

**COMPARISON OF DEFUZZIFICATION METHODS FOR FUZZY
STOCHASTIC LINEAR PROGRAMMING**

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STOCHASTIC LINEAR PROGRAMMING

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ABSTRACT

The present study focused on comparison of three defuzzification methods in transforming fuzzy two-stage stochastic linear programming problem into a crisp problem. The fuzzy transformation techniques that utilized in this study were Yager's robust ranking method, generalized mean integration representation (GMIR) method, and centroid defuzzification method (CDM). Besides that, an assumption that the probability distribution obtained via expert was fuzzy and consisted only partial information was made. Five problems which modified based on Dakota's Furniture Company were presented to give an illustration on how the fuzzy transformations using the three mentioned techniques were carried out. The defuzzified two-stage stochastic linear programming problems from each of the techniques were solved using a modelling system of GAMS, which implemented using a solver called DECIS. The difference between first problem and the rest of the problems was demand levels in first problem were symmetric triangular fuzzy numbers. Transformation of first problem using three different techniques resulted in getting the same model formulation, and hence the result obtained from GAMS/DECIS obviously was similar. The results of Problem 2 and Problem 3 obtained from the GAMS/DECIS showed a slight difference in resource quantities, production quantities, and the total profit, and CDM method showed the best optimum solutions. Meanwhile, GMIR method showed better optimum solutions in Problem 4 and 5. Hence, it can be concluded that CDM and GMIR are best methods of defuzzification for non-symmetric triangular fuzzy numbers problems comparing to Yager's robust ranking method.

ABSTRAK

Kajian ini fokus kepada perbandingan tiga teknik penyahkaburan dalam mentransformasi masalah pengaturcaraan linear stokastik dua peringkat kabur kepada masalah nyata. Teknik transformasi kabur yang digunakan dalam kajian ini adalah kaedah kedudukan teguh Yager, kaedah perwakilan integrasi min umum (GMIR) dan kaedah penyahkaburan sentroid (CDM). Selain itu, andaian dibuat bahawa taburan kebarangkalian diperolehi melalui pakar adalah kabur dan mengandungi hanya separa informasi. Lima masalah yang telah diubah berdasarkan Syarikat Perabot Dakota telah dibentangkan untuk memberi ilustrasi bagaimana transformasi kabur menggunakan tiga teknik yang disebut tadi dilakukan. Masalah pengaturcaraan linear stokastik dua peringkat yang telah dinyahkaburkan dari setiap teknik diselesaikan menggunakan sistem permodelan GAMS, yang mana dilaksanakan menggunakan penyelesaian dipanggil DECIS. Perbezaan di antara masalah pertama dan masalah-masalah yang lain adalah tahap permintaan dalam masalah pertama merupakan nombor kabur segitiga simetri. Transformasi bagi contoh pertama menggunakan tiga teknik yang berbeza menghasilkan pembentukan model yang serupa, dan justeru itu keputusan yang diperolehi melalui GAMS/DECIS semestinya serupa. Keputusan bagi masalah kedua dan ketiga yang diperolehi melalui GAMS/DECIS menunjukkan perbezaan yang sangat sedikit dalam kuantiti sumber, kuantiti pengeluaran dan jumlah keuntungan, dan teknik CDM menunjukkan penyelesaian optimum yang terbaik. Sementara itu, teknik GMIR menunjukkan penyelesaian optimum terbaik dalam masalah ke-empat dan ke-lima. Justeru itu, dapat disimpulkan bahawa CDM dan GMIR merupakan teknik penyahkaburan yang terbaik bagi masalah nombor kabur segitiga tidak simetri berbanding dengan kaedah kedudukan teguh Yager.

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LIST OF ABBREVIATIONS

AMPL	A Mathematical Programming Language
CDM	Centroid Defuzzification Method
GAMS	General Algebraic Modelling System
GMIR	Graded Mean Integration Representation
NP	Non-deterministic Polynomial Time
SLP	Stochastic Linear Programming
2-SSLP	Two-Stage Stochastic Linear Programming

LIST OF SYMBOLS

Ω	-	Finite set
2^Ω	-	Power set of Ω
P	-	Probability
ξ	-	Random variable defined on Ω
x	-	Decision variable in first stage
y	-	Decision variable in second stage
c	-	Fixed cost associated with decision x
A	-	Fixed matrix
b	-	Known vector
E_ξ	-	Mathematical expectation with respect to ξ
q	-	Cost associated with second stage decision
h	-	Vector
T	-	Technology matrix
W	-	Recourse matrix
Q	-	Optimal value of second-stage problem
μ	-	Membership function of fuzzy set
\tilde{A}	-	Fuzzy set of A
\subseteq	-	Subset of
\in	-	Element of
π	-	Probability set
$\tilde{\pi}$	-	Fuzzy probability set
ω	-	Scenario
\sqcup	-	Fuzzy value

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Optimization is a very old and classical term used to describe the best selection of the solution of the particular problem from some set of available alternatives. For instance, supply chain networks optimize the production and distribution to ensure the operating costs (including production costs, transportation costs and distribution costs) are reduce to the lowest and maximize the inventory placement, and vehicle networks find the best route to deliver the goods demanded to ensure optimization of the vehicles available, number of tasks completed in time and vehicles' capacities.

Generally, such problems have been attracting attention of researchers from decades ago. Various theories, techniques, methods, and algorithms as well as computer software have been introduced with the aim of finding the best solution of the particular problem arises. Besides searching for the best solution, researchers also attempt to find the best theories, techniques, methods, algorithms, computer software that can reach the optimal solution as fast as possible.

In this study, we are focusing on optimization of stochastic linear programming (SLP) problem, more specifically fuzzy stochastic linear programming. In comparison to linear programming, network flow programming and integer programming which ignores the impact of uncertainty and the outcomes of the problem is predictable and deterministic, SLP on the other hand has been an attraction to various parties as it takes the uncertainty into consideration. SLP has been extensively studied by various researchers such as Ang, Meng, and Sun (2014), Barbarosoğlu and Arda (2004), Ben Abdelaziz and Masri (2005, 2009), Fábíán and Szóke (2007), Higle (2005), Huang and Ahmed (2008), MirHassani *et al.* (2000), Tsiakis *et al.* (2001), Sakawa, Katagiri and Matsui (2014), Sudhakar and Kumar (2010), and Tan, Huang and Liu (2013).

Fuzzy stochastic linear programming problem includes the characteristics of “fuzziness” into the SLP problem. Fuzzy stochastic linear programming problem has been studied by researchers such as Bag, Chakraborty and Roy (2008), Giri, Maiti and Maiti (2014), Halim, Giri and Chaudhuri (2011), and Hop (2007). Fuzziness can be found in demand levels, probability distribution, production rate, and demand rate of the production inventory model and production planning model. As for the transportation problem, fuzziness probably occurred in sources, demands, capacities of conveyances, transportation cost and transportation time.

The fuzziness in the fuzzy stochastic linear programming problem usually solved using the defuzzification techniques such as center of area, center of gravity, smallest of maximum, mean of maximum, largest of maximum, ranking techniques, bisector method and some meaningful discovered techniques, for instance graded mean integration representation method (GMIR). The most frequently used defuzzification method is centroid method, so called center of area or center of gravity method.

1.2 Motivation of the study

Linear programming has extensively evolved in order to model and tackle the real life problem more accurately and precisely. Linear programming problem assume that all the parameters of the problem, such as the coefficients of the objective function, the inequalities and the availabilities are known numbers (Tintner, 1960). In other words, linear programming is specifically formulated to model deterministic problems. Although linear programming used to be one of the most applicable operational research techniques in real world problems, however due to the fact that linear programming requires much well-defined and precise data that are hardly obtained in real world, numerous amount of effort have been done by researchers to propose new model of optimization. From a general linear programming, SLP has been introduced, and followed by much more complicated yet more precise model have been discovered, such as two-stage stochastic linear programming (2-SSLP), multi-stage SLP, multi-objective SLP, and stochastic convex linear programming.

As we live in the world full of uncertainty, dealing with uncertainty is unavoidable. Majority of concrete real life problems consists of relatively some level of uncertainty about values to be assigned to various parameters. As quoted by Shackle (1961) about the uncertainties in real world:

“In a predestinate world, decision would be illusory; in a world of perfect knowledge, empty; in a world without order, powerless. Our intuitive attitude to life implies non-illusory, no-empty, non-powerless decision...Since decision in this sense excludes both perfect foresight and anarchy in nature, it must be defined as choice in the face of unbounded uncertainty.”

Upon realization of the fact that real world problems almost invariably include some unknown parameters, Dantzig (1955) and Beale (1955) incorporated the influence of “uncertainties” into linear programming problem. In this sense, if

there is an element that is subject to uncertainty, the linear programming is called SLP. SLP undoubtedly has become an essential tool to solve optimization problem since it has been introduced by Dantzig (1955) and Beale (1955). Since then, SLP has been recognized as powerful modelling tool, but under condition that precise probabilistic description of the randomness is presented. However, this information is hardly available for the decision makers, for example information as regards to the demand level of customers on particular product. Although the information is usually obtained by decision makers through past data, yet it is inaccurate as demand might increase or decrease depending on variables such as the launch of new products by a competitor or lack of advertising of the product.

Furthermore, as the studies on stochastic programming progress extensively, researchers come to the realization that in real life situations, information on probability distribution is usually not completely known. In other words, only partial information on the probability distribution is known. The term partial information is sometimes also referred as imprecise information, incomplete knowledge or linear partial information.

Along with the development on stochastic optimization, the concept of fuzziness has also been integrated into the SLP to visualize the problem in a more realistic way to present the real life problem. As stated by Rommelfanger (1996), utilization of fuzzy mathematical programming is highly recommended for the purpose of reducing cost of information and modelling the problem more realistically. The term “fuzzy” is first introduced by Bellman and Zadeh (1970). Fuzzy logic allows the management of uncertainty and vagueness of the information obtained. The fuzzy mathematical programming is first introduced by Bellman and Zadeh (1970), and further studied by researchers such as Luhandjula (2007), Luhandjula and Gupta (1996), Nukala and Gupta (2007), Omar (2012), Veeramani and Sumathi (2014), and Zimmermann (1985).

1.3 Background of Study

Stochastic programming is an essential part of mathematical programming with random parameters, and has been widely applied to various fields such as economic management and optimization control (Birge and Louveaux, 1997). Undeniably, stochastic programming has attracted more attention of researchers from various area of expertise, as it proved to be able to handle the uncertainty scenarios that is unpredictable in real life environment.

Two-stage stochastic linear programming (2-SSLP) and multi-stage stochastic linear programming have also been extensively studied since SLP has been discovered in 1955. Numerous theories, algorithms and methods to solve the 2-SSLP and multi-stage SLP has been introduced since then. However, according to Han and Ma (2012), these theories and algorithm obtained on stochastic linear programming are all based on assumption that the probability distributions of random parameters have complete information. In the mid-1960s, researchers such as Dupačová has discovered the limitations of the expected value paradigm that have been practiced so far, where he concluded that the exact knowledge of underlying probability distribution are difficult to estimate accurately (Thiele, Terry, and Epelman, 2010). In 2005, Ben Abdelaziz and Masri proposed a model of SLP with fuzzy linear partial information on probability distribution, in conjunction with the study of Kofler (2001) on linear partial information with application. In addition to that, various computer softwares including modelling systems such as A Mathematical Programming Language (AMPL) and General Algebraic Modelling System (GAMS), and powerful large scale solvers such as CPLEX, excel solver, LINDO, LINGO, OSL-SE and DECIS have been established to serve as the tool to solve linear programming problem. Despite of that, however most of them are still incapable of solving a stochastic linear programming problem. Only limited number of softwares and solvers are available to model and solve stochastic programming problem.

Fuzzy linear programming has been studied extensively by numerous researchers, however, only few studies on fuzzy stochastic linear programming can be found up to date (Giri, Maiti and Maiti, 2014; Sakawa, Katagiri, and Matsu, 2014; Wang, and Watada, 2011). Various methods of fuzzy transformation and defuzzification methods have been proposed to defuzzify the optimization problem, such as max membership defuzzification method, centroid defuzzification method (CDM), weighted average method, mean max membership method, center of sums method, center of largest area method, first (or last) maxima, Yager's robust ranking method, and GMIR. Although a lot of methods have been proposed to defuzzify the fuzziness in the problems, however up to now, limited studies have been made to search for the best methods among all the proposed methods. Mogharreban and DiLalla (2006) proposed that center of area is the best method of defuzzification, meanwhile Naaz, Alam and Biswas (2011) suggested that the center of gravity, bisector method, and mean of maxima methods were the three best defuzzification methods. There was a proposed method of defuzzification to defuzzify the generalized fuzzy numbers, which was GMIR method. However, no comparison between GMIR method and previously proposed method has been made.

1.4 Problem Statement

Fuzzy transformation is an essential method to defuzzify any fuzzy numbers into a crisp value. Hence, countless methods have been proposed with the aim to be the best defuzzification methods under various circumstances. Although various defuzzification methods have been proposed, limited studies on comparison of defuzzification methods have been. Therefore, the present study would like to compare GMIR defuzzification method with the most frequently used method, which is CDM, and also the most simplest method, which is the Yager's robust ranking method.

1.5 Research Objectives

The objectives of the study are:

1. To defuzzify the fuzzy two-stage stochastic linear programming problem with symmetric fuzzy demand levels using Yager's robust ranking method, GMIR, and CDM.
2. To defuzzify the fuzzy two-stage stochastic linear programming problem with non-symmetric fuzzy demand levels using Yager's robust ranking method, GMIR, and CDM.
3. To solve the defuzzified problems resulting from Yager's robust Ranking, GMIR and CDM using GAMS/DECIS solver.
4. To compare the performance of the three defuzzification methods in symmetric fuzzy demands levels problems.
5. To compare the performance of the three defuzzification methods in non-symmetric fuzzy demands levels problems.

1.6 Scope of the Study

This study aims to transform the fuzzy 2-SSLP problem using different defuzzification methods under the circumstances that the probability distribution is not explicitly known and it is fuzzy. At first, the problem is modelled as fuzzy stochastic problem, then we performed fuzzy transformation process to defuzzify all the fuzzy parameters into a crisp value. The defuzzified problems are solved separately using an optimization system called GAMS and solve using a solver called DECIS. In addition to that, this study would like to investigate how the symmetric and non-symmetric in triangular fuzzy numbers would affect the optimum solutions acquired.

1.7 Significance of the Study

Throughout this study, hopefully more real world problems can be solved under the circumstances that the probability distribution is fuzzy and have linear partial information. As the limitation on the number of research done on fuzzy 2-SSLP problem is countable, this study would like to provide an insight to various parties to solve the real life problems such as transportation problem, replacement problem, supply chain problem and production planning, and products mixing, which subjects to randomness and fuzziness in the parameters. In addition, hopefully this study can provide useful information on which defuzzification methods to be utilized; of course it is problem-dependent. Through this comparison study on the defuzzification methods, we hope it will guide the researchers to choose the best methods to defuzzify the fuzzy problems.

1.8 Organization of the Study

This study can be divided into five chapters. Chapter 1 presented an overview of the study, where the background of the study and problem statement, as well as research objectives, scope of the study and significance of the study are included.

Chapter 2 reviewed the past literature, including an overview of the literature from past studies on stochastic linear programming, particularly in 2-SSLP. Details and literature on fuzzy set theory, triangular fuzzy numbers, fuzzy distribution, linear partial information on probability distribution, fuzzy linear partial information on probability distribution and fuzzy two-stage stochastic linear programming are provided in this chapter. Furthermore, fuzzy transformation is also discussed in this chapter.

Chapter 3 formulated the model of fuzzy 2-SSLP with fuzzy linear partial information on probability distribution. The defuzzification techniques employed in the numerical examples in Chapter 4 are explained in detail.

In Chapter 4, five numerical problems which illustrated the fuzzy 2-SSLP with fuzzy linear partial information on probability distribution are discussed. A detail on defuzzification of fuzzy parameters and probability distribution using the three defuzzification methods, which are Yager's ranking method, GMIR and CDM is provided. Problem 1 tested the three defuzzification methods using the symmetric triangular fuzzy numbers in demand levels, mean while the rest of the problem tested the three defuzzification methods using the non-symmetric triangular fuzzy numbers in demand levels. The solutions generated from GAMS/DECIS solver are also provided. The discussion on the research findings are provided as well.

Chapter 5 included the conclusion on the study, discussion on the findings, and recommendations for future studies.

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