

THE PARALLELIZATION OF HELMHOLTZ EQUATION RELATED TO
BREAST CANCER GROWTH

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requirements for the award of the degree of
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To my beloved parents, siblings and friends.

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ABSTRACT

Detecting breast cancer at an early stage will decrease the mortality rate and improve the cancer treatment successfully. This research focuses on the parallelization of the mathematical modeling on breast cancer growth using one and two dimensional Helmholtz equations. Finite difference method (FDM) is chosen to discretize the Helmholtz equation in order to generate a large sparse grid solution. Some numerical iterative methods are used to simulate the grid solution. The numerical methods under consideration are alternating group explicit (AGE), Red Black Gauss Seidel (RBGS), Gauss Seidel (GS) and Jacobi (JB) method. The alternative numerical method can be detected and quantified by comparing and analyzing the numerical methods under consideration in the aspect of run time, number of iterations, maximum error, root mean square error and computational complexity. Domain decomposition technique of the parallel AGE, RBGS and JB can be applied to decompose the full domain solution into subdomains. The message passing among the neighbourhood of subdomain can be done efficiently using MATLAB Distributed Computing Software. This technique is a straight forward implementation on a distributed parallel computer system (DPCS) because of the non-overlapping subdomain feature. The computer system architecture of DPCS is a single instruction multiple data stream (SIMD) and well suited to support the high computational complexity of a large sparse matrix. The development of DPCS is based on the Linux platform with eight processors of Intel® Core™ Duo Processor architecture and MATLAB Distributed Computing Software version R2011a. The visualization of one and two dimensional of breast cancer growth are captured using Comsol Multiphysic version 4.3a. The parallel performance evaluations of parallel AGE, RBGS and JB are measured in terms of run time, speedup, efficiency, effectiveness and temporal performance. As a conclusion, the parallel algorithm of AGE is superior than RBGS, GS and JB for solving one and two dimensional Helmholtz equations for breast cancer growth early detection.

ABSTRAK

Pengesanan kanser payudara pada peringkat awal akan mengurangkan kadar kematian dan meningkatkan rawatan kanser dengan jayanya. Kajian ini memberi tumpuan kepada penyelarian model matematik ke atas pertumbuhan kanser payudara menggunakan persamaan Helmholtz berdimensi satu dan dua. Kaedah beza terhingga (FDM) dipilih untuk mendiskrit persamaan Helmholtz dengan menjana penyelesaian grid jarang yang besar. Beberapa kaedah lelaran berangka digunakan untuk mensimulasikan penyelesaian grid. Kaedah berangka yang dipertimbangkan adalah kaedah kumpulan selang-seli tak tersirat (AGE), kaedah Gauss Seidel Merah Hitam (RBGS), kaedah Gauss Seidel (GS) dan kaedah Jacobi (JB). Kaedah alternatif berangka dapat dikesan dan diukur dengan membanding dan menganalisis kaedah berangka yang dipertimbangkan dalam aspek masa, bilangan lelaran, ralat maksimum, ralat punca min kuasa dua dan kerumitan pengiraan. Teknik penguraian domain AGE, RBGS dan JB digunakan untuk mengurai penyelesaian domain penuh ke dalam beberapa subdomain. Mesej yang dihantar melalui subdomain berdekatan boleh dilakukan dengan cekap menggunakan Perisian Pengkomputeran Teragih MATLAB. Teknik ini adalah pelaksanaan terus di dalam sistem komputer teragih selari (DPCS) kerana ciri subdomain yang tidak bertindih. Senibina sistem komputer DPCS merupakan arahan tunggal pelbagai aliran data (SIMD) dan didapati sesuai untuk menyokong pengiraan matriks jarang yang besar lagi rumit. Pembangunan DPCS adalah berdasarkan pada platform Linux dengan lapan pemproses senibina Intel ® Core™ Duo dan Perisian Pengkomputeran Teragih versi R2011a MATLAB. Gambaran satu dan dua dimensi pertumbuhan kanser payudara dirakam dengan menggunakan *Comsol Multiphysic* versi 4.3a. Penilaian prestasi selari AGE, RBGS dan JB diukur dari segi masa, kecekapan, keberkesanan dan prestasi sementara. Kesimpulannya, algoritma selari AGE adalah lebih baik daripada kaedah RBGS, GS dan JB untuk menyelesaikan persamaan Helmholtz berdimensi satu dan dua bagi pengesanan awal pertumbuhan kanser payudara.

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LIST OF SIMBOLS

r	-	The current of Ion
t	-	Time
e	-	Elementary Charge
x,y	-	The space at coordinate system
E	-	Electric Field
B	-	Magnetic Induction
∇^2	-	Laplace Operator
$\frac{\partial^2 r}{\partial x^2}$	-	Second order derivative for r at x
$\frac{\partial^2 r}{\partial y^2}$	-	Second order derivative for r at y
u_i^k	-	The value of u at the grid point i at time step k
$u_{i,j}^k$	-	The value of u at the grid point i,j at time step k
Ω^R	-	Domain at red grid
Ω^B	-	Domain at black grid
p	-	Number of processors

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Breast cancer is the most frequent women illness throughout the world as well as in Malaysia. It is about 18.1% of all cases reported in National Cancer Registry Report 2007. According to Dr. Harjit Kaur, Prince Court Medical Center Consultant Breast and Endocrine surgeon, the incidences of breast cancer among women is as high as one in 11 women in Malaysia (Bernama, 2012). Breast cancer is actually a malignant tumor that starts from a group of cancer cells. The group of cells invade surrounding tissues and spread to distant areas of the targeted region. The main purpose of this research is to predict the detection of breast cancer growth using mathematical modelling and give the accurate prediction for future treatment and therapy.

1.1.1 Breast Cancer Growth

Breast cancer is a large collection of out of control cancer cells. Detecting breast cancer as early as possible will improve treatment successfully. People may choose not to attend screening programmes because they perceive the test as painful, expensive, a waste of time and merely inconvenient (Brailsford *et al.*, 2012).

Therefore, this study proposes a mathematical model to predict the breast cancer cell growth. Many mathematical models for early detection of breast cancer growth have been proposed to fit the clinical data to offer growth prediction. The mathematical model using hyperbolic equation is proposed by Bounaim *et al.* (2007). The hyperbolic equation depends on the pressure of the breast tissue. The graph captured by using the hyperbolic equation is as follows.

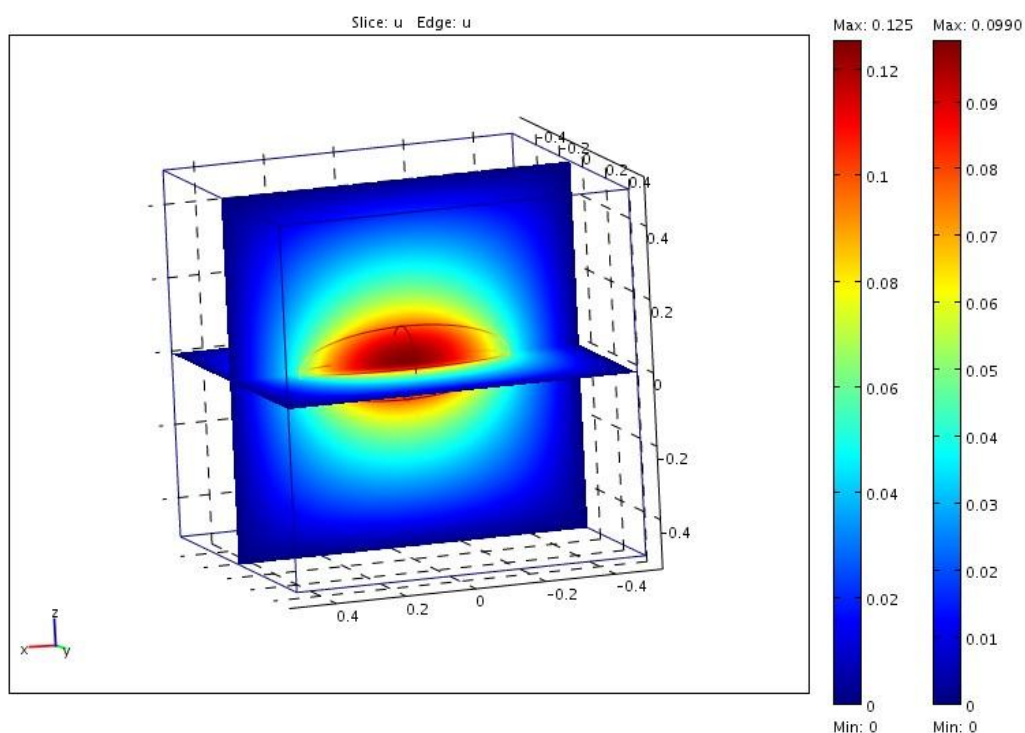


Figure 1.1 The visualization of breast cancer using hyperbolic equation

Figure 1.1 shows the breast cancer growth detection using hyperbolic equation. The colours represent the pressure of the breast tissue. The brown colour shows the highest pressure where the tumor is detected. The yellow and green colour represent the affected breast tissues and the blue colour is normal breast tissues.

Other alternative mathematical models to predict the breast cancer detection are using thermal simulation (Gonzalez, 2007), transport and diffusion model by (Hinow and Gerlee, 2009) and Helmholtz equation (Gunnarsson, 2007). However, this research focuses on mathematical modeling on Helmholtz equation to formulate the early detection of breast cancer growth. The model gives an accurate solution of visual, insight and prediction of breast cancer cell growth. It is highly beneficial of the mathematical modelling to obtain the appropriate visualization, accurate prediction and verifiable detection without disturbing the patient's psychology. Simulation models are capable of modelling complex scenarios with more flexible assumptions than analytical models, but the extra complexity requires more detailed data to inform the model (Stevenson, 1995). The outcome of this research is to generate the visualization and cancer cell detection graphically using Helmholtz equation.

1.1.2 Finite Difference Method

A finite difference method (FDM) is governed by Taylor Series expansion (Teh, 2005). The FDM is used to discretize the Helmholtz equation and generate a full grid solution. The grid solution consists of a large sparse grid and can be expressed in the large scale system of algebraic equations. The function of FDM is to determine the unknown dependent variable. FDM utilize the grid uniformly. At each grid, the derivative is approximated by an algebraic expression. A large scale system of algebraic equations can be obtained by evaluating the previous step of each grid for the dependable variable.

This research focused on the central difference operator to discretize the Helmholtz equation. Distributed parallel computer system (DPCS) platform is used to support the simulation of the large scale system of algebraic equations. The two types of algorithms to execute the large scale system of Helmholtz equation are sequential and parallel algorithms. The serial calculation of large scale system will increase the time execution and memory space (Nagaoka and Watanabe, 2012). To overcome this problem, parallel computing with DPCS will be used to increase the speedup.

1.1.3 Distributed Parallel Computer System (DPCS)

The DPCS platform is the computational tool to simulate the mathematical problem. The parallel computing is run in simultaneous manner and uses multiple computation resource to solve a computational problem (Said, 2006). The parallel computing is done by implement the sequential algorithm into parallel algorithm in solving large scale system. The DPCS is well suited to support the expensive computational of large scale system of algebraic equations. This is because the DPCS consists of number of processors for computing the sub-domain of a full grid solution. DPCS is developed by connecting several processors to a main memory. The main memory will manage the instructions and data storage. Each processor runs its own operating system using local memory and is connected with each other via a communication network (Bader *et al.*, 2005). Multiprocessors give more access memory to solve larger problems with high speedup since the single processors have limited size of memory.

The one and widely used way to classify parallel computers is called Flynn's taxonomy (Flynn, 1972). The taxonomy distinguishes multiprocessor computer architectures according to two independent dimensions of instruction and data. They can have only one of two possible states, either single or multiple. Table 1.1 shows

the four possible classifications of computer architecture. The architecture are single instruction single data (SISD), single instruction multiple data (SIMD), multiple instruction single data (MISD) and multiple instruction multiple data (MIMD).

Table 1.1: Classifications of parallel computer architecture

SISD single instruction single data	SIMD single instruction multiple data
MISD multiple instruction single data	MIMD multiple instruction multiple data

The SISD architecture is a serial or nonparallel computer. The architecture consists only one instruction stream on the processor and one data stream as an input at one clock cycle. This is the oldest architecture, most prevalent form of computer and it has deterministic execution. An example of SISD is illustrated in Figure 1.2.

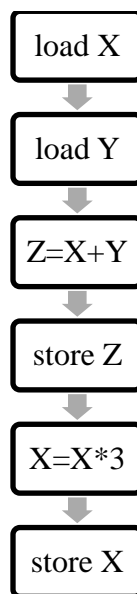


Figure 1.2 An example of SISD architecture

The SIMD is a type of parallel computer with all processors execute the same instruction and they can operate on a different data at any given clock cycle. The

SIMD allows all processors to receive the same instruction and executes simultaneously using different data set (Flynn, 1972). This type of architecture has an instruction dispatcher, high bandwidth internal network and large array of very small-capacity instruction units. The SIMD is best suited for specialized problems characterized by a high degree of regularity. It also consists of synchronous and deterministic execution. An example for this architecture is shown in Figure 1.3. From the figure, one instruction is used by all the processors p_1 , p_2 and p_n but different data are used to compute the solution.

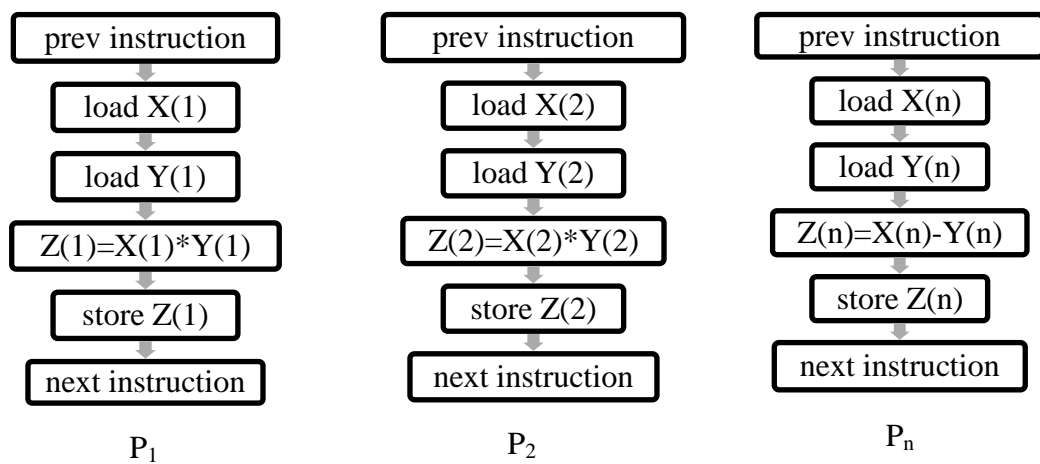


Figure 1.3 An example of SIMD architecture

The MISD theory exists but there are only a few actual examples have ever occurred. The current and most common type of parallel computer is MIMD. The architecture consists of different instruction stream with different data stream for every processor. The execution process can be synchronous or asynchronous, deterministic or non- deterministic. This architecture is used in most current supercomputers, networked parallel computer and symmetric multi processor (SMP) computers. An example for MIMD is shown in Figure 1.4.

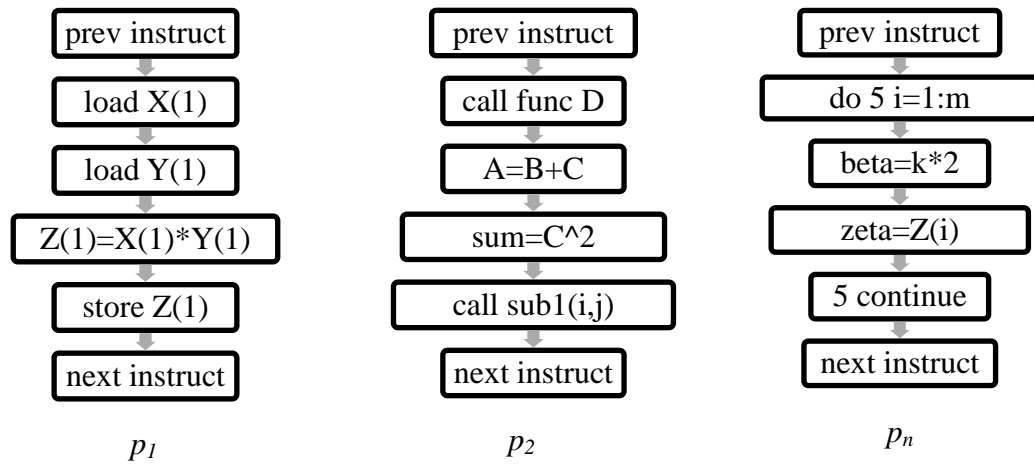


Figure 1.4 An example of MIMD architecture

The simulation of this research is focused on the SIMD architecture. Parallel algorithm of numerical method will be implemented on DPCS in Linux operating system environment.

1.1.4 Parallel Computational Platform

There are some software systems at operating system and programming language level that have been designed for parallel computers (Hamzah, 2011). The software system provides partitioning mechanism to separate and allocate sub-domain to the client processors. Some examples of parallel software are parallel virtual machine (PVM), message passing interface (MPI) and OpenMP. However, MPI is chosen as the communication software for MATLAB distributed computer system (MDCS). MDCS is available for homogenous collection of Linux environment and hooked by a local area network. The large scale system of Helmholtz equation, sequential and parallel algorithms will be solved using this environment system.

The configuration of DPCS contains 8 processors of Intel® *Core™Duo* processor architecture and MDCS version R2011a. The MDCS is oriented by libraries and toolboxes available for PDE. The MDCS is widely used as a mathematical computing environment because it supports multithread parallelism, distributed computing and explicit parallelism. The advantages of this software are lower-level parallel environment, higher-level toolbox, easier coding, users' friendly interface for message passing routines and greater control and performance. The architecture of MDCS is illustrated in Figure 1.5. The MDCS used server-client distributed model for message passing paradigm. The server will start a job manager to distribute domain equally to the clients while the clients will do the calculation until the solution is converged and send back the results to the server. There is also communication involved between clients in sending and receiving neighborhood data. The message passing paradigm between clients is via inter-connection network.

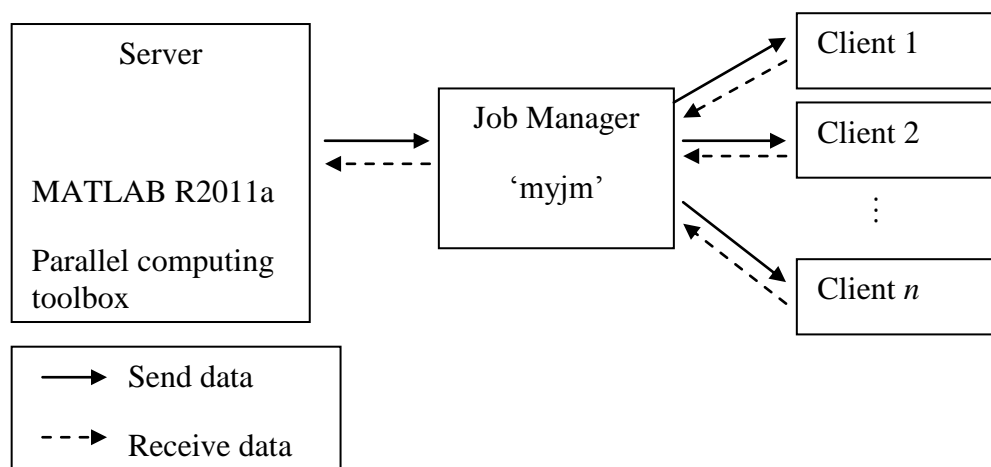


Figure 1.5 The MDCS architecture

The workflow of MDCS can be executed as follows.

- i. Start the MDCS service in cluster processors.
`./mdce start`
- ii. Start job manager on the server processor.

```
./startjobmanager -name <job manager name> -
remotehost<job manager hostname> -v
```

and also can be configured in the admin centre window by navigate to
cd/usr/share/bin/MATLAB/R2011a/bin/admincentre

iii. Start client processors and leech on the job manager.

```
./startworker -jobmanagerhost<job manager
hostname> -jobmanager<job manager name> -
remotehost<client hostname> -v
```

or can be start in the admin centre popup window.

The communication library and its activity in MDCS is based on application programming interface (API). The API is a software library with specification for routines, data structures and variables. By using MDCS, variable declaration is not needed before assigning a value because it has been declared in its library. Table 1.2 represents the communication commands to run the programs using MDCS.

Table 1.2: The parallel command in MDCS

Command	MDCS
Initial	pmode start <number of processors>
Send data	labSend(<data>,<destination>,<tag>)
Receive data	labReceive(<data>,<destination>,<tag>)
End program	pmode close
Reset	mpiprofile reset

There are four steps involved to develop the parallel algorithm of Helmholtz equation. The steps in developing the parallel algorithms are shown in Figure 1.6 (Foster, 1996). The steps are partition, communication, agglomerate and mapping. The partition and communication steps involved simultaneous and scaling characteristics while the agglomerate and mapping steps considered the characteristics of the efficiency and effectiveness of the mapping structure and communication among processors.

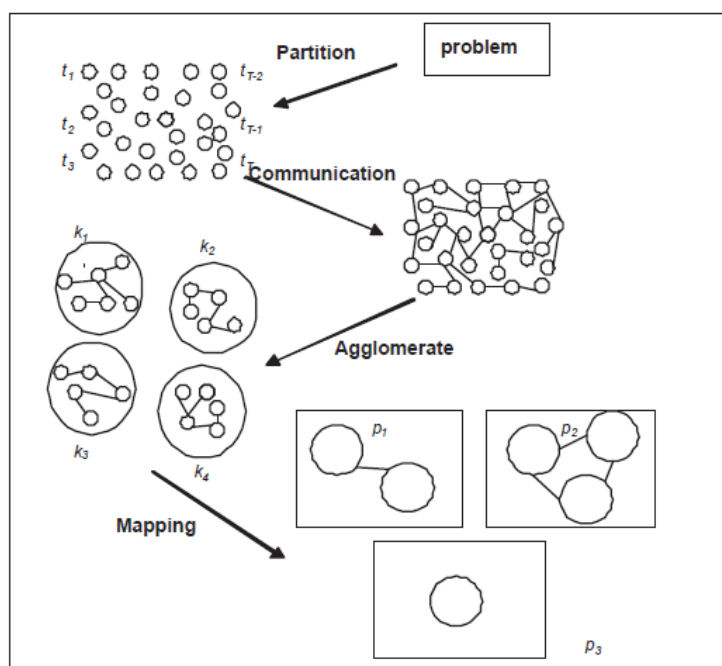


Figure 1.6 Four steps in developing parallel algorithms (Foster, 1996)

1.1.5 Parallel Performance Evaluation

In developing a parallel program, it is necessary to evaluate and validate the performance of the program. The evaluation process will give an insight to the programmer about the parallel program. The evaluation is based on execution time, speedup, efficiency, effectiveness and temporal performance.

Execution time (T_p) is the amount of time needed to complete run of a computer program routine. The notation p represents the number of processors used to compute the program. The execution time is a crucial measurement to be calculated in performance evaluation.

Speedup (S) is a measurement to determine how the programs scale as more processors are used. The speedup is defined as the time taken to complete an algorithm with a single processor divided by the time taken to complete the same algorithm with p processor. It can also be defined as

$$S = \frac{T_s}{T_p} \quad (1.1)$$

where T_s is the execution time using single processor and T_p is the execution time using p parallel processors. Speedup convey how fast the execution of the parallel program relative to the sequential program. The graph for speedup is super linear if the speedup equals to number of processors.

Efficiency is a measure of processor utilization for a parallel program. Efficiency is defined as a measure of the speedup achieved per processor. A hundred percent efficiency means all the processors are fully used all the time. The formula of efficiency is given by

$$E(p) = \frac{S(p)}{p}. \quad (1.2)$$

The value of $E(p)$ practically lies between 0 and 1, $0 \leq E \leq 1$. Overheads will be an obstacle in achieving high level of efficiency in parallel performance. The type of overheads impacts on the parallel performance are communication and idle time. Communication time is time spent to communicate and exchange data during the execution in each processor. The idle time is the time when processors become idle for waiting other processors to send messages. The idle time can happen when there is unbalance workload between processors.

Effectiveness (F_p) in using parallel algorithms can be determined by calculating the speedup and efficiency. The F_p can be computed based on the following formula

$$F_p = \frac{S(p)}{p \cdot T(p)} \quad (1.3)$$

and also can be defined as

$$F(p) = \frac{E(p)}{T(p)} = \frac{S(p)E(p)}{T(1)} \quad (1.4)$$

where $T(1)$ is the execution time of one processor. The formula is dependent on the speedup. When the speedup is increased, the effectiveness also will increase.

Temporal performance (R) is used to measure the performance of parallel algorithms. It is proportional to the execution time of p number of processors. The temporal performance can be defined as

$$R = \frac{1}{T_p}. \quad (1.5)$$

1.2 Helmholtz Equation

This research focuses on the early detection of breast cancer growth using Helmholtz equation with elliptic type (Gunnarsson, 2007; Alias *et al.*, 2009). Gunnarsson (2007) performed microwave imaging using tomography methods where a cross sectional slice of dielectric properties is generated. Cancerous and normal breast tissue are different in terms of water content, cell concentration and pressure

(Bounaimet *al.*, 2007). The high water content in malignant breast tissues cause significantly microwave scattering than low water content in normal fatty breast tissue. The combination of water content and breast cell is related to semi solid and semi liquid characteristic in wave phenomena.

Based on the wave phenomena, the Helmholtz equation is well suited to govern the cancer cell growth detection (Alias, 2008). The Helmholtz equation is a common scalar wave equation which describes the time harmonic electrical field in cancer growth situation of the incidence field is a vertically polarized and the object properties is homogenous along the vertical z-axis. The Helmholtz equation can be expressed as,

$$\left(\nabla^2 + K^2(r)\right)e(r) = 0. \quad (1.6)$$

Where

- ∇^2 : Laplacian operator
- K : The wave number of the electromagnetic
- $e(r)$: Total electric field
- r : Current in the electrical field

The Helmholtz equation and its simulation can express the sequential and parallel breast cancer cell using computational programming on DPCS.

1.3 Research Objectives

The research objectives of this study are as the following.

- i. To develop the sequential algorithms on Helmholtz equation based on some numerical methods such as JB, GS, RBGS and AGE.
- ii. To develop the parallel algorithms on Helmholtz equation based on some numerical methods such as JB, RBGS and AGE.
- iii. To analyze the breast cancer growth based on numerical analysis parallel performance evaluation on solving Helmholtz equation.

1.4 The Scope of Study

This study focused on the early detection of breast cancer growth using PDE of elliptic type. The FDM is chosen to discretize the elliptic Helmholtz equation. The approximation solutions of 1D and 2D problem are solved using MDCS. Sequential and parallel algorithms are developed to solve the problem. The numerical methods under consideration are JB, GS, RBGS and AGE methods. The scope of this research is depicted on Figure 1.7.

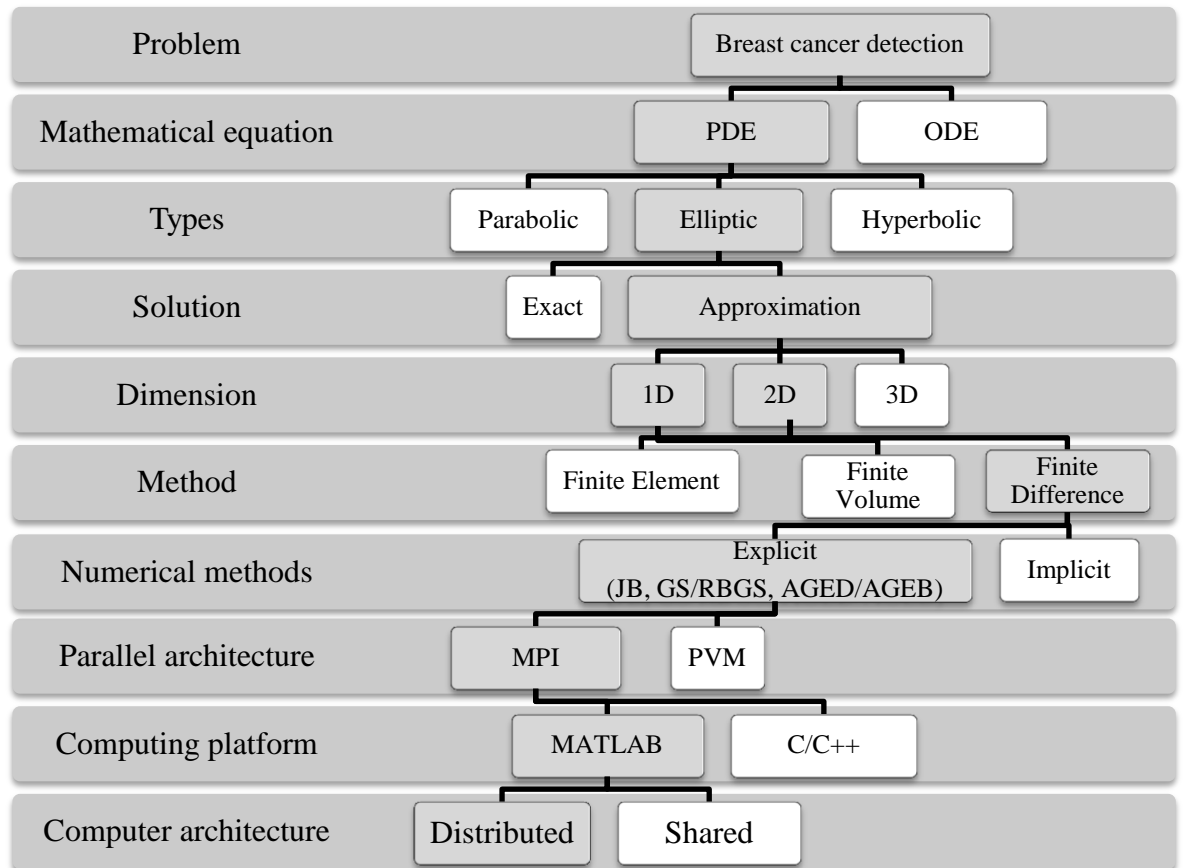


Figure 1.7 The research framework

1.5 The Outline

Chapter 1 is the research framework and discussed briefly about introduction, problem formulating, research objectives, scope of research and also the thesis outlines. The chapter also includes the importance and purpose of mathematical modeling in predicting the breast cancer growth. Some important terms related to numerical analysis and parallel performance evaluation such as convergence, consistency, stability, speedup, efficiency, effectiveness and temporal performance are also presented in this chapter.

Chapter 2 basically reviews on Helmholtz equation concept, corollary and discretization using finite difference method with three points central difference formula. There are multi-dimensional Helmholtz equations under consideration in solving the breast cancer growth detection problem which include 1D and 2D equations. The chapter also presents comparison of Helmholtz equation with medical images in terms of breast cancer growth.

Chapter 3 presents the development of sequential algorithms for the Helmholtz equation. The Helmholtz equation then will be solved using some numerical methods. The numerical methods are JB, GS, RBGS and AGE method. The classic numerical methods, JB, GS and RBGS are used for benchmarking.

Chapter 4 applies the scheme presented in Chapter 3 into parallel algorithms to improve the time execution when deals with large sparse matrices. The parallelization of multi-dimensional equation will used the same numerical methods in Chapter 3 except GS. The parallel computing is implemented on MDCS with distributed memory in message passing environment.

The numerical results obtained from Chapter 3 and parallel performance evaluation from previous chapter will be discussed in Chapter 5. The numerical analysis is based on execution time, computational complexity, consistency, stability, root mean square error (RMSE) and maximum error. For parallel performance evaluation, criterions under consideration are execution time, speedup, efficiency, effectiveness, temporal performance and granularity.

Chapter 6 draws the conclusion of this thesis. Contributions are highlighted and further studies are suggested.

REFERENCES

- Alias, N. (2004). Parallel strategies on a distributed parallel computer system. Project report. Universiti Teknologi Malaysia.
- Alias, N., Rosly, N. S., Ghaffar, Z. S. A., Satam, N., Islam, M. R. (2008). Early Detection of Breast Cancer Using Wave Elliptic Equation with High Performance Computing. *Proceeding of 2008 Student Conference on Research and Development (SCORED 2008)*.
- Alias, N., & Shahril, R. (2011, November). Semi-implicit method of fast active contour models using iterative method for edge detection. In *Proceedings of the 13th IASME/WSEAS international conference on Mathematical Methods and Computational Techniques in Electrical Engineering conference on Applied Computing* (pp. 155-163). World Scientific and Engineering Academy and Society (WSEAS).
- Alias N, Islam M. R., Shahir R, Hamzah H, Satam N, Safiza Z, Darwis R, Ludin E, Azami M. Parallel System for Abnormal Cell Growth Prediction Based on Fast Numerical Simulation. 6083 LNCS 2010: 222-231.
- Alias, N., Islam, M. R., Rosly, S. A. Dynamic PDE Solver for Breasts' Cancerous Visualization on Distributed Parallel Computing Systems. *Proceedings of the IASTED International Conference on Advances in Computer Science and Engineering, ACSE 2009*: 138-143.

- Alias, N., Sahimi, M. S., Abdullah AR. The AGEB Algorithm for Solving The Heat Equation In Two Space Dimensions and Its Parallelization on A Distributed Memory Machine. In :Traff JL, Benkner S, Dongarra J, editors. Recent advances in parallel virtual machine and message passing interface lecture notes in computer science. Springer; 2012. P. 214-221.
- Alias N. Development And Implementation Of Parallel Algorithms In The IADE And AGE Class Of Methods To Solve Parabolic Equations On A Distributed Parallel Computer Systems. Thesis of Doctor of Philosophy 2004. Universiti Kebangsaan Malaysia, Bangi.
- Bader, D. A., Hart, W. E., Phillips, C. A. (2005). Parallel Algorithm Design for Branch and Bound. *Tutorials on Emerging Methodologies and Applications in Operations Research International Series in Operations Research & Management Science*, 76, 5-1-5-44.
- Bounaim, A. (2007). Mathematics in Medicine: Bioacoustics Modeling and Computations for an Ultrasonic Imaging. Department of Informatics, University of Oslo, Norway.
- Bagnara, R., (1995). A Unified Proof for the Convergence Of Jacobi And Gauss-Seidel Methods. *Siam Review*, 37(1), 93-97.
- Bername (2012, October 13). More women may be dying of breast cancer in M'sia than statistics indicate, says expert. *The Star Online*. Retrieved August 19, 2013, from <http://www.thestar.com.my/>
- Brailsford, S. C., Harper, P. R., Sykes, J. (2012). Incorporating Human Behaviour in Simulation Models of Screening for Breast Cancer. *European Journal of Operational Research* 219, 491–507.
- Chaffer, C.L., Weinberg, R.A. (2011). A Perspective on Cancer Cell Metastasis. *Science* 331, 1559–1564.

- Chaplain, M., McDougall, S., Anderson, A. (2008). Mathematical Modeling Of Tumor-Induced Angiogenesis, *Annu.Rev.Biomed.Eng.*,8, 233–257.
- D'Yakonov, E.G., 1963. On the application of disintegrating difference operators, *Z. Vycisl. Mat. Mat. Fiz.*, 3: 385-388.
- Douglas, J., & Rachford, H. H. (1956). On the numerical solution of heat conduction problems in two and three space variables. *Transactions of the American mathematical Society*, 421-439.
- Evans, D. J. (1997). *Group Explicit Methods for the Numerical Solution of Partial Differential Equations*. Netherlands: Gordon and Breach Science Publishers.
- Evans D.J., Bai Z.Z. (1997). Matrix multisplitting relaxation methods for linear complementarity problems. *International Journal of Computer Mathematics* Volume 63, Issue 3-4.
- Evans, D.J., Sahimi, M.S., (1988). The Alternating Group Explicit(AGE) Iterative Method for Solving Parabolic Equations I: 2-Dimensional Problems. *Intern. J. Computer Math.* 24, 311–341
- Evans DJ, Sahimi MS (1989).The Alternating Group Explicit (AGE) Iterative Method to Solve Parabolic and Hyperbolic Partial Differential Equations.*Annual Review of Numerical Fluid Mechanics and Heat Transfer* 1989, 2, 283-389.
- Ferlay J, Bray F, Pisani P, Parkin DM. Globocan 2002: Cancer incidence, mortality and prevalence worldwide. IARC Cancer Base 2004
- Foster, I. (1996). *Designing and building parallel programs: Concepts and tools for parallel software engineering*. Inc: Addison-Wesley Publishing Company.

- Flynn, M. J. (1972). Some Computer Organizations and Their Effectiveness. *IEEE Trans. Comput.*, 948–960.
- Frieboes, H., Edgerton, M, Fruehauf, J., Rose, F., Worrall, L., Gatenby, R., Ferrari, M., Cristini, V. (2009). Prediction of Drug Response in Breast Cancer Using Integrative Experimental/Computational Modeling. *Cancer Res*, 60, 4484–4492.
- Gonzalez, F. J. (2007). Thermal Simulation of Breast Tumors. *Revista Mexicana De Fisica*, 53, 323-326.
- Gunnarsson, T. (2007). Microwave Imaging Of Biological Tissues: Applied Towards Breast Cancer Tumor Growth Detection. Department of Computer Science and Electronics, Mälardalen University Västerås, Sweden.
- Hadley, G. R. (2005). A Complex Jacobi Iterative Method For The Indefinite Helmholtz Equation. *Journal of Computational Physics*, 203, 358-370.
- Hamzah, N. (2011). The Parallelization of Direct and Iterative Schemes For Keller-Box Method Using Distributed Memory System. *Master Degree Thesis*, Universiti Teknologi Malaysia.
- Hinow, P., Gerlee, P. (2009). A Spatial model of tumor-host in interaction: application of chemotherapy. *Mathematical Biosciences and Engineering*, 6, 521-546.
- Konecky, S. D. (2008). Non-Invasive Imaging of Breast Cancer with Diffusing Near-Infrared Light. Physics and Astronomy, Faculties of the University of Pennsylvania.
- Mansor, N. A., (2012). Novel Fourth-order IADE methods for the solution of the heat equations. *PhD Thesis*, Universiti Tenaga Nasional, Malaysia.

- Mitchell, A.R. and G. Fairweather, 1964. Improved forms of the alternating direction methods of Douglas, Peaceman and Rachford for solving parabolic and elliptic equations. *Numer. Math.*, 6: 285-292.
- Nagaoka, T., Watanabe, S. (2012). Accelerating three-dimensional fdtd calculations on gpu clusters for electromagnetic field simulation. *Proceedings of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, 5691–5694.
- Omar ZA., Mohd Ali Z, Tamin INS (2011). National Cancer Registry Report 2007. Ministry of Health, Malaysia. 78.
- Othman MS, Yusuf LM, Basitt UNA, Alias N. The architecture of scientific computing library portal on sequential algorithm based on partial differential equation. *Proceeding 2010 International Symposium on Information Technology – System Development and Application and Knowledge Society, ITSIm'10 2010*:1109.
- Othman MS, Yusuf LM, Basitt UNA, Alias N. Characteristics determination on scientific computing library portal. *International Conference on Computer and Communication Engineering, ICCCE'10 2010*.
- Peaceman, D.W., (1977). *Fundamentals of Numerical Reservoir Simulation*. Elsevier Scientific Publishing Company, Amsterdam.
- Peaceman DW, Rachford HH (1955), The Numerical Solution of Parabolic and Elliptic Differential Equations. *Journal of The Society for Industrial and Applied Mathematics 1955*, 3, 28-41.
- Peskin, A.P., Hoepfner, D.J., Stuelten, C.H. (2011). Segmentation and Cell Tracking of Breast Cancer Cells. *International Symposium on Visual Computing 2011*, Part 1, 381-391.

- Popovic, J., Runborg, O. (2011). Analysis of a Fast Method for Solving the High Frequency Helmholtz Equation in One Dimension. Springer: BIT Numer Math, 721-755.
- Ping, S. Y. (2006). The development of Two Dimensional Brain Tumors' Algorithm On Distributed Parallel Computer Systems. *Master Degree Thesis*, Universiti Teknologi Malaysia.
- Said, N. M. (2006). Mathematical Modelling For Brain Tumour Growth Using High Performance Computing. *Master Degree Thesis*, Universiti Teknologi Malaysia.
- Shahril, R. (2013). Two Dimensional Active Contour Model On Multigrids For Edge Detection Of Images. *Master Degree Thesis*, Universiti Teknologi Malaysia.
- Sinek, J., Sanga, S., Zheng, X., Frieboes, H., Ferrari, M., Cristini, V. (2009). Predicting Drug Pharmacokinetics And Effect In Vascularized Tumors Using Computer Simulation. *J. Math. Biol.*, 58, 485–510.
- Stevenson, C.E. (1995). Statistical Models for Cancer Screening. *Statistical Methods in Medical Research* 4, 18–32.
- Teh, C. R.C., (2005). Kaedahberangka: matematik untuk sains dan kejuruteraan menggunakan MAPLE.
- Varga, S. R., (2000). Matrix iterative analysis, Springer Series in Computational Mathematics, Springer-Verlag.
- Wilkinson, B. and Allen, M. (2004). *Parallel Programming: Techniques and Applications Using Networked Workstations and Parallel Computers*. Prentice-Hall Inc.
- Yanenko, N. N. (1971). *The method of fractional steps* (pp. 2427-2435). Springer-Verlag, New York.

Zubaidin, Z. M. (2013). Alternating Group Explicit Method For Edge Detection On Brain And Breast Tumour Images. *Master Degree Thesis*, Universiti Teknologi Malaysia.