MATHEMATICAL MODELLING OF UNSTEADY BIOMAGNETIC FLUID FLOW AND HEAT TRANSFER WITH GRAVITATIONAL ACCELERATION

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To my beloved mother, father and fiance

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ABSTRACT

Numerical computation and simulation have been carried out on the two-dimensional Navier Stokes equations with a coupling of biomagnetic fluid dynamics and heat transfer. Biomagnetic fluid refers to a fluid that exists in living creature with its flow influenced by the presence of magnetic field. Studies have shown that the flow of biomagnetic fluid under spatially varying magnetic field could be adequately modelled based on the principle of ferrohydrodynamics. The main objective of this research is to employ the model of biomagnetic fluid using the principle of ferrohydrodynamics to investigate the effect of gravitational acceleration on unsteady fluid flow. This study is important because most existing studies on two-dimensional biomagnetic fluids only analyze flows in the steady state conditions and the effect of gravitational acceleration have not been addressed. The governing equations consist of a set of nonlinear partial differential equations which are first non-dimensionalized and then discretized using a finite difference technique on a staggered grid system. The discretized equations are solved using the pressure correction method based on the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm. The numerical results show that the gravitational acceleration has a profound effect on both velocity and temperature profiles. The streamlines plotted show that vortices appear near the lower plate where the magnetic source is located and the distraction becomes greater with the increase of magnetic field strength.

ABSTRAK

Pengiraan berangka dan simulasi telah dilakukan terhadap persamaan Navier-Stokes dua dimensi dengan gandingan dinamik bendalir biomagnet dan pemindahan haba. Bendalir biomagnet merujuk kepada bendalir yang wujud dalam hidupan dengan alirannya dipengaruhi oleh kehadiran medan magnet. Kajian menunjukkan bahawa aliran bendalir biomagnet dengan medan magnet yang berubah terhadap ruang boleh dimodelkan menggunakan prinsip ferohidrodinamik. Objektif utama kajian ini adalah untuk menggunakan model bendalir biomagnet dengan prinsip ferohidrodinamik untuk mengkaji kesan pecutan graviti pada aliran bendalir tak mantap. Kajian ini penting kerana kebanyakan kajian yang telah dijalankan mengenai bendalir biomagnet dua dimensi hanya menganalisis aliran dalam keadaan mantap dan kesan pecutan graviti tidak diambilkira. Sistem persaman yang terdiri daripada satu set persamaan pembezaan separa tak linear diturunkan kepada persamaan tanpa matra, kemudian pendiskretan dilakukan menggunakan kaedah beza terhingga pada sistem grid berperingkat. Persamaan terdiskret ini diselesaikan menggunakan kaedah pembetulan tekanan berdasarkan kepada algoritma Kaedah Separuh Tersirat untuk Persamaan Berkaitan Tekanan (SIMPLE). Keputusan berangka menunjukkan bahawa pecutan graviti mempunyai kesan yang besar terhadap halaju dan suhu. Garis arus yang diplotkan menunjukkan pusaran muncul berhampiran dengan sumber magnet pada plat di sebelah bawah dengan gangguan yang lebih besar apabila kekuatan medan magnet meningkat.

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LIST OF SYMBOLS

SYMBOLS:

ρ	-	Fluid density
ν^*	-	Kinematic viscosity
p	-	Pressure
p'	-	Corrected pressure
p^*	-	Guess pressure
α_p	-	Underrelaxation factor
h	-	Reference height
L	-	Reference length
u_r	-	Maximum velocity
T_u	-	Upper temperature
T_l	-	Lower temperature
β	-	Thermal expansion coefficient
g	-	Acceleration due to gravity
u and v	-	Fluid velocity components
u^* and v^*	-	Guess velocity
u' and v'	-	Corrected velocity
α	-	Thermal diffusivity
Re	-	Reynolds number
Pr	-	Prandtl number

Gr		Grashof number
Mn_f	-	Magnetic number due to FHD
Mn_m	-	Magnetic number due to MHD
H	-	Magnetic field gradient
nx and ny	-	Number of grids

LIST OF ABBREVIATIONS

ABBREVIATIONS:

PDE	-	Partial differential equation
CFD	-	Computational fluid dynamics
FDM	-	Finite difference method
SIMPLE	-	Semi – implicit method for pressure – linked equation
BFD	-	Biomagnetic fluid dynamics
FHD	-	Ferrohydrodynamics
MHD	-	Magnetohydrodynamics

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Biomagnetic fluid dynamics (BFD) is the investigation of biological fluid under effect of a magnetic field. A fluid that is present in a living creature is known as a biomagnetic fluid. Blood is one of the fluid that has characteristic of biomagnetic fluid and is considered a magnetic fluid. Tzirtzilakis and Kafoussias (2003) stated that the characteristics of blood which indicate the nature of a magnetic fluid is caused by the interaction of the cell membrane, intercellular protein and the hemoglobin molecule which is a form of iron oxides that exist at a uniquely high concentration in the mature red blood cells.

Based on the mathematical model of BFD, the flow of biofluid under the effect of magnetic field consider the principles of ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD). FHD is considered as non-conducting electrically magnetic fluid and the flow is influenced by the fluid magnetization in the magnetic field. MHD considers conducting fluids and ignores the effect of polarization and magnetization as stated by Loukopoulos and Tzirtzilakis (2004).

There are numerous applications on BFD study in medicine and bioengineering research work. The magnetic devices development for cell separation, high-gradient magnetic separation, reduction of blood flow during surgeries, targeted transport of drugs using magnetic particles as drug carries, treatment of cancer tumor causing magnetic hyperthermia and magnetic wound treatment and development of magnetic tracers are some applications as stated by Tzirtzilakis (2008).

1.2 Problem Statement

Most previous studies in fluid flow problem only analyze a two-dimensional biomagnetic fluid flow in the steady state condition. Sometimes, the steady state conditions become unstable in some situation. The unsteady state condition is important in the study of fluid flow problem. However, there is no study that has been done in unsteady biomagnetic fluid flow. In medical application, when wound is treated magnetically, the wound closes after 21 – 26 days. The same wound which is not dealt with magnetically, scab and ulcer would be formed even after 50 days. The existence of the magnetic field in fluid will cause a change in the fluids velocity and temperature. Besides, a gravitational acceleration also can affect the velocity and temperature in the flow of biomagnetic fluid and heat transfer. Theoretically, the problem of fluid flow and heat transfer depends on the relative intensity of the flow parameters such as the Reynolds number, the Grashof number and the Richardson number. These parameters will influence the flow of fluid, velocity and the rate of heat transfer. In the problem of biomagnetic fluid flow, the methods that are most used is stream function vorticity formulation. This method eliminates the pressure gradient from the momentum equations and the equations involving the vorticity and the stream function. For vorticity, it is difficult to specify the boundary conditions which will give problem in getting a converged solution. Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm in pressure correction method has been shown to be the appropriate method. This method is suitable for the unsteady simulation. This study investigates the effect of gravitational acceleration on biomagnetic fluid flow and heat transfer in a rectangular channel.

1.3 Research Objectives and Scope

The aim of this research is to investigate the of the biomagnetic fluid flow and heat transfer. The objectives of this study are:

1. To verify and validate the SIMPLE algorithm using problem of lid driven cavity and rectangular channel.
2. To model an unsteady biomagnetic fluid flow and heat transfer in a rectangular channel with gravitational acceleration.
3. To determine the effect of gravitational acceleration on unsteady biomagnetic fluid flow and heat transfer.

The scope of this research is on the Navier Stokes equations coupling with spatially varying magnetic field and heat transfer. The governing equations are assumed to be two-dimensional, unsteady, laminar, incompressible and Newtonian biomagnetic fluid. SIMPLE algorithm in pressure correction method on a staggered grid system is used to obtain the numerical solution. The two problems in which flow of biomagnetic fluid in a rectangular channel and with heat transfer are considered. To induce magnetic force to the flow, the high gradient magnetic field which is spatially varying in nature is used.

1.4 Significance of Research

Many research works have been done on the biomagnetic fluids due to the applications of medical and bioengineering since the last decades. The investigation of the effect of magnetic field on fluids is valuable because there are many applications in a wide range of fields. The research on the interaction of the electromagnetic field and the magnetic field with fluids have been documented such as in chemical engineering, high speed noiseless printing, nuclear fusion and transformer cooling.

The fluid flow processes, heat transfer and chemical reactions occur in the natural environment, in engineering equipments and in living creature. These processes are important in many practical situations. There are numerous applications of fluid flow and heat transfer such as a thermal insulations, hydro-electric power generation, bioheat transfer, temperature control, aircraft and rockets. Heat transfer gives effect to the natural environment pollution.

In medical application, as stated by Tzirtzilakis (2005), hyperthermia or hypothermia is one of the treatments in which body tissue is exposed to slightly higher temperatures to destroy and kill cancer cells. By the magnetic fluid application, on injucted fluid of magnetic, hyperthermia can also be used for the eye injuries treatment. The temperature will increase when magnetic field is applied. Thus, the wound of 12 and 20 cm^2 will close after 21-26 days. But, scabs and ulcers will be formed even after 50 days for wound that are not treated magnetically.

1.5 Overview of Thesis

This thesis contains six chapters. Chapter 1 begins with a short introduction, which consists of the background of the problem, statement of the problem, objectives of the research, scope of the research and significance of the research.

Chapter 2 presents the literature review. This chapter starts with an introduction on biomagnetic fluid dynamics. It focuses on the mathematical model of biomagnetic fluid flow, the magnetization equations, equations of magnetic field intensity and problems related to the biomagnetic fluid dynamics. Next, the problems of heat transfer are presented. The staggered and non-staggered grids system are discussed thoroughly. Previous works on the methods of solution are presented and discussed. There are several numerical methods typically used for the Navier-Stokes equations and heat transfer solution.

In Chapter 3, the governing equations of the Navier Stokes and energy equations are presented. The chapter starts with the conservation of mass, momentum and energy equations. It then follows with the discussion about the biomagnetic fluid dynamincs (BFD) mathematical model.

The numerical procedure will be presented in Chapter 4. This chapter presents the non-dimensionalization of unsteady Navier-Stokes equations. This is followed by discretization of momentum and continuity equations based on a staggered grid system. The equations are discretized using finite difference technique. For the numerical procedure, the pressure correction method using SIMPLE algorithm is adopted. The source code for this algorithm is written in Matlab. In this chapter, verification against benchmark problems is solved in order to experiment the accuracy and dependability of this source code. For this study, we considered a lid driven cavity and a rectangular channel for verification calculations.

Chapter 5 illustrates the biomagnetic fluid flow problem under the influence of a spatially varying magnetic field in a rectangular channel. For the mathematical formulation, the Navier Stokes equations, energy equation and an additional term that describes the magnetic force effect which is consistent with the principles of ferrohydrodynamics (FHD) are employed. The effect of magnetic force arising in Navier Stokes equation is due to the magnetization of the fluid under the action of the spatially varying of magnetic field. Besides, the flow is considerably affected by the magnetic field strength and the magnetic field gradient. This model is solved using finite differences numerical technique based on SIMPLE algorithm on a staggered grid system. The contribution of this chapter is to investigate the effect of the gravitational acceleration on biomagnetic fluid flow and heat transfer in a rectangular channel.

Chapter 6 summarizes the flow of biomagnetic fluid and heat transfer problem. The results from the simulation are concluded and several recommendations for future woks are suggested.

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