# FINITE ELEMENT FORMULATION FOR FREE VIBRATION OF COMPOSITE BEAMS

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## ABSTRACT

Investigations concerning the dynamic response of composite beams with partial interaction are scarce. Derivation of the differential equations describing the interaction between composite elements normally involves with the solution of high order system of equations, which closed form solutions are difficult. This study concerns with the finite element formulation of composite beams for free vibration. The formulation involves with the establishment of the stiffness matrix and the mass matrix of the beam. The former was obtained through extremization of the total potential energy (or Hamilton principle for dynamic) whilst the latter was obtained by lumping the elements mass at nodes. Natural frequencies of the beam were obtained as eigenvalues. These were then verified by existing analytical solution.

**ABSTRAK**

Penyelidian membabitkan respon terhadap daya dinamik pada rasuk komposit yang tidak mempunyai interaksi sepenuhnya diantara dua permukaan bahan adalah terhad. Menerbitkan persamaan terbitan yang melibatkan interaksi diantara dua elemen komposit biasanya memerlukan penyelesaian terhadap persamaan yang mempunyai kuasa yg tinggi. Oleh itu, penyelesaian menggunakan kaedah langsung (terus) adalah sukar. Kajian ini berkaitan dengan formulasi element unsur terhingga (finite element) rasuk komposit yang melibatkan getaran bebas (tiada daya dinamik). Formulasi tersebut memerlukan penggunaan matriks kekukuhan dan juga matriks jisim. Matriks kekukuhan boleh diperolehi setelah membezakan (extremizating) Jumlah Tenaga Keupayaan ( Total Potential Energy ) manakala matriks jisim diperolehi dengan menjumlahkan jisim setiap elemen pada nod masing-masing. Frekuensi rasuk tersebut diperolehi dalam bentuk nilai eigen (eigenvalues). Semua ini kemudiannya akan diverifikasi oleh penyelesaian analitik yang sedia ada.

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| | | |
|---|---|---|
| $M$ | - | Moment |
| $z$ | - | Distance between the neutral axis of concrete and steel |
| $\mu$ | - | Coefficient of friction |
| $T$ | - | Normal stress at the interface |
| $H$ | - | External axial force |
| $w$ | - | Bending deformation |
| $u$ | - | Horizontal displacement |
| $q$ | - | Intensity of loading |
| $\Pi$ | - | Total Potential Energy |
| $U$ | - | Strain energy |
| $V$ | - | Load potential |
| $\omega$ | - | Natural frequency |
| $E$ | - | Modulus of elasticity |
| $I$ | - | Second moment area. |
| $\rho$ | - | Density |
| $A$ | - | Cross-sectional area |
| $k_s$ | - | Modulus of shear connector |
| $L$ | - | Span of the beam |

# CHAPTER 1

# INTRODUCTION

## 1.1 INTRODUCTION

Beams composed of two or more elements are commonly used to span large openings. If the elements of such a beam are interconnected, the elements act simultaneously and thus the load carrying capacity of the beam is greater than the sum of the individual's capacities. A steel-concrete composite beam consists of concrete and steel acting as a beam together. Example of composite beams are reinforced concrete beam (RC), conventional composite beam (i.e. composite beam with concrete slab), RC beam stiffened with tension materials such as steel or composite materials and profiled composite beam. The advantages of composite beams are light weight, high strength, improved durability and the capacity to withstand dynamic loads.

## 1.2 COMPOSITE BEHAVIOUR

Steel-concrete composite structures are defined as structures built up by concrete and steel components connected by shear connectors to form an interacting unit. The behaviour of composite members depends on the degree of shear connection between the components. Rigid shear connectors usually develop full composite action between the individual components whilst, flexible shear connectors generally permit the development of partial composite action. The latter requires consideration of the interlayer slip between the components.

Despite the established studies on the static behaviour, investigations concerning the dynamic responses of composite beams with partial interaction are scarce. A beam is said to be undergoing free vibration when it is disturbed from its static equilibrium position and then allowed to vibrate without any external dynamic excitation. The analytical result describing free vibration provides a basis to determine the natural frequency of a beam. This is important when it comes to forced vibration. The largest response amplitude, defined as resonance occurs when the frequency of the vibration is the same as the natural frequency. As far as composite beams are concerned, investigations are limited only on the dynamic behaviour of beams with full interaction whilst work on partial interaction allowing slip only analytically solved by Wu et.al. (2007), on one hand, and Hamed and Rabinovitch (2005,2007) solved the problems using variational principles, on the other. However, so far there is no finite element formulation for free vibration of composite beams.

## 1.3 RESEARCH BACKGROUND AND RATIONALE

Existing partial interaction theories of composite beams are based on force or direct equilibrium whereby differential equations are derived and solved. As discussed in the literature review, the difficulty of the problem increases as parameters to be considered increases. Increment in the number of parameters results in higher order differential equation which might as well coupled. Newmark (1951), in allowing the effect of slip, has derived and solved a second order differential equation whilst Adekola (1968) dealt with fourth and second order coupled differential equations due to the allowance of uplift. Mohd Yassin (2007) have extra in terms due to friction in their differential equation as compared to Adekola (1968). In the study of vibration, Wu et.al. (2007) derived a sixth order partial differential equation despite the fact that only slip was taken into account. A direct comparison with Newmark (1951) would reveal that the inclusion of vibration modifies the second order ordinary differential equation into sixth order partial differential equation. Hamed and Rabinovitch (2005,2007) solved vibration of composite beams using Hamilton's variational principle for both free and forced vibration.

Obviously, the problems of partial interaction and vibration of composite beams are complex and difficult. Recent works based on variational principle only applied conventional Ritz method. It is therefore the main interest of this study to derive finite element formulation for the problem.

## 1.4 PURPOSE AND OBJECTIVES OF STUDY

The purpose of this study is to provide an alternative approach to the Ritz variational approach in treating the partial interaction and vibration of composite beam by formulating finite element approach to the problem. The objectives of the study are as follows:

1) To establish stiffness matrix and mass matrix of composite beams allowing partial interaction and free vibration
2) To obtain the natural frequencies of the composite beam
3) To verify the formulation by comparing the results (i.e static deflection and natural frequency) with those obtains through analytical solutions

## 1.5 ASSUMPTIONS AND LIMITATIONS

The study is based on a few assumptions and limitations which are:

1) Equal curvature between the two subelements, whereby uplift will not occur.
2) This study is to find the eigenvalues only and not the eigenvectors.
3) Linear and elastic analysis.

## 1.6 OUTLINE OF THESIS

This chapter introduces the study by stating why it is needed, the problem statements, objectives and what are the results that will be obtained. Chapter 2 elaborates more on the topic in terms of the partial interaction and also vibration. Previous studies are analyzed and problems are outlined in this chapter. Finite element (FE) formulations are treated in Chapter 3. Chapter 4 discusses on the results achieved by the FE formulations and being verified by existing solutions while Chapter 5 deals with the conclusion of the study and recommendations that can be made to further improve the study.

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