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Evaluation of N-RTK Interpolation with Location-Based Dependency

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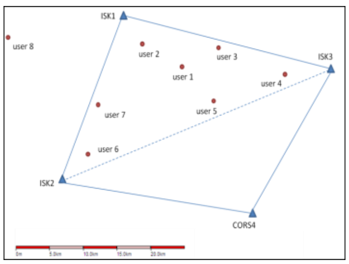

Abstract

This paper presents an evaluation of interpolation methods, that commonly being used in Network-based Real-Time Kinematic (N-RTK) positioning. The interpolation methods attempt to estimate the network residuals of atmospheric and orbital errors for an N-RTK user, by calculating the so-called network coefficient. In this study, a network of GPS stations, known as ISKANDARnet was utilised to calculate the network coefficients at various locations of N-RTK users include inside and outside of the ISKANDARnet area. It was found that all the interpolation methods performed similarly when the user location is nearby to master station. However, the noise of correction terms for each interpolation method was different as the user is situated at various locations especially outside of the network. In addition, the coefficient value indicates more than one (>1) as the user is located outside of the network area, except for the DIM interpolation method.

Keywords: Interpolation Methods, Network-Based RTK, Network Coefficient

Abstrak

Kertas kajian ini membentangkan satu penilaian terhadap kaedah-kaedah interpolasi yang biasanya digunakan dalam penentudududukan rangkaian berasaskan kinematik masa hakiki (N-RTK). Kaedah interpolasi cuba menganggarkan reja rangkaian selisih atmosfera dan orbit bagi pengguna N-RTK dengan mengira pekali rangkaian. Dalam kajian ini, rangkaian stesen GPS yang dikenali sebagai ISKANDARnet telah digunakan untuk mengira pekali rangkaian di pelbagai lokasi pengguna N-RTK termasuk di dalam dan di luar kawasan ISKANDARnet. Kajian mendapati bahawa semua kaedah interpolasi menunjukkan keputusan yang sama apabila lokasi pengguna adalah berdekatan dengan stesen induk. Walau bagaimanapun, ralat dari segi pembetulan untuk setiap kaedah interpolasi adalah berbeza apabila pengguna berada di pelbagai lokasi terutamanya di luar kawasan rangkaian. Tambahan pula, nilai pekali menunjukkan lebih daripada satu (>1) apabila pengguna berada di luar kawasan rangkaian, kecuali untuk kaedah interpolasi DIM.

Kata kunci: Kaedah Interpolasi, Rangkaian berasaskan kinematic masa hakiki, Pekali Rangkaian

■1.0 INTRODUCTION

The essential part of network real-time kinematic (N-RTK) GPS positioning is to estimate network corrections by utilizing the GPS baselines residual vectors, between a master station and few other reference stations once the GPS satellites signal carrier-phase ambiguities have been resolved. Since the residual vectors are dominated by distance-dependent errors i.e., orbit and atmosphere biases, it can be very useful in mitigating these errors as being formulated by an interpolation method. Typically, the interpolation algorithm calculates the 'network coefficients' using a geometric model and spatial locations of the user station with three or more reference stations to represent these distance-dependent errors for the entire network.[1, 2]

The aim of this study is to evaluate the performance of several interpolation methods for the N-RTK system. In this study, each interpolation method has been tested at various user and master stations locations, and with different number of reference stations.

■2.0 N-RTK Interpolation Methods

Interpolation methods can be divided into two main groups: geostatistical and deterministic.[3] Geostatistical methods such as Ordinary Kriging and Least Squares Collocation Method use statistical properties of measured points, whereas deterministic such as Linear Combination Model (LCM), Distance-Based Linear Interpolation Method (DIM), Linear Interpolation Method (LIM), Lower-Order Surface Model (LSM) and Least-Squares Collocation Method (LSC) methods use predefined mathematical functions and calculate the function's coefficients from measured points. This paper focuses on the deterministic group, where the interpolation methods use n-1 independent error vector to model the distance-dependent errors at the user station. It must be noted that, the coefficient is generated from n (at least three) reference stations (Dai et al., 2004).

2.1 Linear Combination Model (LCM)

The early algorithm of LCM is to estimate the effect of orbital errors.[4] Then, this algorithm was extended to reduce the effect of atmospheric delay.[5] Multipath and measurement noises also can be reduced if the user is within the network of reference stations.[2]

In this model, the network coefficient α_i is determined according to the following conditions:[6]

$$\sum_{i=1}^{n}\alpha_i = 1 \tag{3.1}$$

$$\sum_{i=1}^{n}\alpha_i(\hat{X}_u - \hat{X}_i) = 0 \tag{3.2}$$

$$\sum_{i=1}^{n}\alpha_i^2 = Min \tag{3.3}$$

where $\hat{X}_u$ and $\hat{X}_i$ are horizontal coordinate vectors for the user station and the i^{th} reference station, respectively. Specifically to compute the network coefficient (α), the following equation must be considered:

$$\alpha = B^T(BB^T)^{-1}W \tag{3.4}$$

where

$$B = \begin{pmatrix} 1 & 1 & \dots & 1 & 1 \\ \Delta X_{1n} & \Delta X_{2n} & \dots & \Delta X_{n-1,n} & 0 \\ \Delta Y_{1n} & \Delta Y_{2n} & \dots & \Delta Y_{n-1,n} & 0 \end{pmatrix} \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} W = \begin{pmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{pmatrix} \tag{3.5}$$

B is denoted as the difference of coordinates between reference stations and master station, and W describe the difference of coordinate between user and master station. The network coefficient can be computed for n reference stations, but only n-1 coefficients are being used to interpolate the errors. Note that α_n is coefficient related to master reference station that will not be applied in the interpolation. Generally, the coefficients values are always less than one if only the user is located inside the network of reference station.[6]

2.2 Distance-Based Linear Interpolation Method (DIM)

The model of DIM has been proposed initially for ionospheric correction estimation. This model directly relies on distances between the reference stations and the user station. The model can be expressed in the following equations:[6]

$$\Delta\nabla\hat{I}_u = \sum_{j=1}^{n-1}\frac{w_i}{w}\Delta\nabla\hat{I}\,i \tag{3.6}$$

$$w_i = \frac{1}{d_i} \tag{3.7}$$

$$w = \sum_{j=1}^{n-1} w_i \tag{3.8}$$

where n is the number of reference stations in the network, and d_i is the distance between the i^{th} reference station and the user station. Meanwhile, $\Delta\nabla\hat{I}\,i$ denote as the double-differenced ionospheric delay at the i^{th} reference station.

The coefficients calculation for DIM can be determined by Equation 3.9 which specifically shows the n-1 coefficients vector. It should be emphasized that the coefficients value are always less than one either the user is located inside or outside of the reference stations network.[6]

$$\vec{\alpha} = \left[\frac{w_1}{w}\,\frac{w_2}{w}\dots\frac{w_{n-1}}{w}\right] \tag{3.9}$$

2.3 Linear Interpolation Method (LIM)

The linear interpolation approach represents a plane fits residuals of reference stations to user station. The residuals are derived on a satellite-by-satellite and epoch-by-epoch basis after network ambiguities have been solved. The distance-dependent errors for any user within the network area can be interpolated based on approximate user coordinate and 'known' coordinates of the reference stations. The equation can be expressed by:[6]

$$\begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} = \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} \\ \Delta X_{2n} & \Delta Y_{2n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \tag{3.10}$$

It can be seen that the residual vector (V) only involve plane coordinates (2D) as ΔX and ΔY are coordinate differences between reference stations and the master station, and the parameters a and b represent the coefficients for ΔX and ΔY respectively.

The coefficients (a and b) can be derived through the satellite-by-satellite and epoch-by-epoch to reduce atmospheric biases. If only three reference stations are used, the LIM coefficients are identical to LCM except when the number of reference stations is greater than three.[6] In the case of utilizing three or more reference stations, the coefficients can be estimated using Least Square Estimation (LSE) as follows:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(A^T A\right)^{-1} A^T V \tag{3.11}$$

Correspond to the Equation 3.10, A and V can be define as:

$$A = \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} \\ \Delta X_{2n} & \Delta Y_{2n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \quad V = \begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} \tag{3.12}$$

Next, the coefficients can be applied at the user location for interpolation of biases:

$$\hat{V}_{1u} = [\Delta X_{un} \quad \Delta Y_{un}] \cdot \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = [\Delta X_{un} \quad \Delta Y_{un}] \cdot \left(A^T A\right)^{-1} A^T V \tag{3.13}$$

In this case, the n-1 coefficient vector can be expressed as follow:

$$\vec{\alpha} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un}\end{bmatrix} \cdot \left(A^T A\right)^{-1} A^T \tag{3.14}$$

Note that only two coefficients for each satellite pair are needed in this method (see Equation 3.11), which can be transmitted to user station for practical implementation of LIM in real-time application.

2. 4 Lower-Order Surface Model (LSM)

The extended surface fitting by using polynomials was introduced[7] to model the distance-dependent biases over the CORS network. The advantage of this model is variables of the fitting function could be two dimensions (i.e. horizontal coordinates) as expressed in Equations 3.15 and 3.16, or three dimensions (i.e. horizontal coordinate and height) as expressed in Equations 3.17 and 3.18.

$$V = a \cdot \Delta X + b \cdot \Delta Y + c \tag{3.15}$$

$$V = a \cdot \Delta X + b \cdot \Delta Y + c \cdot \Delta X^2 + d \cdot \Delta Y^2 + e \cdot \Delta X \Delta Y + f \tag{3.16}$$

$$V = a \cdot \Delta X + b \cdot \Delta Y + c \cdot \Delta H + d \tag{3.17}$$

$$V = a \cdot \Delta X + b \cdot \Delta Y + c \cdot \Delta H + d \cdot \Delta H + e \tag{3.18}$$

The parameters *a*, *b* and *c* can be estimated by using LSE as in Equation 3.19 if four or more reference stations were used. Note that, this equation is based on Equation 3.15.

$$\begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = \left(A^T A\right)^{-1} A^T V \tag{3.19}$$

$$\text{Where A=} \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} & 1 \\ \Delta X_{2n} & \Delta Y_{2n} & 1 \\ \vdots & \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} & 1 \end{bmatrix} \text{ V=} \begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} \tag{3.20}$$

Next, by using estimated parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$, biases at the user location can be expressed as:

$$\hat{V}_{1u} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un} & 1\end{bmatrix} \cdot \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un} & 1\end{bmatrix} \cdot \left(A^T A\right)^{-1} A^T V \tag{3.21}$$

In this case, the n-1 coefficient vector $\vec{\alpha}$ can be written as follow:

$$\vec{\alpha} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un} & 1\end{bmatrix} \cdot \left(A^T A\right)^{-1} A^T \tag{3.22}$$

The LSM is merely a generalisation of the LIM where the relationship between the biases and the horizontal components of the positions may be polynomial rather than linear.[8] It is emphasised that in LSM, utilising fitting variable and fitting order is associate with number of reference stations required. For instance, the plane-fitting function is only can be used with minimum of four reference stations.

■3.0 THE EXPERIMENTS

The experiments have been conducted by utilizing coordinates of ISKANDARnet - a GPS continuously operating reference station (CORS) located at Southern part of Peninsular Malaysia and being maintained by GNSS & Geodynamics Research Group, Universiti Teknologi Malaysia. In this study, the network coefficients were calculated for user's positions with its locations were placed 'inside' and 'outside' of the ISKANDARnet (Fig. 1). The LCM, LIM, DIM and LSM interpolation methods were selected for these experiments due to the fact that the calculation only require coordinates of the user, master and reference stations (Table 1), or distance between the user and the reference stations (Table 2).

The condition of each experiment can be described as follows;

Experiment 1 - evaluates the coefficient values according to various locations of user stations. In this experiment, reference station of ISK1 was selected as a master station and ISK2, ISK3 and CORS4 as the reference stations.

Experiment 2 - evaluates the coefficient values according to various locations of master station. In this experiment, the reference station of ISK1, ISK2, ISK3 and CORS4 were selected as the master station.

Experiment 3 - evaluates the coefficient values according to the number of reference stations: three (i.e. ISK1, ISK2 and ISK3) and four (i.e. ISK1, ISK2, ISK3 and CORS4).

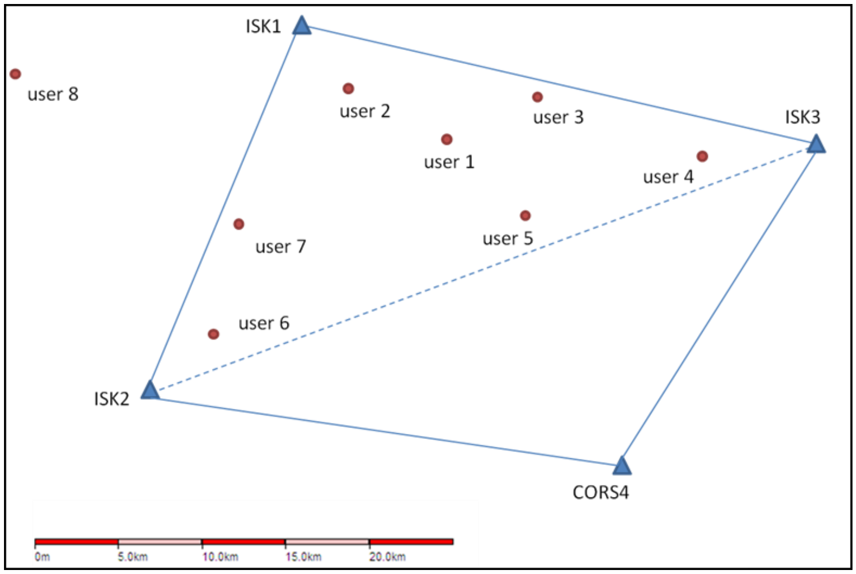

Figure 1 Distribution of the reference stations and user locations.

Table 1 The Cartesian coordinates of user and reference stations.

Stations	Cartesian Coordinate		
	X (m)	**Y (m)**	**Z (m)**
ISK1	-1503102.987	6196139.591	172488.822
ISK2	-1494311.946	6198831.419	150598.239
ISK3	-1533049.538	6188979.381	165328.685
CORS4	-1518022.731	6175576.682	145799.481
User 1	-1511870.484	6194142.763	166307.723
User 2	-1506596.304	6195379.524	167894.582
User 3	-1517017.668	6192811.190	168539.575
User 4	-1526264.182	6190659.324	164474.907
User 5	-1516187.189	6193216.991	160979.430
User 6	-1498329.297	6197766.896	153919.546
User 7	-1499781.035	6197230.622	160588.136
User 8	-1486463.640	6200267.613	169444.171

Table 2 Distance between user and reference stations.

Stations / Users	ISK1	ISK2	ISK3	CORS4
User 1	10.9 km	24 km	21.8 km	22.8 km
User 2	5.8 km	21.5 km	27.3 km	26.7 km
User 3	14.8 km	29.6 km	16.8 km	22.9 km
User 4	25.1 km	35.8 km	7.0 km	18.8 km
User 5	17.7 km	24.9 km	17.9 km	15.8 km
User 6	19.2 km	5.3 km	37.6 km	25.6 km
User 7	12.4 km	11.5 km	34.6 km	26.9 km
User 8	24.6 km	32.3 km	60.7 km	55.3 km

3.1 Experiment 1: Results & Analysis

Tables 3 – 6 show the network coefficient values as calculated by using LCM, LIM, DIM and LSM interpolation methods. Generally, the results in these tables have indicated that most of the coefficient value of a user is the largest once the user is located near to any reference station. For instance, the user4 and user6 ('corner' users) have the largest coefficient values as indicated by $\alpha 2$ and $\alpha 1$, which are highly influenced by the reference station of ISK3 and ISK2, respectively. One can also be noticed that the coefficient value is more than one (>1) for user which is located outside (see user8 in Tables 3, 4 and 6) and at the edge (see user3 and user7 in Table 6) of the network. In contrast, the coefficient values in DIM remain less than one (<1) although the user is located outside of the network area. These conditions show that the coefficient value is also dependent on the interpolation algorithm being applied. Interestingly, it was found that all coefficients value ($\alpha 1$, $\alpha 2$ and α3) have been properly distributed for the user which is located almost at the centre of the network despite of interpolation methods being used. This is shown for user1 and user5, given that the results of its coefficient values are comparatively equivalent in Tables 3 – 6.

Table 3 Coefficients from LCM interpolation method.

Table 4 Coefficients from LIM interpolation method.

		LIM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	A1	0.011	0.110	-0.129	-0.178	0.073	**0.540**	0.350	1.475
ISK3	A2	0.185	0.064	0.319	**0.524**	0.265	-0.198	-0.133	-1.504

		LCM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.151	0.229	0.014	-0.102	0.153	**0.617**	0.481	1.708
ISK3	α2	0.324	0.234	0.460	**0.600**	0.344	-0.121	-0.004	-1.273
CORS4	α3	0.021	-0.079	0.013	0.226	0.217	0.226	0.054	-0.273
ISK1 (master)	α4	0.504	0.617	0.513	0.275	0.286	0.279	0.470	0.838
	$\sum_{i=1}^{4}\alpha_i$	1	1	1	1	1	1	1	1
	$\sqrt{\sum_{i=1}^{3}\alpha_i^2}$	0.358	**0.337**	0.461	0.649	0.435	0.668	0.484	**2.147**

CORS4	A3	0.176	0.057	0.170	0.310	0.305	0.311	0.198	-0.016
	$\sum_{i=1}^{3}\alpha_i$	0.371	0.231	0.361	0.656	0.643	0.652	0.414	-0.045
	$\sqrt{\sum_{i=1}^{3}\alpha_i^2}$	0.255	**0.139**	0.384	0.635	0.411	0.654	0.423	**2.107**

Table 5 Coefficients from DIM interpolation method.

		DIM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.314	0.387	0.247	0.125	0.253	**0.665**	0.569	0.187
ISK3	α2	0.348	0.304	0.434	**0.636**	0.351	0.197	0.189	0.462
CORS4	α3	0.336	0.310	0.319	0.239	0.396	0.138	0.242	0.352
	$\sum_{i=1}^{3}\alpha_i$	1	1	1	1	1	1	1	1
	$\sqrt{\sum_{i=1}^{3}\alpha_i^2}$	**0.578**	0.581	0.593	0.691	0.587	**0.707**	0.647	0.610

Table 6 Coefficients from LSM interpolation method.

		LSM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.718	0.921	0.590	0.208	0.474	**0.930**	1.008	-0.303
ISK3	α2	0.886	0.922	1.032	**0.908**	0.664	0.190	0.520	2.990
CORS4	α3	-0.604	-0.844	-0.622	-0.115	-0.138	-0.120	-0.528	-1.687
	$\sum_{i=1}^{3}\alpha_i$	1	1	1	1	1	1	1	1
	$\sqrt{\sum_{i=1}^{3}\alpha_i^2}$	1.290	1.553	1.800	0.938	**0.827**	0.957	1.252	**3.446**

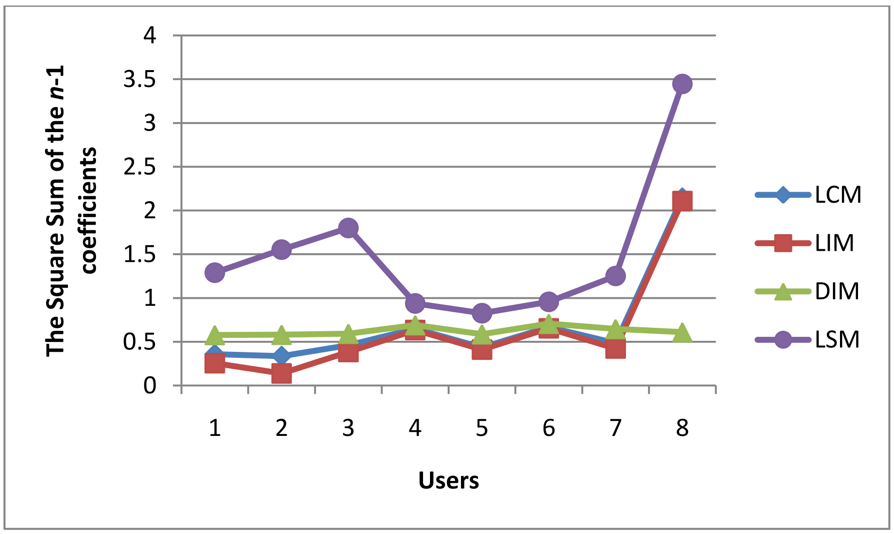

Figure 2 The square sum of the n-1 coefficients or noise for the correction terms of LCM, LIM, DIM and LSM.

Figure 2 shows the plots for the last row in Tables 3 – 6, that is representing the square sum of the n-1 coefficients value of LCM, LIM, DIM and LSM, respectively. According to Dai et al., (2001), the square sum of the n-1 coefficients is an indicator of noise for the correction terms, hence the smaller the better. Based on Figure 2, this experiment clearly indicates that LSM has the highest noise value (of correction terms) for all network users. In addition, the user8 which is located outside of the network has shown the highest noise value for all the interpolation methods except for DIM.

Significantly, the noise value of the DIM and LSM is the smallest at user1 and user5, respectively. Thus, it is suggested that by locating user at the centre of the network, the noise will be reduced by these two interpolation methods. On the other hand, the LCM and LIM interpolation methods have the smallest noise value at user2 which is located nearest to the master station. Results from the LCM and LIM will be further verified in the next experiment by selecting various location of master station.

3.2 Experiment 2: Results & Analysis

Figures 3 – 6 show the noise value (or square sum of the n-1 coefficient) of user stations according to the various location of master station (i.e. ISK1, ISK2 and ISK3) for each interpolation method (i.e. LCM, LIM, DIM and LSM).

From the Figures 3.3 and 3.4, both LCM and LIM interpolation methods have shown the smallest noise value for the user2, user6 and user4 once the ISK1, ISK2 and ISK3 were set as a master station respectively. This verified the finding in Experiment 1 which indicated that the noise value is getting smaller when the user nearer to a master station. Thus, it can be suggested that amongst the reference stations, the criteria to set for a master station in both LCM and LIM methods must consider the nearest user location. Results in the same figures also indicate that the noise values are large at the user8 although different master station was selected. It confirms that LCM and LIM methods are not suitable for the user which is located outside of the network.

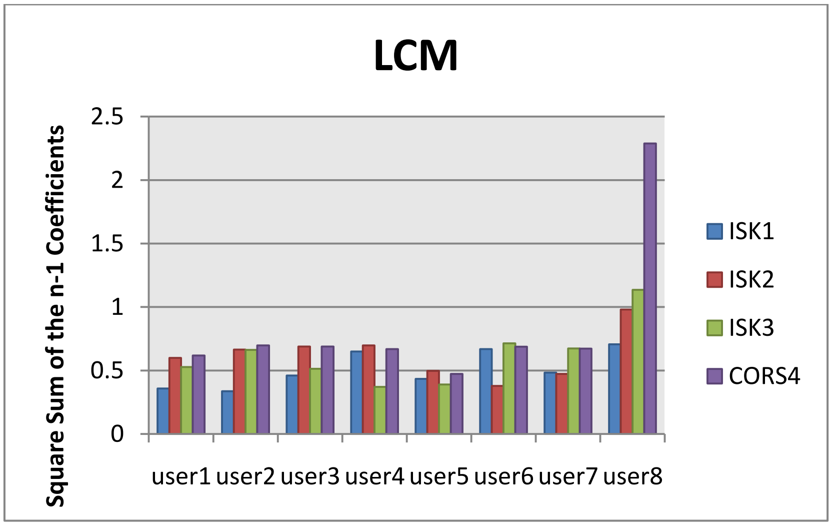

Figure 3. The square sum of the n-1 coefficients or noise for the correction terms of LCM using different location of master station.

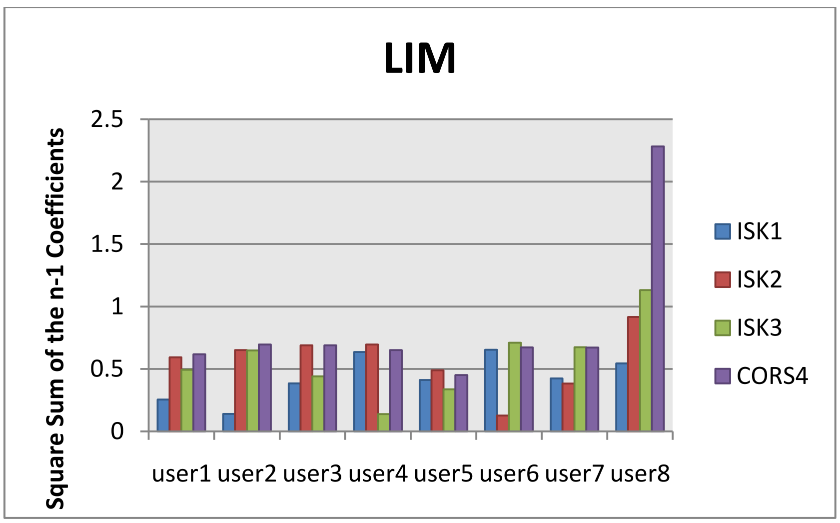

Figure 4 The square sum of the n-1 coefficients or noise for the correction terms of LIM using different location of master station.

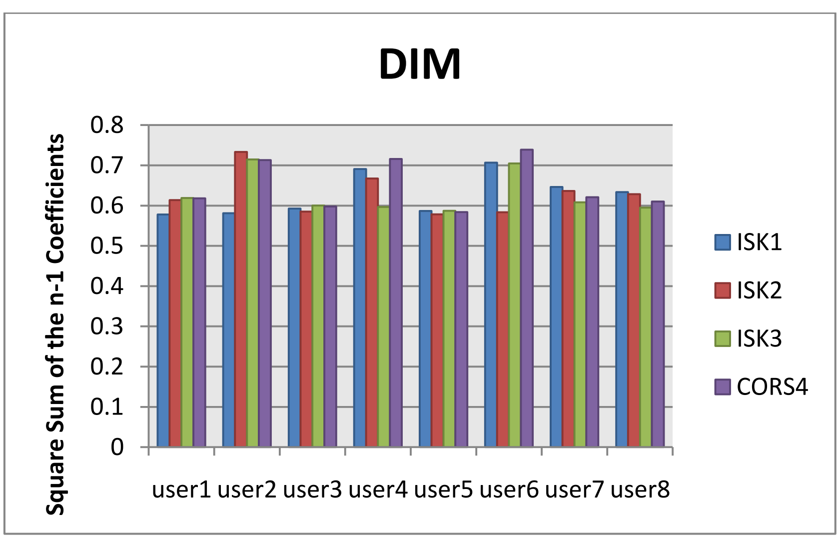

Figure 5 The square sum of the n-1 coefficients or noise for the correction terms of DIM using different location of master station.

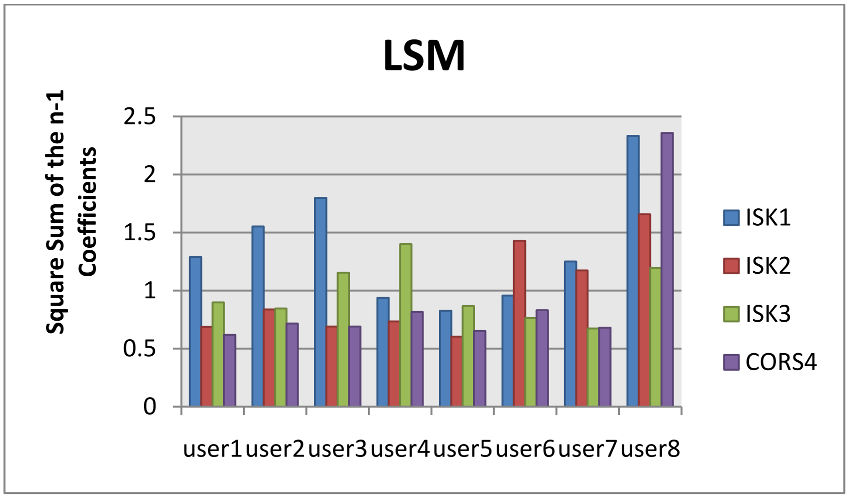

Figure 6 The square sum of the *n*-1 coefficients or noise for the correction terms of LSM using different location of master station.

3.3 Experiment 3: Results & Analysis

Figures 7 – 9 show the noise value by using three and four reference stations in LCM, LIM and DIM respectively. It must be mentioned that the LSM could not be included in this experiment since this method only can be used with minimum of four reference stations.

From Figures 7 and 8, it can be seen that the LCM and LIM have slight difference in the noise values between three and four reference stations for the user located within the network area. Thus, it implies that by using at least three reference stations in LCM and LIM, it is sufficient enough for interpolating corrections to the network user due to insignificant improvement of the noise shown with adding the fourth reference station. However, the results by DIM interpolation method in Figure 9 show that the noise values are slightly increased when applying three rather than four reference stations. Thus, it is recommended that to use LCM or LIM interpolation method if deploying only three reference stations in the N-RTK system.

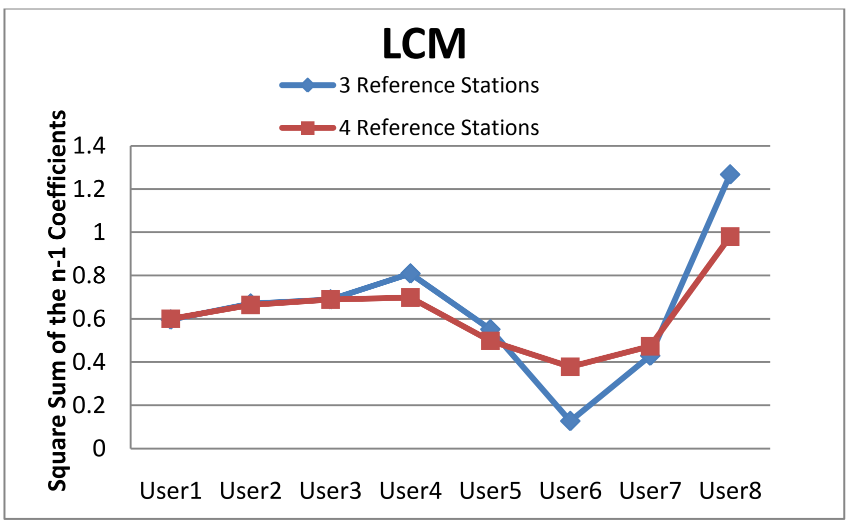

Figure 7 The square sum of the n-1 coefficients by using three and four reference stations in LCM.

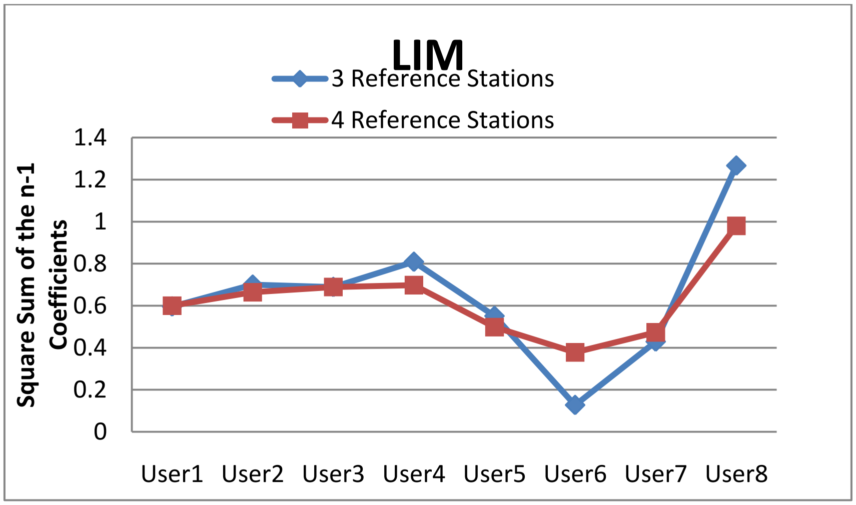

Figure 8 The square sum of the n-1 coefficients by using three and four reference stations in LIM.

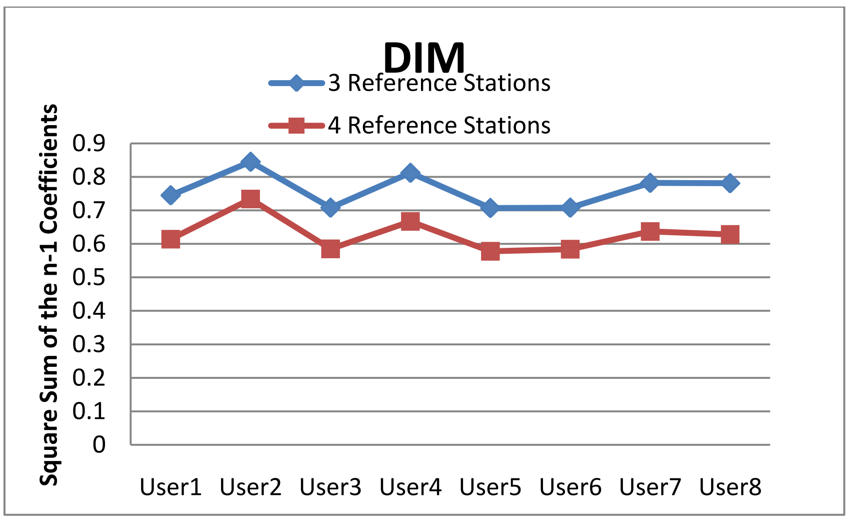

Figure 9 The square sum of the n-1 coefficients by using three and four reference stations in DIM.

Tables 7 – 9 show the numerical results of the coefficients by using three reference stations for LCM, LIM and DIM interpolation methods, respectively. From these tables, it can be seen that the coefficient values of these interpolation methods are identical when using three reference stations except the DIM. Generally, these interpolation methods have no major difference in their characteristic especially LCM and DIM. These methods show that the sums of generated coefficients remain equal to 1 even though using three reference stations.

Table 7 Coefficients by using three reference stations in LCM.

		LCM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.1702	0.1566	0.0259	0.1032	0.3493	0.8215	0.5291	0.2927
ISK3	α2	0.3427	0.1628	0.4724	0.8037	0.5394	0.0817	0.0445	-0.4695
ISK1 (master)	α4	0.4871	0.6806	0.5018	0.0931	0.1113	0.0967	0.4264	1.1768
	$\sum_{i=1}^{4}\alpha_i$	1	1	1	1	1	1	1	1
	$\sqrt{\sum_{i=1}^{3}\alpha_i^2}$	0.3826	0.2259	0.4731	0.8103	0.6426	0.8256	0.5310	0.5533

Table 8 Coefficients by using three reference stations in LIM.

		LIM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.1702	0.1566	0.0259	0.1032	0.3493	0.8215	0.5291	0.2927
ISK3	α2	0.3427	0.1628	0.4724	0.8037	0.5394	0.0817	0.0445	-0.4695
	$\sum_{i=1}^{4} \alpha_i$	0.5129	0.3194	0.4983	0.9069	0.8887	0.9032	0.5736	-0.1768
	$\sqrt{\sum_{i=1}^{3} \alpha_i^2}$	0.3826	0.2259	0.4731	0.8103	0.6426	0.8256	0.5310	0.5533

Table 9 Coefficients by using three reference stations in DIM.

		DIM							
		user1	user2	user3	user4	user5	user6	user7	user8
ISK2	α1	0.476	0.5598	0.3623	0.1645	0.4189	0.7711	0.7505	0.7437
ISK3	α2	0.524	0.4402	0.6377	0.8355	0.5811	0.2289	0.2495	0.2563
	$\sum_{i=1}^{4} \alpha_i$	1	1	1	1	1	1	1	1
	$\sqrt{\sum_{i=1}^{3} \alpha_i^2}$	0.7079	0.7121	0.7334	0.8515	0.7163	0.8044	0.7909	0.7866

■4.0 CONCLUDING REMARKS

This paper has reviewed several interpolation methods that utilise n-1 coefficients to generate correction terms at any N-RTK user stations. Generally, all interpolation methods have shown that the coefficient values are highly influenced by the nearest reference station. For instance, user4 and user6 have the largest coefficients values due to be located nearby to ISK3 and ISK2 respectively. Additionally, it emphasised that the users, whose location at almost the centre of the network (user1 and user5) have proper weighting coefficients, which leads to better positioning performance.

Although the characteristic of all interpolation methods are almost the same, in terms of noise for the correction terms, it turns out that the ability of LCM and LIM is better as the user is located close to a master station. Therefore, for the LCM and LIM, a master station should be selected nearest to a user station. On the other hand, there is no significant effect of various master locations by using either DIM or LSM interpolation method. Both of interpolation methods also show that the noise of the correction terms will be reduced by locating user at the centre of the network. Furthermore, it would be cost-effective by using three reference stations in LCM or LIM, since the coefficient results were found compatible with the addition of the fourth reference stations. It is expected that the knowledge of these interpolation methods will be useful to design reference stations network and to select proper interpolation method for the implementation of N-RTK system.

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