

HYBRID LATTICE-BOLTZMANN SIMULATION OF CONVECTIVE FLOW IN
A CHANNEL WITH EXTENDED SURFACES

ALIREZA FAZELI

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To my beloved parents and wife

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ABSTRACT

Laminar convective flow in a channel with extended surfaces mounted at the bottom wall is investigated by using two methods. First, by the usual double-population SRT lattice Boltzmann method (LBM) and second by a hybrid scheme in which the flow is solved by single population LBM and the thermal field by the finite-difference (FD) technique with considering an appropriate coupling among them. Here, the iterative method has been chosen in order to solve the discretized energy equation with finite-difference. The transient Reynolds number for the condition of this study was determined to be 600 and all simulations were conducted in the laminar range of Reynolds numbers. It is shown that for CFD problems in which the steady state solution is desired or for those with time consistency it is possible to save computation time of the simulation remarkably by employing the aforementioned hybrid scheme. For the case study of this work, the hybrid scheme resulted in reduction of 18 percent of total simulation time.

ABSTRAK

Aliran perolakan lamina di dalam saluran dengan permukaan lanjutan di dasar saluran dikaji dengan menggunakan dua kaedah. Pertama, dengan kaedah biasa iaitu double population SRT melalui kaedah Lattice Boltzmann (LBM) dan yang kedua adalah dengan menggunakan kaedah hibrid di mana medan aliran diselesaikan dengan menggunakan single-population LBM manakala medan haba diselesaikan dengan menggunakan teknik perbezaan terhingga (FD) dengan mempertimbangkan gandingan yang sesuai untuk mereka. Di sini, kaedah iteratif telah dipilih untuk menyelesaikan persamaan tenaga dengan menggunakan kaedah perbezaan terhingga. Nombor Reynolds sementara untuk keadaan kajian ini telah ditentukan pada 600 dan semua simulasi telah dijalankan di julat nombor Reynolds yang lamina. Ia menunjukkan bahawa untuk masalah CFD yang memerlukan penyelesaian keadaan tenang atau bagi mereka dengan memerlukan konsistensi masa, ia menunjukkan penjimatan masa pengiraan simulasi yang banyak dengan menggunakan skim hibrid tersebut. Bagi kajian kes ini, skim hibrid menghasilkan pengurangan 18 peratus daripada jumlah masa simulasi.

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LIST OF SYMBOLS

C_i	Discrete particle velocity in each discrete direction
C_s	Speed of sound
$D2Q9$	Two-dimension nine-velocity LBM
f	Distribution function in discrete Boltzmann equation
FD	Finite-difference method
FE	Finite-element method
FV	Finite-volume method
LBE	Lattice Boltzmann equation
LBM	Lattice Boltzmann method
LBM -FD	Hybrid scheme of LBM and FD
m	Physical molecular mass
Ma	Mach number
Re	Reynolds number
t	Time
u	Macroscopic flow velocity in the LBM
ν	Kinematic viscosity in the LBM
ρ	Dimensionless fluid density in the LBM
τ	Single relaxation time of Boltzmann equation

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CHAPTER 1

INTRODUCTION

1.1 Background

Conventional sources of energy have been depleting at an alarming rate, which makes future sustainable development of energy use very difficult. This concern led to a recent expansion of efforts to produce efficient heat transfer equipment and generated increasing needs for understanding of fluid flow. Thus, heat transfer enhancement technology plays an important role and it has been widely applied to many applications. Suitable heat transfer augmentation techniques can result in considerable technical advantages and savings of costs. Channels with extended surfaces, because of their effectiveness in heat transfer are suitable candidates for engineering applications [1-3].

Figure 1.1 shows a rectangular channel with extended surfaces (ES). The ES's can be applied with different sizes and shapes on the inner surface of the channel. The transverse turbulators such as ribs or grooves break the laminar sub-layer and create local wall turbulence due to flow separation and reattachment between successive ribs, which reduce the thermal resistance and significantly enhance the heat transfer [3-4].

Geometrical characteristics such as duct aspect ratio, ES height, ES angle-of-attack, ES shape, and relative arrangement of the ES's (in-line, staggered, etc.), play significant role in the rate of heat transfer [5]. By use of suitable ES geometry, a large amount of heat can be transferred between a wall and a fluid in the channel

with less unit size and even with very small temperature difference. During the work, using ES's enhances not only the heat transfer, but also results in considerable pressure loss and pumping power as well.

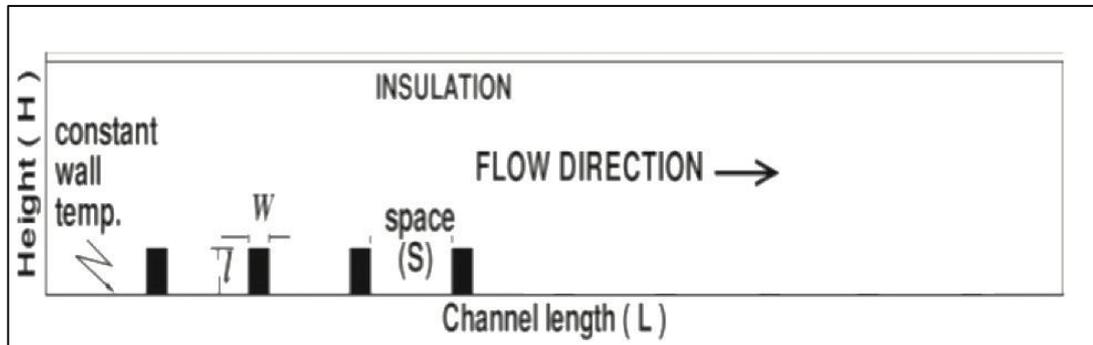


Figure 1.1 Two-dimensional channel with extended surfaces

Therefore, such channels are of interest for practical problems and here are taken as the case study to predict the behavior of the working fluid i.e., air, using three different CFD techniques, which are a mix of conventional, and the Lattice-Boltzmann (LBM) methods in order to determine the least computational time consuming one. LBM has been proven as a reliable CFD method but due to certain inherent limitations when it comes to simulation of coupled-variables problems e.g. fluid flow and temperature field, it has been suggested to employ the hybrid scheme of the LBM in which flow is solved by the LBM while the temperature gradient by a conventional CFD method e.g., finite-difference, and an appropriate coupling among them is implemented. Although this hybrid scheme is widely used but no literature address their required computational time compared with that problems that are solved by the LBM for the both variables i.e., velocity and temperature fields.

1.2 Problem statement and scope of the study

The laminar forced convective flow in a channel with inner extended surfaces on the lower wall is studied numerically. The industry requirements and demands for higher heat transfer devices have become very important factor in the design of

engineering systems. In the field of heat exchanger performance, it is known that utilization of ES's can potentially lead to better heat transfer performance due to mixing of the fluid and increasing of heat transfer area [6]. Many investigations have been already performed in order to study heat transfer rate/enhancement in such geometries but here the focus is on the CFD method by a tradeoff between LBM-based CFD methods with considering their computational resources to have them compared. While researchers use hybrid scheme of the LBM it is of interest to compare its computational time compared with that of where both field are solved by the LBM say, LBM-LBM. If the computational time of the hybrid scheme i.e., LBM-FD, is bigger than LBM-LBM, finding a way to decrease the LBM-FD computational time is of interest since, by its implication we not only have been freed of the LBM limits (for coupled flows) but also have reduced the required computational time that makes the proposed method an asset.

The numerical simulation in this study is performed first by the usual LBM for both the velocity and temperature gradients along the channel (LBM-LBM) then while LBM is used again to simulate the velocity field, a conventional CFD technique i.e., finite difference (FD), is used to solve the unsteady advection-diffusion equation to obtain the temperature field (LBM-Unsteady-FD) and lastly, the flow is solved by the LBM while FD is used to solve the steady advection-diffusion equation (LBM-Steady-FD) and finally their required computational recourses are compared.

1.3 Application of the study

Due to limitations on solving fluid related problems analytically and importance of having prediction of the fluid's behavior for any practical use before build, computational fluid dynamics (CFD) places at the center of attention. Due to the vast usage of CFD in engineering problems, researches in the field have been always trying to propose new methods in order to have the simulations in less computation time and have a more accurate result. LBM was proposed about twenty years ago and due to its calculation that is done locally in the grid, it is a suitable

method for parallel computing as it is of second order accuracy. Problems have been experienced that when it comes to simulation of coupled problems LBM cannot handle the problem for the second variable (dependent variable) i.e., here is temperature, because the range of the relaxation time (RT) for the second variable is imposed by the condition of the first variable i.e., here is velocity, as result although we can define the range of the RT in a way that locates in the valid range for Single Relaxation Time Lattice-Boltzmann scheme (SRT-LBM) but, the RT for the second variable is out of our control and may become invalid due to the condition of the first variable so, it is proposed to use the hybrid scheme of LBM where, the velocity field is handled by the usual SRT-LBM and the temperature field by a conventional CFD method that here finite-difference is chosen. Hence, by having a technique that comprises SRT-LBM and FD, it becomes possible to take advantage of the LBM model to handle the velocity the negatives sides of the SRT-LBM are eliminated by replacing an FD technique. If we could propose a hybrid scheme that also be capable to offers less computation time comparing with that of LBM-LBM, we have taken one step further in enhancement of current available CFD techniques that can handles the coupled problems more efficiently and requires less computational resources.

1.4 Objectives

The objectives of this study are as follows:

1. To obtain 2-D simulation for the channel by D2Q9-SRT-LBM for both the velocity field and temperature distribution.
2. To obtain 2-D simulation for the channel by D2Q9-SRT-LBM for the velocity field and finite-difference (FD) for unsteady advection-diffusion equation to acquire temperature distribution.
3. To obtain 2-D simulation for the channel by D2Q9-SRT-LBM for the velocity field and FD for steady-state advection-diffusion equation to acquire temperature distribution.

4. To compare the results and specifically computational time required for among the three above-mentioned simulation methods and determine the least time consuming one.

1.5 Thesis outlines

This thesis is divided into five chapters as follows:

Chapter 1 presents the problem statement, scope of this study, applications of the study and the objectives of the project.

Chapter 2 contains the literature review that is related to the fluid flow, heat transfer problems in rectangular channels and numerical studies with different CFD methods. This chapter covers the available literature on hybrid schemes of the Lattice-Boltzmann method in specific.

Chapter 3 provides the implementation methodology over each of the simulation methods concerns here i.e., 1- LBM-LBM, 2- LBM-unsteady-FD and 3- LBM-steady-FD.

Chapter 4 discusses the results obtained from each methods and compare them followed by Chapter 5 where the conclusions of this study are made.

REFERENCES

1. Shokouhmand, H., Vahidkhah, K. and Esmaeili M.A. Numerical Analysis of Air Flow and Conjugated Heat Transfer in Internally Grooved Parallel-Plate Channels. World Academy of Science, Engineering and Technology, 2011. 73.
2. Manca, O., Nardini, S., Ricci, D. A. Two-Dimensional Numerical Investigation on Forced Convection in Channels With Transversal Ribs. ASME Conference Proceedings. 2009.
3. Lienhard, J.H., John, H., Lienhard, V.A. Heat transfer textbook third edition. Phlogiston press Cambridge, Massachusetts. USA. 2004.
4. Kamali, R. and Binesh, A.R. The importance of rib shape effects on the local heat transfer and flow friction characteristics of square ducts with ribbed internal surfaces. International Communications in Heat and Mass Transfer, 2008. 35:1032-1040.
5. Jansangsuk, D., Khanoknaiyakarn, C. and Promvong, P. Experimental study on heat transfer and pressure drop in a channel with triangular V-ribs. Energy and Sustainable Development: Issues and Strategies (ESD). Proceedings of the International Conference on Heat Transfer, 2010. 1-8.
6. Webb, R.L. Principles of enhanced heat transfer. John Wiley & Sons, 1994.
7. Alamyane, A.A., Mohamad, A.A. Simulation of forced convection in a channel with extended surfaces by the lattice Boltzmann method. Computers and Mathematics with Applications, 2010. 59:2421-2430.
8. Noble, D.R. Comparison of accuracy and performance for lattice-Boltzmann and finite-difference simulations of steady viscous flow. Int. J. for Numerical Methods in Fluids, 1996. 23:1-18.
9. Mezrhab, A. Analysis of radiation–natural convection in a divided enclosure using the lattice Boltzmann method. Computers & Fluids, 2007. 36:423-434.

10. Lallemand, P. Theory of the lattice Boltzmann method: Acoustic and thermal properties in two and three dimensions. *Physical Review E*, 2003. 68:1-25.
11. Lallemand, P., Hybrid finite-difference thermal lattice Boltzmann equation. *Int. J. of Modern Physics B*, 2003. 17:41-47.
12. Sloot, P.M.A. Boundary conditions for thermal lattice Boltzmann simulations. *ICCS 2003, LNCS 2657*, 2003. 977-986.
13. Mezrhab, A. Hybrid lattice-Boltzmann finite-difference simulation of convective flows. *Computers & Fluids*, 2004. 33:623-641.
14. Yong, S. Finite difference based lattice Boltzmann simulation of natural convection heat transfer in a horizontal concentric annulus. *Computers & Fluids*, 2006. 35:01-15.
15. Albuquerque, P. A hybrid lattice Boltzmann finite difference Scheme for the diffusion equation. PhD Thesis, University of Geneva, 2005.
16. Tölke, J. A thermal model based on the lattice Boltzmann method for low Mach number compressible flows. *J of Computational and Theoretical Nanoscience*, 2006. 3:1-9.
17. Huang, H. Hybrid lattice Boltzmann finite-difference simulation of axisymmetric swirling and rotating flows. *Int. J for Numerical Methods in Fluids*, 2007. 53:1707-1726.
18. Mohammed Jami, Lattice Boltzmann method applied to the laminar natural convection in an enclosure with a heat-generating cylinder conducting body. *Int. J. of Thermal Sciences*, 2006. 46:38-47.
19. Mohammed Jami, Numerical study of natural convection in a cavity of high aspect ratio by using the lattice Boltzmann method. *Int. J. for Numerical Methods in Engineering*, 2008. 73:1727-1738.
20. Liou, T.M., Chen, S.H. and Shih K.C. Numerical simulation of turbulent flow field and heat transfer in a two-dimensional channel with periodic slit ribs. *International Journal of Heat and Mass Transfer*, 2002. 45:4493-4505.
21. Won, S.Y., Mahmood G.I. and Ligrani, P.M. Flow structure and local Nusselt number variations in a channel with angled crossed-rib turbulators. *International Journal of Heat and Mass Transfer*, 2003. 46:3153-3166.
22. Korichi, A., Oufer, L. Numerical heat transfer in a rectangular channel with mounted obstacles on upper and lower walls. *International Journal of Thermal Sciences*, 2005. 44:644-655.

23. Chaube, A., Sahoo, P.K. and Solanki, S.C. Analysis of heat transfer augmentation and flow characteristics due to rib roughness over absorber plate of a solar air heater. *Renewable Energy*, 2006. 31:317-331.
24. Korichi, A. and Oufar, L. Heat transfer enhancement in oscillatory flow in channel with periodically upper and lower walls mounted obstacles. *International Journal of Heat and Fluid Flow*, 2007. 28:1003-1012.
25. Promvonge, P. and Thianpong, C. Thermal performance assessment of turbulent channel flows over different shaped ribs. *International Communications in Heat and Mass Transfer*, 2008. 35:1327-1334.
26. Thianpong, C., Chompookham, T., Skullong, S. and Promvonge, P. Thermal characterization of turbulent flow in a channel with isosceles triangular ribs. *International Communications in Heat and Mass Transfer*. 2009. 36:712-717.
27. Chompookham, T., Thianpong, C., Kwankaomeng, S. and Promvonge P. Heat transfer augmentation in a wedge-ribbed channel using winglet vortex generators. *International Communications in Heat and Mass Transfer*, 2010. 37:163-169.
28. Alamgholilou, A. and Esmailzadeh, E. Experimental investigation on hydrodynamics and heat transfer of fluid flow into channel for cooling of rectangular ribs by passive and EHD active enhancement methods. *Experimental Thermal and Fluid Science*, 2012. 38:61-73.
29. Satta, F., Simoni, D. and Tanda G. Experimental investigation of flow and heat transfer in a rectangular channel with 45° angled ribs on one/two walls. *Experimental Thermal and Fluid Science*, 2012. 37:46-56.
30. Eiamsaard, S. and Promvonge, P. Numerical study on heat transfer of turbulent channel flow over periodic grooves. *International Communications in Heat and Mass Transfer*, 2008. 35:844-852.
31. Adachi, T., Tashiro, Y., Arima, H. and Ikegami, Y. Pressure drop characteristics of flow in a symmetric channel with periodically expanded grooves. *Chemical Engineering Science*, 2009. 64:593-597.
32. Eiamsaard, S. and Promvonge, P. Thermal characteristics of turbulent rib-grooved channel flows. *International Communications in Heat and Mass Transfer*, 2009. 36:705-711.

33. Trisaksri, V. and Wongwises, S. Critical review of heat transfer characteristics of nanofluids. *Renewable and Sustainable Energy Reviews*, 2007. 11:512-523.
34. Ahmed, M.A., Shuaib, N.H., Yusoff, M.Z. and Al-Falahi A.H. Numerical investigations of flow and heat transfer enhancement in a corrugated channel using nanofluid. *International Communications in Heat and Mass Transfer*, 2011. 38:1368-1375.
35. Mohammed, H.A., Al-Shamani, A.N. and Sheriff, J.M. Thermal and hydraulic characteristics of turbulent nanofluids flow in a rib-groove channel. *International Communications in Heat and Mass Transfer*, 2012. 39:1584-1594.
36. Versteeg, H.K. and Malalasekera W. *An introduction to computational fluid dynamics. the finite volume method*. Harlow, Longman, 1996.
37. Mohamad, A.A. *Lattice Boltzmann Method - Fundamentals and engineering applications with computer codes*. Springer, 2011.
38. Griebel, M., Dornseifer, T. and Neunhoefer, T. *Numerical simulation in uid dynamics: a practical introduction*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1998.
39. Al-Jahmany, Y.Y. Comparative study of lattice-Boltzmann and finite volume methods for the simulation of non-Newtonian fluid flows through a planar contraction. PhD thesis, 2004.
40. Zienkiewicz, O.C., Taylor, R.L., and Nithiarasu, P. *The Finite Element Method for Fluid Dynamics, Sixth Edition*. Butterworth-Heinemann, Oxford, 6th edition, December 2005.
41. Canuto, C., Quarteroni, A., Hussaini, M.Y. and Zang, T. *Spectral methods in fluid dynamics*. Berlin, Germany, 1988.
42. Succi, S. *The lattice Boltzmann equation for fluid dynamics and beyond*, Oxford University Press, 2002.
43. Peters, A., Melchionna, S., Kaxiras, E. and Succi, S. Multiscale Simulation of Cardiovascular flows on the IBM Bluegene/P: Full Heart-Circulation System at Red-Blood Cell Resolution. SC '10 Proceedings of the 2010 ACM/IEEE International Conference for High Performance Computing, Networking, Storage and Analysis, 2010.

44. Frisch, U., Hasslacher, B., and Pomeau, Y. Lattice-gas automata for the Navier- Stokes equation. *Phys. Rev. Lett.*, 1986. 56:1505-1508.
45. Hardy, J., Pomeau, Y., De Pazzis, O. Time evolution of a twodimensional model system. I. Invariant states and time correlation functions. *J. Math. Phys.*, 1973. 14:1746-1755.
46. McNamara, G.R. and Zanetti, G. Use of the Boltzmann Equation to Simulate Lattice-Gas Automata. *Phys. Rev. Lett.*, 1988. 61:2332-2335.
47. Frisch, U., d'Humieres, D., Hasslacher, B., Lallemand, P., Pomeau, Y., Rivet, J.P. Lattice gas hydrodynamics in two and three dimensions. *Complex Systems*, 1987. 1:649-707.
48. Higuera, F.J., and Jimenez, J. Boltzmann approach to lattice gas simulations. *Europhys. Lett.*, 1989. 7:663-668.
49. Koelman, J., A simple lattice Boltzmann scheme for Navier-Stokes fluid flow. *EPL (Europhysics Letters)*, 1991.
50. Lallemand, P., Luo, L.S. Theory of the lattice Boltzmann method: Dispersion, dissipation, isotropy, Galilean invariance, and stability. *Phys. Rev. E*, 2000. 61:6546-6562.
51. He, X., and Luo, L.S. Theory of the lattice Boltzmann method: From the Boltzmann equation to the lattice Boltzmann equation. *Phys. Rev. E*, 1997. 56:6811-6817.
52. Ansumali, S. and Karlin, I.V. Single relaxation time model for entropic lattice Boltzmann methods. *Phys. Rev. E*, 2002. 65:0563-0573.
53. Humieres, D., Ginzburg, I., Krafczyk, M., Lallemand, P. and Luo, L.S. Multiple- relaxation-time lattice Boltzmann models in three dimensions. *Phil. Trans. R. Soc. A*, 2002. 360:437-451.
54. Eggels, J. Direct and large-eddy simulation of turbulent fluid flow using the lattice- Boltzmann scheme. *International Journal of Heat and Fluid Flow* Volume, 1996. 17.
55. Ladd, A.J.C. Lattice-Boltzmann simulations of particle-fluid suspensions. *Journal of Statistical Physics*, 2001.
56. Gunstensen, A., Rothman, D. and Zaleski, S. Lattice Boltzmann model of immiscible fluids. *Physical Review A*, 1991.
57. Grunau, D., Chen, S. and Egger, K. A Lattice Boltzmann Model for Multi-phase Fluid Flows, 1993.

58. Shan, X. and Chen, H. Lattice Boltzmann model for simulating flows with multiple phases and components. *Phys. Rev. E*, 1993. 47:1815-1819.
59. Peters, A., Melchionna, S., Kaxiras, E., Sircar, J., Bernaschi, M., Bison, M. and Succi, S. Multiscale Simulation of Cardiovascular flows on the IBM Bluegene: Full Heart-Circulation System at Red-Blood Cell Resolution. *SC '10 Proceedings of the 2010 ACM/IEEE International Conference for High Performance Computing, Networking, Storage and Analysis*, 2010.
60. Wolf-Gladrow, D. *Lattice-Gas Cellular Automata and Lattice Boltzmann Models - An Introduction*. Springer, 2005.
61. Dubois, F. and Lallemand, P. Towards higher order lattice Boltzmann schemes. *J. Stat. Mech*, 2009.
62. Gellera, S., Krafczyka, M., Turekb, S. and Hronb, J. Benchmark computations based on lattice-Boltzmann, finite element and finite volume methods for laminar flows. *Computers and Fluids*, 2006. 35(8).
63. Yu, H. and Zhao, K. Lattice Boltzmann method for compressible flows with high Mach numbers. *Phys. Rev. E*, 2000. 61:3867-3870.
64. Sbragaglia, M., Benzi, R., Biferale, L., Chen, H., Shan, X. and Succi, S. Lattice Boltzmann method with self-consistent thermo-hydrodynamic equilibria. *Journal of Fluid Mechanics*, 2009.
65. Prasianakis, N.I. *Lattice Boltzmann method for thermal compressible flows*. PhD thesis, ETH, 2008.
66. Filippova, O. and Hanel, D. A novel lattice BGK approach for low Mach number combustion. *J. Comput. Phys.*, 2000. 158:136–160.