Blood pressure will always be highest in the two main arteries, just outside the heart, but, because the pulmonary circulation is inaccessible, blood pressure is measured in the systemic circulation only.

According to Razia and Atanu (2013), the blood pressure is the pressure of blood fluid exerted upon the walls of blood vessels. During each cardiac cycle, the pressure rises to the maximum level called the systolic blood pressure and as it relaxes, it falls to the minimum level called the diastolic blood pressure.

Abnormally raised blood pressure is commonly known as hypertension, the cause of half of all deaths from stroke and heart diseases, worldwide. The World health statistics (Gupta, 2004) indicates startling finding, i.e., one in three adults worldwide have hypertension. Cardiovascular diseases are the top most cause of mortality in India. In India, in year 1990, death due to stroke and 24% of death due to coronary heart disease in India is linked to hypertension (Gupta and Kasliwal, 2004).

1.9 Viscosity

According to Mazumdar (1989), fluids are "sticky" to greater or lesser extents, and this property is denoted by the term viscosity. Because of viscosity, when a viscous fluid flows across a wall, the fluid in immediate contact with the wall is at rest. As we know from watching the flow of streams or rivers, the flow velocity is greater, the greater is the distance from the riverbank, and it reaches its maximum in the middle. One reason for this is that there exists a frictional force between neighboring elements of a viscous fluid. We say that there is a shearing stress between such elements. Fluids that are not viscous cannot support shearing stresses. For newtonian fluids, the stress, τ on a volume element of fluid is proportional to the rate of deformation or strain rate of volume element, and this constant of proportionality is called the coefficient of viscosity μ.

The coefficient of viscosity μ in a flowing liquid was defined earlier as the constant of proportionality between the stress applied τ and the velocity gradient , or rate of shear, dv/dr, of the liquid laminae (Wilmer and Michael, 2005). If the viscosity is measured in a concentric cylinder viscometer, the rate of shear is approximately constant throughout the liquid and is measured directly, as is the applied stress (Cokelet et al., 1963). In order to be able to compare results from the two methods directly, it is necessary to define stress and rate of shear in terms of the pressure gradient, the rate of flow and the tube radius. From equation of velocity gradient which is

$$\frac{dv}{dr} = -\frac{r\left(p_1 - p_2\right)}{2\mu L}. \qquad (1.1)$$

where v is velocity, r is radius, p is pressure and L is length. We can write

$$\tau = \mu\left(\frac{dv}{dr}\right) = -\frac{r\left(p_1 - p_2\right)}{2L}. \tag{1.2}$$

It is usual in these formulations to use Δp as the pressure gradient, so that $\Delta p = \left(p_1 - p_2\right)/L$, and we write

$$\tau = r\Delta p / 2 \tag{1.3}$$

where r is the radius of a given lamina of liquid.

1.10 Newtonian Fluid and Non-Newtonian Fluid

Newton defined the viscosity of a fluid as a lack of slipperiness between the layers of the fluid, of course, in doing so he implied that there was such a thing as a "layer of fluid" or "laminae" of fluid and the viscosity arises because of rubbing one lamina upon the other (Mazumdar, 1989). Suppose we have two laminae, which are in contact with one another [see Figure 1.4]. Suppose some force F parallel to the x-axis acts and produces relative motion between the two laminae, such as the top lamina moves with velocity dv relative to the bottom lamina. Hence, there is a rate of change of velocity with distance in the y direction (there exists a velocity gradient dv/dy). It is hypothesized that the force, F is directly proportional to $A\frac{\partial v}{\partial y}$, or

$$F \alpha A \frac{dv}{dy}, \tag{1.4}$$

where A is the area of contact between the laminae. The proportionality constant is then defined to be the viscosity of the fluid, and is usually denoted by μ, such as

$$F = \mu A \frac{dv}{dy}. \tag{1.5}$$

The dimension of the viscosity is given by

$$[\mu] = \frac{[force]}{[area] \times \frac{[velocity]}{[length]}} = \frac{MLT^{-2}}{L^2 \frac{LT^{-1}}{L}} = \frac{M}{LT}. \tag{1.6}$$

Equation (1.6) is a linear relation, the behavior of the fluid is called Newtonian. Any fluid for which the relation is non linear is called non-Newtonian.

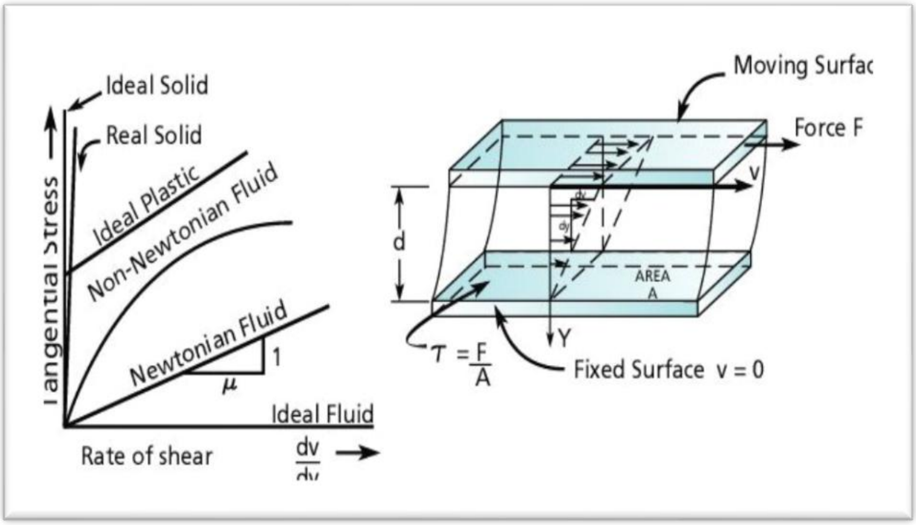

Figure 1.4: Newtonian concept of viscosity

1.11 Is Blood a Newtonian Fluid?

A fluid is said to be Newtonian if the viscous stresses that arise from its flow, at every point, are proportional to the local strain rate, the rate of change of its deformation over time (Batchelor, 2000). That is equivalent to saying that those forces are proportional to the rates of change of the fluid's velocity vector as one moves away from the point in question in various directions. More precisely, a fluid is Newtonian if, and only if, the tensors that describe the viscous stress and the strain rate are related by a constant viscosity tensor that does not depend on the stress state and velocity of the flow (Kirby, 2010).

Newtonian fluids are the simplest mathematical models of fluids that account for viscosity. While no real fluid fits the definition perfectly, many common liquids and gases, such as water and air, can be assumed to be Newtonian for practical calculations under ordinary conditions.

Poiseuille’s law is also so well established experimentally, that is often used in order to determine the vicosity coefficient μ of viscous fluids. When blood is examined in the manner, the viscosity coefficient of blood is found to be about five times the value for water ($\mu_b = 5\mu_w$), if the diameter of the tube is relatively large. Thus at normal physiological temperature of 37^0C, the viscosity of water μ_w is 0.007P (P = Poise), and the viscosity of blood μ_b as determined by Poiseuille’s law in large tubes is about 0.035P.

The fact that the effective viscosity coefficient of blood according to Poisuelli’s law depends on the radius of the cube in which it is measured indicates that blood is not a Newtonian fluid, for which μ is a constant. Rather, blood is said to behave as non-Newtonian fluid. Fluids, which have elaborate molecular structure, in particular those consisting of long chain molecules, are in general non-Newtonian. Thus, biological fluids such as cytoplasm can be expected to be non-Newtonian (Mazumdar, 1989).

In the case of Newtonian fluids

$$\tau = \mu\gamma, \tag{1.7}$$

where τ is the shear stress and γ is shear strain rate. A simple model for non-Newtonian behavior is the power law model given by

$$\tau = \mu\gamma^{n}, \tag{1.8}$$

where n is the power law index. Non-Newtonian fluids viscosity is not independent of the applied shear stress. Figure 1.5 shows the relationship between Newtonian and non-Newtonian fluids.

Most of the biological fluids including blood, lymph and ‘semi fluids’ such as cytoplasm are in fact non-Newtonian. Atlhough adequate mathematical theory of such fluids exists at present, its discussion is beyond the scope of this project. Paterson (1983) in a study of the non-Newtonian behavior of blood observed that the non-Newtonian fluid flow involved many new features not found in the Newtonian fluid flow. However for most purposes, blood can be treated theoretically as an ordinary Newtonian fluid with an appropriate “effective” viscosity coefficient that is constant.

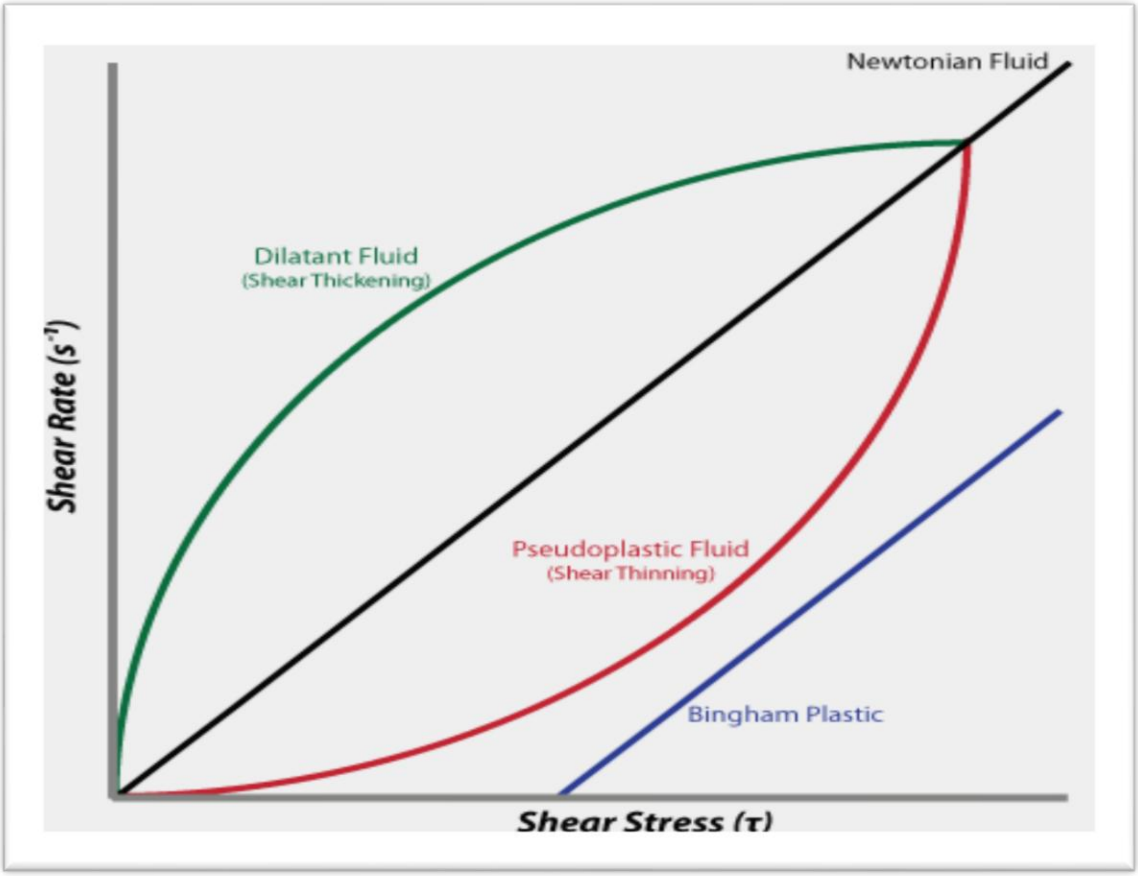

Figure 1.5: Typical shear stress rate relationships for non-Newtonian fluids (Mazumdar, 1989)

1.12 No-Slip Condition

To solve the governing Navier-Stokes equations and continuity equation, appropriate boundary and initial conditions must be supplied. Fluid flow is influenced by the presence of solid boundaries in two manners. First, no fluid can cross a solid boundary, so that the velocity normal to the surface is zero. Second, viscous forces make the fluid ‘stick’ to the wall, so that the tangential velocity at the fluid equal the velocity of the wall that is called the no-slip condition. The tangential component of boundary velocity U_T

$$u_T = U_T. \quad (1.9)$$

If a solid surface is not moving, $U_T = 0$, and the no-slip condition becomes $u_T = 0$ (Shaughnessy, Katz, and Schaffer, 2005).

1.13 Problem Statement

This study will focus on mathematical modeling of blood blow. Although blood is the non-Newtonian fluid, in this study will consider the blood as a Newtonian fluid which is governed by the Navier-Stokes equation for solving blood flow problems. In this study, we will focus on the derivation of cardiovascular system equation or called as master equation with the help of continuity equation and Navier-Stokes equation in order to developed general equation of normal blood flow and extended normal blood pressure equation. The problems of this study will be solved using the fluid dynamic assumption assist by differential equation. At the end of this study, some analysis had been performed to determined the validity of proposed model by using MAPLE 13.

1.14 Objective of the Study

The objectives of the research are:

1. To derive the cardiovascular system equation.
2. To develop the models for blood flow and blood pressure in the body.
3. To analyze the blood flow rate and blood pressure in order to determine the validity of the proposed model.

1.15 Scopes of the Study

The scope of the research are:

1. Consider the blood flow as Newtonian fluid which governed by Navier-Stokes and continuity equation.
2. All the vessels are assumed to be same in nature excluding their size, length and cross-sectional area.
3. Blood has both the radial and axial flow in only one direction, which is z-direction in a three-dimensional system.
4. Cross-sectional area of blood vessels is constant over time and distance.
5. Pressure gradient is constant over distance.

1.16 Significance of the Study

The results of this study will give benefits to the fields of mathematics, physics, engineering, biology and medical. This research will lead to further investigation in the mathematical modeling of blood flow theory and method which is frequently used in mathematics, physics, and mostly in medical field. Besides that, the result of the research can be a guide line to determine the cardiovascular system equation, blood flow rate equation and normal blood pressure equation in getting the strong solution for blood flow problems. This investigation also gives a chance for readers to know more about benefits of this equations model in solving blood flow problems.

1.17 Outline of Dissertation

This report divided to five chapters. Chapter 1 is the introduction of the study, some basic concept of blood flow theory, problem statement, objective of the study, scopes of the study, significance of the study, and outline of thesis in this report.

Chapter 2 is literature review, will include some introduction and will be depicts several of applications and contributions that have been done by some researchers for blood flow problems. Moreover, some introduction of the Navier-Stokes equation, continuity equation, and poisuelli's equation will also include in this chapter.

Chapter 3 is about the research methodology of this project. This will include the derivation of the governing differential equations. In this chapter, we use the knowledge on the previous chapter to derive differential equations of Navier-Stokes equation. Next, i will discuss and derive in detail the Navier-Stokes equation in Cartesian coordinates and Cylindrical coordinate that will be used to solve the Newtonian fluid of blood flow problem. The problem of steady flow in a Cylindrical coordinate is known as Poisuelli's flow. The solution of the Poisuelli's flow problem will the produce Poisuelli's law. The poisuelli's equation is then applied to develop the mathematical model of the blood pressure from cardiovascular system equation. Beside that, the concept of boundary conditions for axial velocity component, $w(\gamma, z, t)$ and derivation of continuity equation will also be discussed in this chapter.

Chapter 4 is about results and discussion of this project. I will derive in detail and solve the mathematical modeling of blood flow from Rahman and Haque (2012). Moreover, the analisis of blood flow rate and blood pressure for determine the validity of the proposed model will also be discussed in this chapter.

Finally, the last chapter of this report is conclusion and further research. This chapter makes a conclusion to the whole investigation and a suggestion for further research on mathematical modeling of blood flow problem.

1.18 Summary

In this chapter, we discuss about the introduction of this study. We also explained about some basic concept of blood flow theory. The objective and scopes of the study is being defined as a guideline of the research. The details of the study will be discussed in following chapter.

REFERENCES

Acheson, D. J. (1990). *Elementary Fluid Dynamics*. Oxford Applied Mathematics and Computing Science Series. Oxford University Press.

Batchelor, G. K. (1967). *An Introduction to Fluid Dynamics*. Cambridge University Press.

Batchelor, G. K. (2000). *An Introduction to Fluid Dynamics*. Cambridge Mathematical Library Series. Cambridge University Press.

Bender, E. A. (2000). *An Introduction to Mathematical Modeling*. New York: Dover.

Campbell, N. A., Reece, J. B. (2005). *Biology*. 7th Edition. Pearson Education, Inc.

Clancy, L. J. (1975). *Aerodynamics*, Section 3.3. Pitman Publishing Limited, London.

Cokelet, G. R. (1963). *The Rheology of Human Blood Measurement Near and at Zero Shear Rate*. Transaction of the Society Rheology 7, 303-17.

Ehrlish, A., Schroeder, C. L. (2004). "*Medical Terminology for Health Professions*". 5th Edition. Thomson Delman Learning. 131-132.

Faris, O., Evans, F., Ennis, D., Helm, D., Taylor, J., Chesnick, A., Guttman, M., Ozturk, C., McVeigh, E. (2003). "*Novel Technique for Cardiac Electromechanical Mapping with Magnetic Resonance Imaging Tagging and An Epicardial Electrode Sock*". Ann. Biomed. Eng. Vol 31, No. 4. 430-440.

Fieldman, J. S., Phong, D. H., Aubin, Y. S., Vinet, L. (2007). *"Rheology". Biology and Mechanics of Blood Flows*, Part II: Mechanics and Medical Aspects. Springer. 119-123.

Gerard, J. T., Bryan, D. (2012). "*The Cardiovascular System: The Blood*". Principles of Anatomy & Physiology. 13th. John Wiley & Sons.

Gupta, R. (2004). *Trends in Hypertension Epidemiology in India*. Journal of Human Hypertension 18: 73-78.

Gupta, R., Kasliwal, R. R. (2004). *Understanding Systolic Hypertension in the Elderly*. JAPI 52: 479-485.

Guyton, A. C., Hall, J. E. (2006). *Textbook of Medical Physiology*. 11^{th} Edition Philadelphia: Elsevier Saunder.

Hinghofer- Szalkay, H. G. and Greenleaf, J. E., (1987). "*Continuous Monitoring of Blood Volume Changes in Humans*". Journal of Applied Physiology. Vol. 63: 1003-1007.

Imaeda, K., Goodman, F. O. (1980). "*Analysis of Nonlinear Pulsatile Blood Flow in Arteries*". Journal of Biomechanics 13. 1007-1022.

Innerbody. Com. (1999-2011). *Arteries*. How To Media, Inc.

Jennifer, S. (2009). *Physiological Fluid Mechanics*. New York: Oxford University Press.

Kilner, P., Yang, G., Wilkes, A., Mohiaddin, R. (2000). "*Asymmetric Redirection of Flow Through The Heart*". Nature. Vol. 404. 759-761.

Kirby, B. J. (2010). *Micro and Nanoscale Fluid Mechanics*: Transport in Microfluidic Devices. Cambridge University Press.

Lauralee Sherwood. (2005). "*Fundamentals of Physiology: a human perspective*". 3rd Edition. Thomson Brooks. 276.

Liu, C. H., Niranjan, S. C., Clark, J. W., San, K. Y., Zwischenberger, J. B., Bidani, A. (1998). "*Airway Mechanics, Gas Exchange, and Blood Flow In a Nonlinear Model of The Normal Human Lung*". Journal of Applied Physiology 84: 1447-1469.

L. Cromwell, Fred, J. Weibell, and Erich, A. Pfeiffer. (2004). "*Biomedical Instrumentation and Measurement*". 2nd Edition. Pearson Education: Singapore. 84-95.

Masood, S., Yang, G., Pennell, D., Firmin, D. (2000). "*Investigating Intrinsic Myocardial Mechanics: The Role of MR Tagging, Velocity Phase Mapping and Diffusion Imaging*", J. Magn. Reson. Imag. Vol. 12, no. 6. 873-883.

Massey, B. S. (2006). *Mechanics of Fluids*. 8th Edition. London & New York: Taylor & Francis.

Mazumdar, J. (1989*). An Introduction to Mathematical Physiology and Biology. New York*: Cambridge University Press.

Paterson, A. R. (1983). *A First Course in Fluid Dynamics*. New York: Cambridge University Press.

Pedlosky, Joseph (1987). *Geophysical fluid dynamics*. Springer. pp.10-13. ISBN 978-0-387-96387-7.

Pfitzner, J. (1976). "*Poisuelli's an his law*". Anaesthesia 31 (2). pp. 273-5.

Rahman, M. S., Haque, M. A., Asaduzzaman, M. (2010). "*Mathematical Modeling of the Heart*". IEEE International Conference on Electrical and Computer Engineering.626-629.

Rahman, M. S. (2011). "*Computational Design of Cardiac Activity*". International Journal of Medicine and Medical Sciences. Vol. 3(10). 321-330.

Rahman, M. S., Haque, M. A. (2012). *Mathematical Modeling of Blood Flow*. IEEE International Conference on Informatics, Electronics & Vision. 672-676.

Razia, S., Atanu, K. P. (2013). "*Blood Pressure and Heart Rate Variability and Diagnosis*". Review Article. Biological Rhythm Research.

Rolf H. Sabersky, Allan J. Acosta, Eward G. Hauptmann. (1989). *Fluid Flow, a First Course In Fluid Mechanics*, 3rd Edition. Macmillian Publishing Company, a division of Macmillian, Inc New York, United State.

Shaughnessy. E. J., Katz, L. M., Schaffer, J. P. (2005). *Introduction to Fluid Mechanics*. New York: Oxford University Press.

Sermesant, M., Delingette, H., and Ayache, N. (2006). "*An Electromechanical Model of The Heart for Image Analysis and Simulation*". IEEE Trans. Med. Imag. Vol. 25: 612-625.

Wikipedia (2010). *Cardiovascular System*. Wkimedia Foundation, Inc.

Wilmer, W. N., Michael, F. O. (2005). *McDonald's Blood Flow In Arteries*. 5th Edition. London & New York: Hodder Arnold.

Yamaguchi, H. (2008). "*Engineering Fluid Mechanics*". Springer.