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A class of proportional-integral sliding mode control with application to active suspension system

Yahaya Md. Sam[a,*], Johari H.S. Osman[a], M. Ruddin A. Ghani[b]

[a]Faculty of Electrical Engineering, University Technology of Malaysia, Skudai, 81310 UTM Johor, Malaysia
[b]Kolej Universiti Teknikal Kebangsaan Malaysia, 75450 Melaka, Malaysia

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Abstract

The purpose of this paper is to present a new robust strategy in controlling the active suspension system. The strategy utilized the proportional-integral sliding mode control scheme. A quarter-car model is used in the study and the performance of the controller is compared to the linear quadratic regulator and with the existing passive suspension system. A simulation study is performed to prove the effectiveness and robustness of the control approach.

Keywords: Automotive control; Quarter-car suspension; Sliding mode control; Robust control; Mismatched uncertainties

1. Introduction

Several performance characteristics have to be considered in order to achieve a good suspension system [2]. These characteristics deal with regulation of body movement, regulation of suspension movement and force distribution. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration. Furthermore, the suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. These objectives can be achieved by minimizing the vertical car body acceleration. An excessive wheel travel will result in non-optimum attitude of tyre relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tyre-to-road contact must be maintained on four wheels.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [8]. Thus the passive suspension system, which approach optimal characteristics had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers [1,5,7]. Active suspension systems dynamically respond to changes in the

* Corresponding author.
E-mail address: yahaya@fke.utm.my (Y.Md. Sam).

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road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, wheel velocity and wheel or body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control strategies such as optimal state feedback [1], backsteeping method [7], optimal state feedback [5], fuzzy control [10] and sliding mode control [9] have been proposed in the past years to control the active suspension system. The sliding mode control has relatively simpler structure and it guarantees the system stability.

In this paper we will consider a control scheme that can improve further the ride comfort and road handling of the active suspension system. The proposed control scheme differs from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral (PI) sliding mode control. The additional integral in the proposed sliding surface provides one more degree of freedom and also reduce the steady-state error. A computer simulation will be performed to demonstrate the effectiveness and robustness of the proposed control scheme.

2. Dynamic model of the suspension

Most of the past active suspension designs were developed based on the quarter-car model as shown in Fig. 1. From the figure the following state-space model of the quarter-car model can be easily obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -K_a/M_s & -C_a/M_s & 0 & C_a/M_s \\ 0 & 0 & 0 & 1 \\ K_a/M_{us} & C_a/M_{us} & -K_t/M_{us} & -C_a/M_{us} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_s \\ 0 \\ -1/M_s \end{bmatrix} u_a + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r}, \tag{1}$$

where M_s and M_{us} are the masses of car body and wheel, respectively, x_s and x_w are the displacements of car body and wheel respectively, K_a and K_t are the spring coefficients, C_a is the damper coefficient, $\dot{r}$ is the road disturbance and u_a is the control force from the hydraulic actuator and assumed as the control input. The following terms are defined as the state variables: $x_1 = x_s - x_w$ for suspension travel, $x_2 = \dot{x}_s$ for car body velocity, $x_3 = x_w - r$ for wheel deflection and $x_4 = \dot{x}_w$ for wheel velocity. Eq. (1) shows that the disturbance input is not in phase with the actuator input, therefore the system suffers from mismatched condition.

Thus the proposed controller must be robust enough to overcome the mismatched condition so that the disturbance would not have significant effect on the performance of the system. Hence, contribution of this paper is presented in the following.

Eq. (1) can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and the continuous function $f(t)$ represents the uncertainties with the mismatched condition, i.e. $\text{rank}[B|f(t)] \neq \text{rank}[B]$. The following assumptions are taken as standard:

Assumption 1. There exists a known positive constant such that $\|f(t)\| \leqslant \beta$, where $\|\cdot\|$ denotes the standard Euclidean norm.

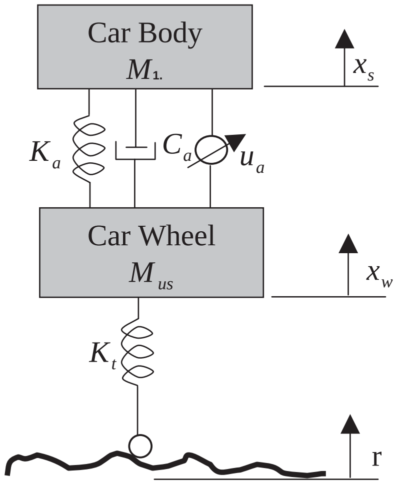

Fig. 1. A quarter-car model.

Assumption 2. The pair (A, B) is controllable and the input matrix B has full rank.

3. Switching surface and controller design

In this study, we utilized the PI sliding surface defined as follows:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau)\,\mathrm{d}\tau, \tag{3}$$

where $C \in \mathbb{R}^{m\times n}$ and $K \in \mathbb{R}^{m\times n}$ are constant matrices. The matrix K satisfies $\lambda(A+BK) < 0$ and C is chosen so that CB is non-singular. It is well known that if the system is able to enter the sliding mode, hence $\sigma(t)=0$. Therefore, the equivalent control, $u_{\mathrm{eq}}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ [6], i.e.

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0. \tag{4}$$

If the matrix C is chosen such that CB is non-singular, this yields

$$u_{\mathrm{eq}}(t) = Kx(t) - (CB)^{-1}Cf(t). \tag{5}$$

Substituting Eq. (5) into system (2) gives the equivalent dynamic equation of the system in sliding mode as

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C\}f(t). \tag{6}$$

Theorem 1. *If $\|\tilde{F}(t)\| \leqslant \beta_1 = \|I_n - B(CB)^{-1}C\|\beta$, the uncertain system in Eq. (6) is boundedly stable on the sliding surface $\sigma(t) = 0$.*

Proof. For simplicity, we let

$$\tilde{A} = (A + BK), \tag{6a}$$

$$\tilde{F}(t) = \{I_n - B(CB)^{-1}C\}f(t) \tag{6b}$$

and rewrite (6) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(t). \tag{7}$$

Let the Lyapunov function candidate for the system is chosen as

$$V(t) = x^{\mathrm{T}}(t)Px(t). \tag{8}$$

Taking the derivative of $V(t)$ and substituting into Eq. (6), gives

$$\begin{aligned}\dot{V}(t) &= x^{\mathrm{T}}(t)[\tilde{A}^{\mathrm{T}}P + P\tilde{A}]x(t) + \tilde{F}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\tilde{F}(t)\\ &= -x^{\mathrm{T}}(t)Qx(t) + \tilde{F}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\tilde{F}^{\mathrm{T}}(t),\end{aligned} \tag{9}$$

where P is the solution of $\tilde{A}^{\mathrm{T}}P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q. It can be shown that Eq. (10) can be reduced to

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\,\|x(t)\|. \tag{10}$$

Since $\lambda_{\min}(Q) > 0$, consequently $\dot{V}(t) < 0$ for all t and $x \in \mathrm{B}^{\mathrm{c}}(\eta)$, where $\mathrm{B}^{\mathrm{c}}(\eta)$ is the complement of the closed ball $\mathrm{B}(\eta)$, centered at $x = 0$ with radius $\eta = 2\beta_1\|P\|/\lambda_{\min}(Q)$. Hence, the system is boundedly stable. □

Remark. For the system with uncertainties satisfy the matching condition, i.e. $\mathrm{rank}[B|f(t)] = \mathrm{rank}[B]$, Eq. (6) can be reduced to $\dot{x}(t) = (A + BK)x(t)$ [4]. Thus asymptotic stability of the system during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system in Eq. (2) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter.

For the uncertain system in Eq. (2) satisfying Assumptions 1 and 2, the following control law is proposed:

$$u(t) = -(CB)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB)^{-1}\frac{\sigma(t)}{\|\sigma(t)\| + \delta}, \tag{11}$$

where $\phi \in \mathbb{R}^{m\times m}$ is a positive symmetric design matrix, k and δ are the positive constants.

Theorem 2. *The hitting condition of the sliding surface* (3) *is satisfied if*

$$\|A + BK\|\,\|x(t)\| \geqslant \|f(t)\|. \tag{12}$$

Proof. In the hitting phase $\sigma^{\mathrm{T}}(t)\sigma(t) > 0$; using the Lyapunov function candidate $V(t) = \frac{1}{2}\sigma^{\mathrm{T}}(t)\sigma(t)$, we obtain

$$\begin{aligned}\dot{V}(t) &= \sigma^{\mathrm{T}}(t)\dot{\sigma}(t)\\ &= \sigma^{\mathrm{T}}(t)\left[-(CA + CBK)x(t) - \phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} + Cf(t)\right]\\ &\leqslant -\left[\left\{\|\phi\| + \left\|\frac{k}{\|\sigma(t)\| + \delta}\right\|\right\}\|\sigma(t)\|^2 + \{\|C\|\,\|A + BK\|\,\|x(t)\| - \|C\|\,\|f(t)\|\}\|\sigma(t)\|\right].\end{aligned} \tag{13}$$

It follows that $\dot{V}(t) < 0$ if condition (12) is satisfied. Thus, the hitting condition is satisfied. □

4. Simulation and discussion

The mathematical model of the system as defined in Eq. (2) and the proposed proportional-integral sliding mode controller (PISMC) in Eq. (11) were simulated on computer. For comparison purposes, the performance of the PISMC is compared to the linear quadratic regulator (LQR) control approach. We assume a quadratic performance index in the form of

$$J = \frac{1}{2}\int_0^{\infty} (x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru)\,\mathrm{d}t, \tag{14}$$

where the matrix Q is symmetric positive semi-definite and R is symmetric positive definite. Then the optimal linear feedback control law is obtained as

$$u = -Kx, \tag{15}$$

where K is the designed matrix gain.

Numerical values for the model parameters are taken from Alleyne and Hedrick [1], and are as follows:

$$M_{\mathrm{s}} = 290\ \mathrm{kg}, \quad M_{\mathrm{us}} = 59\ \mathrm{kg}, \quad K_{\mathrm{a}} = 16812\ \mathrm{N/m},$$

$$K_{\mathrm{t}} = 190,000\ \mathrm{N/m}, \quad C_{\mathrm{a}} = 1000\ \mathrm{N/(m/s)}.$$

Let the set of typical road disturbance be in the form of

$$r(t) = \begin{cases} a(1-\cos(8\pi t))/2 & \text{if } 0.50 \leqslant t \leqslant 0.75 \text{ and } 3.00 \leqslant t \leqslant 3.25, \\ 0 & \text{otherwise,} \end{cases}$$

where a denotes the bump amplitude (see Fig. 2). This type of road disturbance has been used by Lin and Kanellakopoulos [7] and D'Amato and Viasallo [3] in their studies. Furthermore, the maximum travel distance of the suspension travel is as suggested by Lin Kanellakopoulos [7] that is ± 8 cm has been used.

In the design of the LQR controller, weighting matrices Q and R are selected as $Q = \mathrm{diag}[q_1, q_2, q_3, q_4]$ where $q_1 = q_2 = q_3 = q_4 = 1 \times 10^4$ and $R = 1 \times 10^{-4}$, respectively. Thus, the designed gains of the LQR controller are $k_1 = 2750$, $k_2 = 9720$, $k_3 = -206400$ and $k_4 = 8240$. The values of the matrix K for the PISMC is similar to the values of the designed gains in the LQR controller, i.e. $K = [2750, 9720, -206400, 8240]$ such

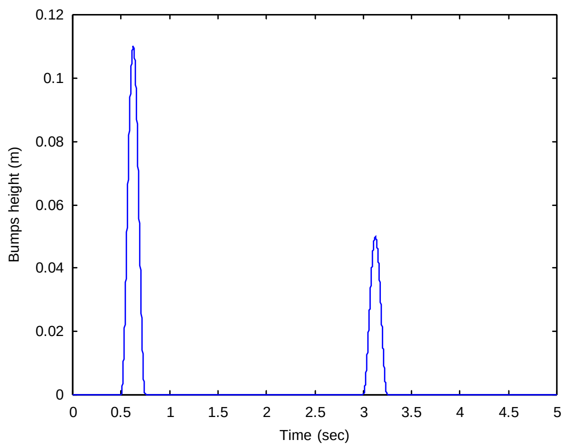

Fig. 2. Typical road disturbance.

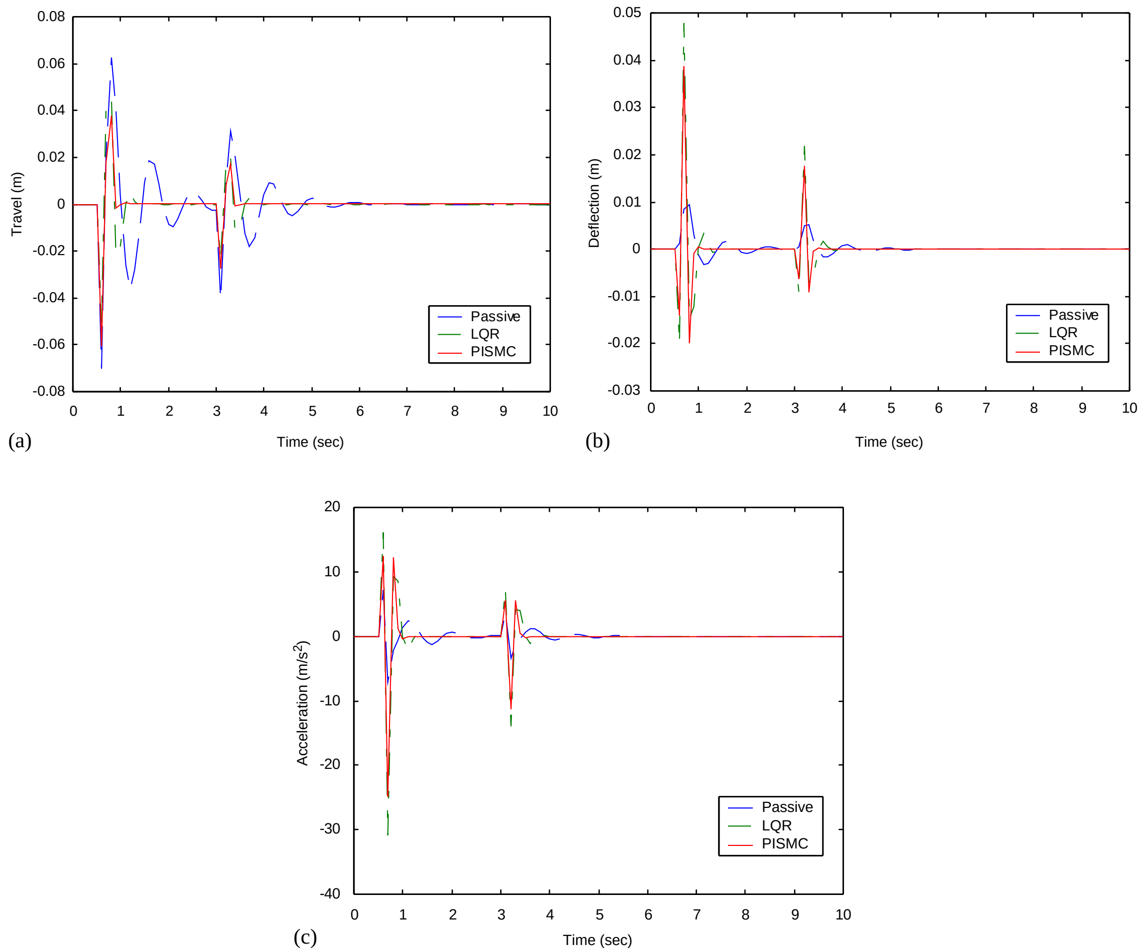

Fig. 3. (a) Suspension travel; (b) wheel deflection; (c) car body acceleration.

that $\lambda(A+BK) = \{-1.5168, -0.1987 \pm j0.1727, -0.0207\}$. In this simulation the following values are selected for the PISMC: $C = [100, 50, 40, 200]$, $\phi = 1000$, $k = 1$ and $\delta = 0.001$.

In order to fulfill the objective of designing an active suspension system, i.e. to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and the wheel deflection. Fig. 3a shows the suspension travel of both controllers for an active suspension system and a passive suspension system for comparison purposes. The result shows that the suspension travel within the travel limit, i.e. ± 8 cm, and the result also shows that the active suspension utilizing the PISMC technique perform better as compared to the others. Fig. 3b and c illustrates clearly how the PISMC can effectively absorb the vehicle vibration in comparison to the LQR method and the passive system. The body acceleration in the PISMC design system is reduced significantly, which guarantee better ride comfort. Moreover, the wheel deflection is also smaller using the proposed controller. Therefore it is concluded that the active suspension system with the PISMC improves the ride comfort while retaining the road handling characteristics, as compared to the LQR method and the passive suspension system.

5. Conclusion

The paper presents a robust strategy in designing a controller for an active suspension system which is based on variable structure control theory, which is capable of satisfying all the pre-assigned design requirements within the actuators limitation. The mathematical model of a quarter car is presented in a state space form. A detailed study of the proportional-integral sliding mode control algorithm is presented and solved the mismatched condition problem in the mathematical model. The performance characteristics and the robustness of the active suspension system are evaluated by two types of controllers, and then compared with the passive suspension system.

The result shows that the use of the proposed proportional-integral sliding mode control technique proved to be effective in controlling vehicle and more robust compared to the linear quadratic regulator method and the passive suspension system.

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