APPLICATION OF FEM AND FDM IN SOLVING 2D IRREGULAR GEOMETRY HEAT TRANSFER PROBLEM

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Dear my beloved family

Abah, Mak, Along, Abang Apis, Yan, Anip, Adik Nani
Alfiy Hafeez, Aisya Hannah

Dear my committed supervisor

Dr Yeak Su Hoe

Dear all precious friends

Thank you

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ABSTRACT

The study will focus on the application of finite element method (FEM) in solving two dimensional irregular geometry heat transfer problem. FEM is known as one of the numerical technique for finding approximate solutions to boundary value problems for differential equation. Therefore this method is a perfect application to solve a parabolic partial differential equation such as heat equation that describes the distribution of heat in a given region over time. In order to minimize an error function and produce a stable solution, FEM uses variation method or also known as the calculus of variations. All the solutions to this method related to the given problem will be compared with the solution from another popular numerical method which is finite difference method (FDM). FDM is also very useful in approximating the solutions to differential equation. However the application is limited to regular geometry and simple irregular geometry problems. The comparison of the solutions from these two methods will confirm that FEM is a better choice in solving two dimensional irregular geometry problems involving heat transfer.

ABSTRAK

Kajian ini akan bertumpu kepada penggunaan kaedah unsur terhingga untuk menyelesaikan masalah yang melibatkan perpindahan haba dalam geometri dua dimensi yang tidak sekata. Kaedah unsur terhingga adalah salah satu kaedah berangka bagi mencari penyelesaian anggaran untuk masalah nilai sempadan bagi persamaan pembezaan. Maka dengan itu kaedah ini adalah pilihan yang tepat untuk menyelesaikan masalah berkaitan perpindahan haba kerana persamaannya turut melibatkan nilai sempadan untuk persamaan pembezaan. Bagi meminimumkan kesalahan fungsi and mendapatkan penyelesaian yang lebih stabil, kaedah unsur terhingga ini menggunakan kaedah variasi atau turut dikenalai sebagai kalkulus variasi. Semua penyelesaian terhadap masalah yang diberi akan dibandingkan dengan penyelesaian daripada satu lagi kaedah berangka yang popular iaitu kaedah perbezaan terhingga. Kaedah perbezaan terhingga juga sangat berguna untuk mendapatkan anggaran jawapan kepada persamaan pembezaan. Tetapi aplikasi kaedah ini limit hanya untuk dimensi yang sekata dan dimensi tidak sekata yang mudah sahaja. Perbandingan antara dua kaedah ini akan membuktikan bahawa kaedah unsur terhingga merupakan pilihan ynag lebih baik untuk menyelesaikan masalah perpindahan haba dalam geometri dua dimensi yang tidak sekata.

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LIST OF SYMBOLS

δ	delta
ψ^T	global virtual temperature vector
T^e	displacement vector
K^e	the stiffness matrix
F^e	load vector
Q	heat generation
E_i	truncation error
π	pi
n	normal flux
T	temperature
A_e	area of the element
det(J)	determinant of Matrix J
S_T	specified temperature
S_q	specified heat flux
S_c	convection

LIST OF ABBREVIATIONS

FEM	Finite Element Method
FDM	Finite Difference Method
2D	Two Dimensional
BVP	Boundary Value Problem

CHAPTER 1

INTRODUCTION

1.0 Introduction

A numerical method is a technique for obtaining approximate solutions of many types of science and engineering problems. The need for numerical methods arises from the fact that for most practical engineering problems their analytical solutions do not exist. While the governing equations and boundary conditions can usually be written for these problems, difficulties introduced by either irregular geometry or other discontinuities are difficult to be solved analytically.

Finite Element Method (FEM) and Finite Difference Method (FDM) are the examples of numerical methods that can be used to solve this kind of problems. FEM is a numerical method for solving problems of engineering and mathematical physics. It is useful for problems with complicated geometries, loadings and material properties where analytical solutions cannot be obtained. It is a numerical technique in finding approximate solutions to boundary value for differential equations problems of engineering and mathematical physics.

Heat transfer is the study of thermal energy transport within a medium or among neighbouring media by molecular interaction, fluid motion, and electro-magnetic waves, resulting from a spatial variation in temperature (Long and Seyma, 2009). This variation in temperature is governed by the principle of energy conservation, which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system, and the energy stored and converted within the system, is zero. Heat transfer finds application in many important areas, namely design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipment, catalytic convertors, heat shields for space vehicles, heaters, electronic equipment, internal combustion engines, refrigeration and air conditioning units, design of cooling systems for electric motors generators and transformers, heating and cooling of fluids in chemical operations, construction of dams and structures, minimization of building heat losses using improved insulation techniques, thermal control of space vehicles, heat treatment of metals and dispersion of atmospheric pollutants.

Basically, the two-dimensional heat equation can be solved theoretically and also numerically by using numerical method such as the finite difference method. Heat equation is one of the most important partial differential equation which describes the distribution of heat (or variation in temperature) in a given region over time.

1.1 Background of the Study

This research is focusing on solving a two-dimensional irregular geometry heat transfer equation by applying the finite element method. Normally, computations of this method will be performed using Matlab program since it is impossible to do the calculations manually for large scale problem and also in order to assure the accuracy of the solution. Finite element method involves tedious and complex calculation. Thus, it is ideal to be applied numerically in Matlab program.

1.2 Problem Statement

Grid or mesh based numerical methods such as finite difference methods (FDM) and finite element methods (FEM) have been widely applied to various areas such as fluid dynamics and solid mechanics. These methods are currently the dominant methods in solving problems of engineering and science.

Problems related to heat transfer can be solve by using finite difference method (FDM). However this approach will cause a few flaws in the findings mainly because the method itself is low in accuracy. Although FDM is easier to compute as well as to code, some problems that involved complex geometry is either difficult or completely cannot be solved by FDM. Generally FDM is a simple method to use for common problems defined on regular geometries.

The two sources of error in FDM are round-off error which is the loss of precision due to computer rounding of decimal quantities and the local truncation error typically expressed using Big-O notation that refers to the error from a single application of the method. Besides, the majority of heat transfer problem is related to engineering and the problem domain is in irregular geometry. It is not ideal to solve this kind of problems using FDM since it is only applicable for problems that involve regular geometry and simple irregular geometry.

Even though FEM is a complex method but it can be used to solve complex geometry problems. Industrial or engineering heat transfer problems normally involve irregular geometry domain. The need for numerical methods arises from the fact that analytical solutions does not exist for most practical problems.

On the other hand FEM uses variation methods to minimize an error function and produce a stable solution. Therefore this method is mostly ideal for problems with complicated geometries when analytical solutions cannot be obtained. For that reason, this research will focus on how to solve heat transfer problems in two–dimensional irregular geometry by using the application of FEM.

1.3 Objectives of the Study

The main objectives of this research are:

1. To study the basic concept of FEM, FDM and heat transfer.
2. To associate the application of FEM and FDM in solving problems related to heat transfer.
3. To solve heat transfer problems in simple 2D irregular geometry.
4. To develop algorithm using Matlab to get numerical results.
5. To compare the solutions from the application of FEM and FDM.

1.4 Scope of the Study

This research will focus on the application of FEM with the approach of Galerkin's method to solve heat transfer problems in simple 2D irregular geometries while comparing the solutions with FDM.

1.5 Significance of the Study

The results of this research can be used for further research in related areas. One of the goals of this research is that, this research will gives a clearer view of FEM, where it will finally lead to further understanding on applying the FEM in solving related problems especially in the science and engineering fields.

1.6 Report Organization

In this report of dissertation, there will be five chapters which are introduction, literature review, method of solution, results and discussion and conclusion and recommendation.

As for Chapter 1 there will be seven subtopics which are introduction, background of study, problem statement, objective of the study, scope of the study, significant of the study and organization of report.

Chapter 2 is going to introduce a literature review on FEM and FDM also its application in solving heat transfer problem.

The formulation and the methods that will be used are discussed in Chapter 3. This chapter provides the modeling and derivation of FEM, FDM and heat transfer equation.

Then all the results that are obtained from Matlab will be discussed in Chapter 4. The results will be presented in figures and table.

Finally Chapter 5 is for the conclusion of this dissertation and recommendation for further research.

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