EXACT SOLUTIONS OF FREE CONVECION FLOW IN ROTATING MAGNETOHYDRODYNAMICS SECOND GRADE FLUID PAST AN INFINITE VERTICAL PLATE

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”TO MY BELOVED FAMILY ESPECIALLY ABAH, MA, ABANG AND ANGAH”

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In the name of Allah, the Most Gracious, and the Most Merciful. All praises belong to Him, the Lord of the Worlds. May shalawat and peace be upon the Prophet Muhamad SAW, his family and companions, and also for the people who are following and continuing the right path. I am so grateful to Allah SWT, for helping me, His weak servant to finish my Master. Thanks to my supervisor Assoc. Prof. Dr Sharidan Shafie, Dr Ilyas Khan, parent, family and friends. They are truly supporters, best advisors and the great partners.

ABSTRACT

The second grade fluid is one of the most popular subclass of non-Newtonian fluids. Due to their importance in industry and engineering, most of researchers have concentrated on the flows dealing with these fluids. Therefore, the main purpose of this thesis is to obtain exact solutions for unsteady free convection flows of rotating second grade fluid with the effects of isothermal and ramped wall temperatures. Specifically, a free convection flow is studied in the presence of magnetohydrodynamics and porosity. Using the constitutive equations, the governing equations are modeled. Some suitable non-dimensional variables are used to write these equations into non-dimensional form. Laplace transform method is used to solve these equation with imposed initial and boundary conditions. Solutions for velocity and temperature fields are obtained. Skin friction and Nusselt number are also evaluated. For the sake of physical understanding, analytical results for velocity are plotted graphically for the emerging flow parameters. As the velocity is a complex function, the graphs for both real and imaginary parts are shown separately. Both cases, isothermal and ramped wall temperatures, are discussed. It is observed that when the second grade parameter increases, the velocity shows an oscillating behavior which decreases and then increases, for both real and imaginary parts. For larger values of rotation parameter, the fluid velocity decreases for the real part whereas it increases for the imaginary part. It is further noted that velocity decreases when magnetic parameter increases for both real and imaginary parts. However, an opposite behavior is observed when the porosity parameter is increased. Both real and imaginary parts of the velocity are found to increase with increasing values of the porosity parameter. An interesting result for the velocity is observed from the comparison of ramped and isothermal temperatures. It is found that fluid moves slowly in case of ramped wall temperature compared to isothermal case. In limiting cases, the present solutions are found identical to published results.

ABSTRAK

Bendalir gred kedua adalah salah satu subkelas bendalir bukan Newtonian yang paling popular. Disebabkan oleh kepentingannya dalam industri dan kejuruteraan, kebanyakkan penyelidik lebih tertumpu kepada aliran yang berkaitan dengan bendalir ini. Oleh itu, tujuan utama tesis ini adalah untuk mendapatkan penyelesaian tepat bagi aliran olakan bebas tak mantap bendalir gred kedua yang berputar dengan kesan sesuhu dan suhu tanjakan dinding. Secara khususnya, aliran olakan bebas dikaji dengan kehadiran hidrodinamik magnet dan keliangan. Menggunakan persamaan juzuk, persamaan menakluk dimodelkan. Beberapa pembolehubah tak bermatra digunakan untuk menulis persamaan ini ke dalam bentuk tak bermatra. Kaedah penjelmaan Laplace digunakan bagi menyelesaikan persamaan tersebut beserta syarat awal dan syarat sempadan yang dikenakan. Penyelesaian bagi medan halaju dan medan suhu diperoleh. Geseran kulit dan nombor Nusselt juga dinilai. Untuk memahami secara fizikal, keputusan analitik bagi medan halaju dan medan suhu diplot secara grafik bagi parameter aliran yang terlibat. Oleh kerana halaju adalah fungsi kompleks, maka graf untuk kedua-dua bahagian nyata dan khayalan ditunjukkan secara berasingan. Kedua-dua kes iaitu sesuhu dan suhu tanjakan dinding adalah dibincangkan. Dapat diperhatikan bahawa apabila parameter gred kedua meningkat, halaju menunjukkan tingkah laku yang berayun dimana pada mulanya berkurangan dan kemudian meningkat untuk kedua-dua bahagian nyata dan khayalan. Bagi nilai parameter putaran yang besar, halaju bendalir berkurangan untuk bahagian nyata manakala meningkat untuk bahagian khayalan. Selanjutnya, dapat dilihat bahawa, halaju berkurangan apabila parameter magnet meningkat untuk kedua-dua bahagian nyata dan khayalan. Namun, tingkah laku yang bertentangan diperhatikan apabila parameter keliangan bertambah. Kedua-dua bahagian nyata dan khayalan bagi halaju didapati meningkat apabila nilai parameter keliangan meningkat. Satu keputusan yang menarik bagi halaju dapat dilihat daripada perbandingan antara suhu tanjakan dan sesuhu. Bendalir didapati bergerak dengan perlahan bagi kes suhu tanjakan dinding berbanding dengan sesuhu. Dalam kes terhad, penyelesaian yang diperoleh dalam kajian ini didapati secaman dengan keputusan yang telah diterbitkan.

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LIST OF SYMBOLS

Roman Letters

$\mathbf{A_1}$	-	first Rivilin-Ericksen tensor
$\mathbf{A_2}$	-	second Rivilin-Ericksen tensor
$\mathbf{B}$	-	Total magnetic field
$\mathbf{B_0}$	-	applied magnetic field
B_0	-	magnitude of applied magnetic field
$\mathbf{b_1}$	-	induced magnetic field
c_p	-	specific heat at constant pressure
$\frac{d}{dt}$	-	material time derivative
$\mathbf{div}$	-	divergence
$\mathbf{E}$	-	electric field
erf	-	error function
$erfc$	-	complementary error function
$\exp$	-	exponential function
$\mathbf{b}$	-	gravitational acceleration
Gr	-	thermal Grashof number
$H(.)$	-	Heaviside function
$\mathbf{I}$	-	identity tensor
$\mathbf{J}$	-	current density
K	-	dimensionless porosity parameter
k	-	thermal conductivity

$\mathcal{L}$	-	Laplace transform
$\mathcal{L}^{-1}$	-	Inverse Laplace transform
M	-	dimensionless magnetic parameter
K	-	dimensionless porosity parameter
Nu	-	Nusselt number
p	-	scalar pressure
p^*	-	modified pressure gradient
Pr	-	Prandtl number
$\mathbf{R}$	-	Darcy's resistance
q	-	Laplace transform parameter
$\mathbf{T}$	-	Cauchy stress tensor
T	-	temperature of the fluid near the plate
t	-	dimensionless time
t_0	-	characteristic time
F	-	complex velocity
u	-	velociy in x-direction
v	-	velociy in y-direction
$\mathbf{V}$	-	velocity vector field
$\bar{\Omega}$	-	angular velocity vector
Ω	-	constant angular velocity
x	-	dimensionless coordinate axis of the plate
y	-	dimensionless coordinate axis of the plate
z	-	dimensionless coordinate axis normal to the plate

Greek Letters

α_1, α_2	-	material moduli or normal stress moduli
α	-	dimensionless second grade parameter
β	-	volumetric coefficient of thermal expansion
ρ	-	density
σ	-	electrical conductivity
ν	-	kinematic viscosity
μ	-	dynamic viscosity
ω	-	dimensionless rotating parameter
ϕ	-	porosity of the medium
τ	-	dimensionless skin friction
$I_0(.)$	-	modified bessel function of order zero
$I_1(.)$	-	modified bessel function of order one

Subscripts

w	-	conditions on the wall
∞	-	free stream condition

Superscript

T	-	transpose operation

Abbreviations

ram	-	ramped temperature
iso	-	isothermal temperature

CHAPTER 1

INTRODUCTION

This chapter discusses the research background, problem statement, research objectives, scope of the research, significance of the study, research methodology and thesis outline.

1.1 Research Background

Generally there are mainly two types of fluids namely Newtonian and non-Newtonian. Newtonian fluids are obeying Newtons law of viscosity. Their shear stress at each point is linearly proportional to the strain rate at that point and the constant of proportionality is known as viscosity. This concept was first introduced by Isaac Newton. Mathematically the relationship of shear stress to the shear rate in Newtonian fluid is given by

$$\tau_{yx} \propto \frac{du}{dy} \tag{1.1}$$

equivalently

$$\tau_{yx} = \mu \frac{du}{dy} \tag{1.2}$$

where τ_{yx} is shear stress, μ is dynamic viscosity and du/dy is the rate of strain or velocity gradient. In common terms, this means the fluid continues to flow, regardless of the forces acting on it. Examples of Newtonian fluids include gases and liquids such as water, hydrocarbons, oils and air.

On the otherhand, there are several fluids where shear stress is not proportional linearly to the velocity gradient are called non-Newtonian fluids. For example ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo. This behavior may be represented by the power law model as:

$$\tau = k\left(\frac{du}{dy}\right)^n ; n \neq 1, \tag{1.3}$$

where n is called flow behavior index and k is the flow consistency index. When the values of $n = 1$ and $k = \mu$, equation (1.3) reduces to equation (1.2) called Newtons law of viscosity. Non-Newtonian fluids are usually divided into three main categories which are differential type, rate type and integral type. Differential and rate type models are used to describe the response of fluids that have slightly memory such as dilute polymeric solutions, while the integral models are used to describe materials such as polymer melts that have considerable memory. One of the most popular subclass of differential type of fluids is called the second grade fluid also known as a viscoelastic fluid. This fluid model was first proposed by Coleman and Noll in (1960). It is found in polymer fluids where these fluids exhibit both the viscous and elastic characteristics. Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied. Whereas, the elastic materials strain instantaneously when stretched and quickly return to their original state once the stress is removed.

The study of free convection flow of second grade fluid has been carried out as an important application in many industries. Free convection or natural convection is the flow that induced by buoyancy forces which arises from density differences caused by temperature variations in the fluid. An example is the free convection heat transfer that occurs from hot components on a hot egg in still air as depicted in Figure 1.1. In this situation, cool air that makes contact with the hot components experiences an increase in temperature and therefore reduction in density. Since the warm air is now lighter than surrounding air, buoyancy forces induce a vertical motion and the hot air rising from the egg is replaced by the inflow of air at room temperature.

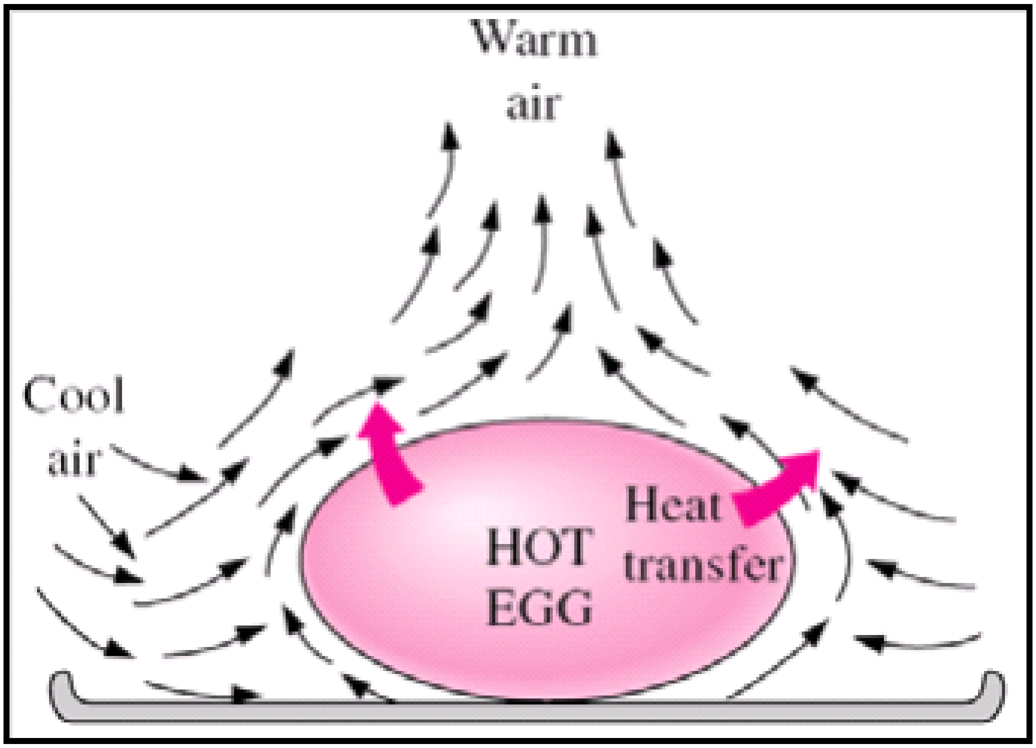

Figure 1.1: Free Convection Phenomena

In this case of heat transfer process, buoyancy forces are responsible for the fluid motion and viscous forces oppose the fluid motion. Having such motivation in mind, several authors have discussed the free convection flow in their research. Some recent attempts are made by Lahurikar (2010) and Vijayalakshm(2010).

Research works in magnetohydrodynmics (MHD) free convection flows has been conducted extensively in recent years. The MHD flows is the study of the interaction of conducting fluids with electro magnetic phenomena. The flow of an electrically conducting fluid in the presence of a magnetic field is importance in various areas of technology and engineering such as liquid metals, plasma and salt water or electrolytes. Several solutions in this case were obtained by many researchers such as Sajid *et al.* (2008), Das *et al.* (2009) and Rajesh (2010) where different boundary conditions have been considered as well as physical situation on flow formation. The word MHD is derived from magneto means magneti field, hydro is for liquid and dynamics is for movement. The concept of MHD was introduced by Hannes Alfven for which he received the Nobel prize in physics in 1970.

Further, the effect of magnetic field on flows through a porous medium have gained the interest of researhers in fluid studies such as Hayat *et al.* (2008a), Hayat *et al.* (2008b), Khan *et al.* (2010) and Salah *et al.* (2011). The interest in this field is due to the wide range of applications either in engineering or in geophysics, such as the optimization of the solidification process of the metals and metal alloys, the study of geothermal sources, treatment of nucleur fuel debris, the control of under ground spreding of chemical wastes and pollutants. By porous medium we mean the materials containing pores or voids (like a hole) on the skeletal portion of the material is often called the matrix or frame. The pores are typically filled with a fluid like liquid or gas. Usually, solid be a skeletal material, but in certain case, foams also can be a structure to analyse the concept of porous medium. The concept of porous medium is usually to characterised the permeability and tensile strength of porosity in the skeletal portion.

In addition, the study of the fluid flow in rotating frame has drawn considerable interest in recent years due to its wide range of applications in designing thermo syphon tubes, in cooling turbine blades, jet engines, pumps and vacuum cleaners, as well as geophysical flows. Rotating plate such that both the fluid and the plate rotate in uniform angular velocity with Coriolis effect is acting on it has investigated by researchers before this. In physics, the Coriolis effect is a deflection of moving objects in the frame rotating in the opposite direction. For example, when the frame rotates in clockwise direction, the moving object will deflect to the left. If the frame rotates in counter-clockwise, then the deflection of object will move to the right. This effect is very important for earths rotation causing freely moving objects to veer toward the right in the Northern Hemisphere and to the left in the Southen Hemisphere. This phenomenon happened in wind, ocean currents, airplanes and missiles.

Moreover, the analysis of the temperature effect on the plate especially when the plate rotates is very important in industry activities for producing the product in good quality. Example of temperatures are constant temperature, constant heat flux, Newtonian heating and ramped wall temperature. Ramped wall temperature is one of the most popular thermal conditions for energy equation to investigate the problems subject to step change in wall temperature. This problem has been discussed by Chandran *et. al* (2005), Seth *et al.* (2011), Deka (2011) and Samiulhaq *et al.* (2014).

Based on the above discussion, it is interesting to study the behavior of the fluid motion influenced by the ramped wall temperature. Therefore, the present study aims to investigate the unsteady free convection flow of an incompressible second grade fluid in rotating infinite vertical plate with the effects of constant and ramped wall temperatures. In addition, the problem of rotating MHD free convection flow of second grade fluid in a porous medium is also studied.

1.2 Statement of Problem

Previous study shows that the flow of Non-newtonian fluid play an important role in fluid flow problems compared to the Newtonian fluid. Non-newtonian fluid such as second grade fluid has attracted many researches to study their fluid behavior when various embedded fluid flow characteristic parameters are considered. Mostly, the theoretically study of unsteady free convection flow of second grade fluid have been conducted in vertical plates. However, only a few researchers considered the problem of convective flow involve with rotating plate. Even, the expressions of the exact solutions obtained in the previous study for the problem of the flow in infinite rotating vertical plates are conducted only in viscous fluid. Therefore, study to explore the mathematical model for the problem of unsteady free convection flow of a second grade fluid in a vertical rotating frame is significant. This thesis emphasized on this matter. The derivation of the mathematical model also included with magnetic field and porosity effects. The analytical solution involving with constant and ramped wall temperature are obtained. The influenced of parametric physical conditions that affected the fluid flow and heat transfer characteristics are analyze and discussed.

1.3 Objectives of the Study

This research aims to construct a mathematical model by formulating an appropriate governing equation, to solve the governing equations analytically using Laplace transform method and to analyze the obtained results for the considered flow problem. The main objectives of this study are to find exact solutions of the velocity and temperature profiles for the problems of unsteady free convection flow of second grade fluid in a rotating infinite vertical plate and rotating MHD second grade fluid in porous medium. Both problems are solved subjected to constant and ramped wall temperatures, respectively.

1.4 Scope of the Study

This research focused on two dimensional unsteady free convection flow of incompressible second grade fluid in rotating frame. Specifically, the infinite vertical plate with constant temperature and ramped wall temperature are considered. The first two problems are only focus on the fluid motion induced by the infinite vertical rotating plate. Whereas, the third and fourth problems are considered the problem of rotating fluid in the presence of MHD flow in a porous medium. Even, these problems are able to solve by Fourier transform or other analytical method, but only Laplace transform method is applied to obtain the exact solutions in this study. MATHCAD software is used for graphical results.

1.5 Significance of Study

The results obtained from this research are significant due to some reasons. This research is enable to enhance the knowledge of the MHD flow and heat transfer characteristics through a porous medium for rotating second grade fluids in vertical infinite plate. This research also provided accurate exact solutions for the mathematical models involving constant temperature and ramped wall temperature. These exact solutions can be used to check the accuracy of the results obtained through numerical schemes.

1.6 Research Methodology

This section discussed the present development of research which are including research design and procedures of the study. This project will undertake the following research methodology.

1.6.1 Mathematical Analysis

The governing equations are obtained according to the proposed problems those have been mentioned in objectives of the study by adopting the continuity, momentum and energy equations. The modified Darcy's law for electrically conducted second grade fluids are used in governing equations. The incompressible second grade fluid in rotating frame has been chosen in solving isothermal and ramped wall temperatures problems. The partial differential equations governing the flow and the appropriate initial and boundary conditions are transformed into non-dimensional forms by using some suitable dimensionless variables. The dimensionless system of equations are solved analytically by using the Laplace transform method. After that, the inverse

Laplace transforms are used to obtain the exact solution of the problems.

1.6.2 Numerical Computation

The exact solutions of the problems are analyzed through graphs involving emerging parameters such as second grade α, rotation ω, magnetic field M, porosity K, Prandtl number $\Pr$ and Grashof number Gr. All graphs are plotted by using MATHCAD software. Moreover, the graphical results are also used to ensure the solutions are satisfied with the imposed initial and boundary conditions. The results are verified by comparing the limiting cases of the present work with the existing solutions available in the literature especially focus on rotating parameter. Non-rotating fluid can be obtained as a limiting case from our general solutions when the rotation parameter approaches zero.

1.7 Thesis Organization

This thesis consists 7 chapters. First, we discuss Chapter 1 that involves research background, statement of problem, objectives of study, scope of study, significance of study, research methodology, mathematical analysis and numerical computation. The details literature review regarding our problems identified in the objectives of research are discussed in Chapter 2. In Chapter 3, the problem of unsteady free convection flow of rotating second grade fluid in infinite vertical plate is presented. The case of isothermal temperature is considered. The governing equations of the problem are formulated by using body force and stress tensors. Dimensionless variables are introduced to simply the dimensional governing equations as well as appropriate initial and boundary conditions. Laplace transform method is used to obtain the analytical solutions. The velocity and temperature profiles are plotted with the effects of embedded flow parameters. Finally, the results of skin friction and Nusselt number are displayed in tables.

Chapter 4 investigates the effect ramped wall temperature on unsteady free convection flow of rotating second grade fluid in infinite vertical plate. Expressions for velocity profiles are obtained by using Laplace transform method and inverse Laplace transforms. Velocity profiles are plotted to investigate the behavior of the fluid flow for various emerging parameters. In this case, we have compared the ramped wall temperature with isothermal temperature. Afterwards, problem in Chapter 5 is extended from problem in Chapter 3. The effect of MHD flows in porous medium is considered. The modified Darcy's law for the second grade fluid is also incorporated. The Laplace transform method is used to obtain the solutions of velocity and temperature. As in Chapter 3, the velocity profiles are plotted according to parameters involved in this problem. The values of skin friction and Nusselt number are discussed in tables. Chapter 6 discussed the problem of Chapter 5 under the influence of ramped wall temperature. Similar to the previous chapter, the governing equations are solved and plotted for the emerging flow parameters. The comparison of isothermal and ramped wall temperature for both real and imaginary parts of velocity is also shown. Further the variation of skin friction for different parameters in real and imaginary is also discussed.

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