SOLVING DAMPED WAVE EQUATION USING FINITE DIFFERENCE METHOD AND INTERPOLATION USING CUBIC B-SPLINE

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To my beloved mother and father,

Rohani Binti Ibrahim and Arzmi Bin Deraman

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ABSTRACT

Damped wave equations have been used particularly in the natural sciences and engineering disciplines. The purpose of this study is to apply the technique of finite difference and cubic B-spline interpolation to solve one dimensional damped wave equation with Dirichlet boundary conditions. In this study, the accuracy of numerical methods are compared with exact solution by computing their absolute error and relative error. The computational experiments are conducted using Matlab 2008 and visualisation using Microsoft Excel 2010. As the result, finite difference method and cubic B-spline interpolation are found to give good approximation in solving damped wave equation. In addition, the smaller time step size, T gives better approximations for both finite difference and cubic B-spline interpolation.

ABSTRAK

Persamaan gelombang lembap telah digunakan terutamanya dalam bidang sains semula jadi dan kejuruteraan. Kajian ini adalah bertujuan untuk menggunakan kaedah beza terhingga dan interpolasi B-splin kubik bagi menyelesaikan persamaan gelombang lembap satu dimensi dengan syarat sempadan Dirichlet. Dalam kajian ini, ketepatan kaedah-kaedah berangka dibandingkan dengan penyelesaian tepat dengan mengira ralat mutlak dan ralat relatif masing-masing. Keputusan pengiraan dijalankan dengan menggunakan Matlab 2008 dan visualisasi dengan menggunakan Microsoft Excel 2010. Hasilnya, kaedah beza terhingga dan interpolasi B-splin kubik memberikan anggaran yang baik dalam menyelesaikan persamaan gelombang lembap. Tambahan lagi, saiz langkah masa, T yang lebih kecil memberikan anggaran yang lebih baik kepada kedua-dua kaedah beza terhingga dan interpolasi B-splin kubik.

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

A partial differential equation (PDE) is a mathematical equation in the form of

$$F(x, y, \ . \ . \ . \ , u, \ u_x, \ u_y, \ . \ . \ . \ , u_{xx}, \ u_{yy}, \ . \ . \ . \ ) = 0,$$

that involves two or more independent variables $x, y, \ . \ . \ .$, an unknown function u and partial derivatives $u, \ u_x, \ u_y, \ . \ . \ . \ , u_{xx}, \ u_{yy}, \ . \ . \ .$ of the unknown function u with respect to the independent variables. The order of a partial differential equation is the order of the highest derivative involved. A solution (or a particular solution) to a partial differential equation is a function that solves the equation or, in other words, turns it into an identity when substituted into the equation [1]. PDE is used to formulate problems involving functions of several variables, and are either solved by hand, or used to create a relevant computer model. PDE can be used to describe a wide variety of phenomena such as sound, wave, heat, electrostatics, electrodynamics, fluid flow, elasticity and quantum mechanics. These seemingly distinct physical phenomena can be formalised similarly in terms of PDE [2].

In PDE, there are three types of equation namely elliptic, parabolic and hyperbolic equation. The example of elliptic equations are Poisson equation and Laplace equation that arise in the temperature or voltage distribution while the simplest example of parabolic equation is heat equation. Hyperbolic equations arise in wave mechanic, gas dynamics, vibrations and other areas [3]. The most frequent hyperbolic equation that always discussed is one dimensional wave equation which is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \ .$$

However, this research is focused on damped wave equation which include the damping factor in the above equation. Hence, the damped wave equation is given as

$$\frac{\partial^2 u}{\partial t^2} = \ c^2 \frac{\partial^2 u}{\partial x^2} - a \frac{\partial u}{\partial t} .$$

To be specific, this research is about solving damped wave equation of PDE using finite difference method (FDM) and interpolation using cubic B-spline.

1.2 Problem Statement

In this research, the finite difference method and cubic B-spline interpolation are used to solve the damped wave equation since the analytical solutions are usually difficult to obtain. Among several numerical methods, finite difference method is widely used for its simplicity. The cubic B-spline interpolation is applied in order to overcome the weakness of polynomials which have oscillating properties and consist of a high number of arithmetical operations involved in the calculations of polynomial.

1.3 Objectives of the Study

The objectives of this study are:

1. To study the finite difference method (FDM) and cubic B-spline interpolation.
2. To study the algorithm of FDM and cubic B-spline interpolation.
3. To solve damped wave equation using FDM and cubic B-spline interpolation.
4. To determine the accuracy of FDM and cubic B-spline interpolation by computing their absolute error and relative error.

1.4 Scope of the Study

In this research, the method of finite difference and cubic B-spline are applied which is focuses in one dimensional damped wave equation. The derivatives in finite difference method is approximated to centered difference and forward difference only. Beside that, this study is only tested on Dirichlet boundary conditions.

1.5 Significance of the Study

The results of finite difference method and cubic B-spline interpolation are compared with exact solutions. The method is very useful to solve real-world application since analytical solution is normally hard to obtain.

REFERENCES

1. Berg, P. W. and McGregor, J. L. *Elementary Partial Differential Equations*. United States of America: Holden-Day. 1966

2. Evans, L. C. *Partial Differential Equations*. Providence: American Mathematical Society. 1998

3. Teh, C. R. C. *Numerical Method*. Jabatan Matematik, Fakulti Sains: Universiti Teknologi Malaysia. 2009

4. Martin, W. T. and Spanier, E. H. *Introduction to Partial Differential equations and Boundary Value Problems*. United States of America: McGraw-Hill. 1968

5. John, F. *Partial Differential Equations*. 3rd. ed. United States of America: Springer-Verlag. 1978

6. Kothari, S. J. *Classification Of Partial Differential Equations And Their Solution Characteristics*. 8th. ed. IIT Rookee, India: Indo-German Winter Academy. 2009

7. Boyce, W. E. And Dilprima, R. C. *Elementary Differential Equations and Boundary Value Problems*. 9th. ed. New York: John Wiley & Sons. 2010

8. Lu, X. *The Applications of Microlocal Analysis in ¾-evolution Equations*. Ph.D. Thesis. Zhejiang University, Hangzhou. 2010

9. Jradeh, M. On the damped wave equation. *12th Intern. Conference on Hyperbolic Problems*. June 9-13, 2008. Maryland, USA.

10. Hancock, M. J. *Solutions to Problems for the 1-D Wave Equation*. United States of America: McGraw-Hill. 2004

11. Atkinson, K. E. *An Introduction to Numerical Analysis*. 2nd. ed. Canada: John Wiley & Sons. 1988

12. Anderson, J. D. *Computational Fluid Dynamics: The Basics with Application*. New York: McGraw-Hill. 1995

13. Urroz, G. E. *Numerical Solution to Ordinary Differential Equations*. United States of America: Holden-Day. 2004

14. Lee, E. T. Y. A Simplified B-Spline Computation Routine: *Computing A Practical Guide to Splines*, 1982. 29 (4): 365–371.

15. Lee, E. T. Y. Comments on some B-spline Algorithms: *Computing A Practical Guide to Splines*, 1986. 36 (3): 229–238.

16. Zhu, C. G. and Kang, W. S. Applying Cubic B-Spline Quasi-Interpolation to Solve Hyperbolic Conservation Laws. *U.P.B. Sci. Bull, Series D*. 2010. 72(4): 49-58.

17. Jiang, Z. and Wang, R. An Improved Numerical Solution Of Burger's Equation by Cubic B-Spline Quasi-Interpolation. *Journal of Information and Computational Science 7*. 2010. 5: 1013-1021.

18. Goh, J., Majid, A. A. and Ismail, A. I. M. Numerical Method using Cubic B-Spline for the Heat and Wave Equation. *Computers and Mathematics with Application*. 2011. 62: 4492-4498.

19. Liu, X., Huang, H. and Xu, W. Approximate B-Spline Surface Based on Radial Basis Function (RBF) Neural Networks. *Springer-Verlag*. 2005. LNCS 3514: 995-1002.

20. Chen, Q. *et al*. A B-Spline Approach for Empirical Mode Decompositions. *Advanced in Computational Mathematics*. 2006. 24: 171-195.

21. Minh, T. N., Shiono, K. and Masumoto, S. Application of General Cubic B-Spline Function (GCBSF) for Geological Surface Simulation. *International Symposium on Geoinformatics for Spatial Infrastructure Development in Earth and Allid Sciences*. 2006

22. Dauget, J. *et al*. Alignment of large image Series using Cubic b-Spline Tessellation: Application to Transmission Electron Microscopy Data. *Spriger-Verlag*. 2007. LNCS 4792: 710-717.

23. Kadalbajoo, M. K., Tripathi, L. P. and Kumar, A. A Cubic B-Spline Collocation Method for a Numerical Solution of the Generalized Black-Scholes Equation. *Mathematical and Computer Modelling*. 2012. 55: 1483-1505.

24. Goh, J., Majid, A. A. and Ismail, A. I. M. Cubic B-Spline Collocation Method for One Dimensional Heat and Advection-Diffusion Equations. *Journal of Applied Mathematics*. 2012. 10: 1155-1163.

25. Greville, T. N. E. *Theory and applications of Spline Functions*. New York: Academic Press. 1975