NUMERICAL SOLUTION OF A MATHEMATICAL MODEL FOR HOLLOW-FIBER MEMBRANE GAS SEPARATION SYSTEM

by

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ABSTRACT

A mathematical model has been developed for the separation of CO_2 from natural gas (CH_4) using hollow-fiber membrane module. Numerical solution technique using Runge-Kutta-Merson method, available from the NAG FORTRAN Library as the subroutine D02BBF, is used to solved the mathematical model. The model is used to predict the effects of the operating conditions, namely, feed pressure and feed composition on the performance of the separation of CO_2 from natural gas (CH_4). The solution strategy employs initial value problem estimate and establishment of feed pressure profile and gas composition profile .

The model developed was used to predict the performance of a commercial size membrane permeator with a diameter and length of 5.08 cm and 304.8 cm respectively. The membrane area utilized was 232877.97 cm^2 with 7200 active fibers. The volumetric feed flow rate was 1.83×10^6 cm^3(STP)/s. The ranges of pressure and feed composition used are 2068-5170 cm Hg and 5-50% respectively.

The model predicted the carbon dioxide purity in the permeate stream 65.54% and retentate 0.146% respectively. The results obtained was based on feed pressure, feed composition and temperature of 5170 cm Hg, 5% carbon dioxide and 25°C respectively. The model also found

that the Mechanical Energy Balance equation (Bernoulli equation) which was incorporated in the mathematical model satisfactorily predict the feed pressure drop along the fiber length. The predicted feed pressure drop was found to significantly affect the performance of a single-stage hollow-fiber membrane permeator.

1.0 INTRODUCTION

The separation of CO_2 from natural gas using non-porous polymeric membrane is one of the most recent advance technology and now recognized as a viable and economical unit operation, offering lower overall cost compared to the coventional separation technique such as absorption, adsorption, amine process, cryogenic system and etc. The CO_2 separation from natural gas is one of the area that have been widely used all over the world today as an alternative to the conventional separation technique. This system is at present used to remove CO_2 from natural gas in order to increase the heating value of the sales gas, recovery of hydrogen from purge gas in ammonia production and in refinery operations, recovery of helium from natural gas, oxygen enrichment (McRenolds, 1986), natural gas dehydration and sweetening (with a view to its transportation by pipeline especially in offshore production), CO_2 removal from fractured wells (Schell, 1982), air fractionating from the production of blanketing nitrogen and landfill gas upgrading (Backhouse, 1986).

Reliability, simplicity, cost effective and less operator attention are the advantages of this viable technology to compete with the conventional separation technique. With the recent development in membrane manufacturing technique to produce membrane with both higher permeability and higher selectivity will further reduce the overall investment cost as well as the operational cost.

2.0 OBJECTIVE OF THE STUDY

The performance of membrane gas separation system not only depends on the intrinsic properties and the structure of the membrane used but also the operating conditions namely feed pressure and feed composition. In order to get the optimum operating conditions of the separation system, this study aims at developing a mathematical model to predict the effects of feed pressure and feed composition on the performance of the separation of CO_2 from natural gas. The performance of the membrane system is measured in terms of PERMEATION rates and selectivities.

3.0 MATHEMATICAL MODELLING

Assumptions

Mathematical model is limited by the assumptions used in formulation and deriving the mathematical model for the system of interest. Generally, constant permeation rate and negligible feed and permeate pressure drops along the membrane surface are usually assumed in formulation of the mathematical model for membrane gas separation system. The model developed in this study will be similar to the one used by Pan (Pan, 1983), the model will also take into consideration the feed pressure drop. The mathematical formulation developed in this study are based on the following assumptions. 1) The feed are on the skin side of the asymmetric membrane 2) The feed pressure and flux vary with position on the membrane surface (Saidi, 1988). 3) No mixing of permeate fluxes of different composition occurs inside the porous supporting layer of the membrane, 4) The porous supporting layer has negligible resistance to permeate fluxes and the diffusion along the pore path is insignificant due to high permeate fluxes, and 5) The feed pressure drop equation was derived from the mechanical energy balance as given by Dodge (Dodge, 1944) for flow of gases under isothermal condition.

4.0 THE GOVERNING EQUATIONS

The governing equations developed in this study comprises three major equations; namely, material balance equations, permeation rate equations and feed pressure drop equation. These governing equations are applicable for a hollow-fiber membrane module. The following formulation is set up in terms of the cross flow pattern. This flow pattern occurs when the feed flows on the skin side of the membrane and the flux permeates through the porous support in perpendicular direction to the feed flow.

4.1 Material Balance Equations

The overall material balance for the separation of binary mixtures in a hollow-fiber asymmetric membrane permeator is given in equation (1).

$$\frac{dV}{dl_h} = u_f - u_r \qquad \text{......(1)}$$

where is l_h is the hollow-fiber length variable, V the volumetric permeate gas flow rate, u_f and u_r are the feed and retentate gas flow rate per unit length of the hollow-fiber, respectively.

The overall material balance for component 1 is given by

$$\frac{d(V y_1)}{d l_h} = u_f x_{1f} - u_r x_{1r} , \qquad \text{....... (2)}$$

where u_f is the feed gas flow rate per unit length of the hollow-fiber, u_r is the residue gas flow rate per unit length of the hollow fiber, x_{1f}, x_{1r} and y_{1r} are the mole fraction of component 1 in the feed , retentate and permeate stream, respectively.

Stage cut, θ is defined as the fraction of feed permeated through the membrane and is given by

$$\theta = V/Lu_f = V/U_f \qquad \text{...... (3)}$$

where L is the hollow-fiber length and U_f is the feed gas flow rate per hollow-fiber module in cocurrent flow mode. Since in any operation the fraction of feed permeated (θ) which is defined in equation (3) is less than 1 , equation (1) and (2) can be rewritten in terms of θ according to equation (4) and (5) respectively,

$$d\theta/dl_t = (1-u_r/u_f) \qquad \text{...... (4)}$$

$$d(\theta y_1)/dl_t = \{x_{1f} - x_{1r}(u_r / u_f)\} \qquad \text{...... (5)}$$

where $l_t = l_h / L$

The overall permeation process is described schematically in Figure 4.2. In Figure 4.3 the permeation of the feed flow in a single hollow-fiber membrane in the cross flow mode is illustrated.

4.2 Permeation Rate Equations.

In a cross flow pattern, the concentration of the local permeate stream leaving the membrane active layer (skin), y', is generally different from the bulk permeate stream, y. The total permeate flux of component 1 and 2 can be represented by the equations (6) and (7) respectively:

$$\frac{d(ux_1)}{dl_h} = -\pi D_o N P_H (P_1 / l_m)(x_1 - \gamma y') \quad \text{... (6)}$$

$$\frac{d\{u(1 - x_1)\}}{dl_h} = -\pi D_o N P_H (P_2 / l_m)\{(1 - x_1) - \gamma(1 - y')\} \quad \text{... (7)}$$

where P_H and P_L are the pressure at the feed and permeate streams respectively, $\gamma = P_L / P_H$, P_1 / l_m and P_2 / l_m are the permeation rates for component 1 and 2, respectively, D_o is the hollow-fiber membrane outside diameter, N is the number of active fiber and then local permeate concentration, y', is defined as follows:

$$y' = \frac{d(ux_1)}{du} \quad \text{... (8)}$$

Equations (6) and (7) show that the permeation driving force across the membrane is dependent on the feed concentration and the local permeate concentration y'. Equation (8) states that the mole fraction of a component in the permeate leaving the membrane active layer is equal to the flow fraction of the same component in the permeate flux through the membrane.With the aid of equation (8), the ratio of equations (6) to equation (7) becomes:

$$\frac{y'}{1 - y'} = \frac{\alpha(x - \gamma y')}{1 - x - \gamma(1 - y')} \quad \text{.... (9)}$$

where α is the selectivity.

Solving for y' from equation (9) yields

$$y' = \frac{1+(\alpha-1)(\gamma+x)-\{[1+(\alpha-1)(\gamma+\alpha)]^2-4\gamma\alpha x(\alpha-1)\}^{1/2}}{2\gamma(\alpha - 1)} \qquad \text{... (10)}$$

4.3 Feed Pressure Drop Equation.

Feed pressure drop is taken into consideration in the mathematical model. The present model predicts pressure drop along the membrane surfaces of the feed stream. The derivation of the feed pressure drop equation assumes a compressible gas flowing under pressure in a tube using a mechanical energy balance, and the equation for feed pressure drop, after simplifications, is reduced to (Dodge, 1944)

$$\frac{d(P_H)}{d l_h} = -\frac{2 z R T F G^2}{\{P_H R_H \Omega_m g_c\}} \qquad \text{..... (11)}$$

where F is the friction factor which can be estimated from equation (12), g_c is the Newton's law conversion factor, G is the mass flow rate per cross-sectional area of the membrane, R_H is given in equation (14), R is the gas constant, T is the feed temperature, z is the compressibility factor and Ω_m is the molecular weight of the gas mixtures in the retentate stream as defined by equation (14).

$$F = 0.008(D_{eq})^{-1/3} \qquad \text{..... (12)}$$

where $D_{eq} = 4R_H$, where R_H is given in equation (13):

$$R_H = \frac{\text{Cross sectional area}}{\text{Wetted perimeter}} = 0.25\,(D^2_{mm}\,/\,ND_o - D_o) \qquad \text{..... (13)}$$

$$\Omega_m = \Omega_1 y_1 + \Omega_2 (1 - y_2), \qquad \text{..... (14)}$$

where y_1 and y_2 are the mole fractions of components 1 and 2 in the mixture, respectively, D_o is the outer diameter of the hollow-fiber membrane and D_{mm} is the membrane module outer diameter.

Equations (4), (5), (6), (7) and (11) are the governing equations for the asymmetric hollow-fiber membrane permeator when feed pressure drop for retentate stream is considered. The solution strategy are described in the following section.

Practically the feed flow rate is much higher than the permeate flow rate so that u, x_1 and P_H can be assumed to vary in the direction of L (refer to Fig. 4.3), so equations (6), (7) and (11) can be numerically integrated along the membrane length. Equations (15), (16) and (17) represent equation (6), (7) and (11) after descretization for numerical solution:

$$\frac{\Delta(u x_1)}{\Delta l_h} = -\pi D_o N P_H (P_1/l_m)(x - \gamma y') \qquad \text{..... (15)}$$

$$\frac{\Delta\{u(1-x)\}}{\Delta l_h} = -\pi D_o N P_H (P_2/l_m)\{(1-x_1) - \gamma(1-y')\} \qquad \text{..... (16)}$$

$$\frac{\Delta P_H}{\Delta l_h} = -\frac{2\,z\,R\,T\,F\,G^2}{P_H\, R_H\, \Omega_m\, g_c} \qquad \text{..... (17)}$$

For the permeate stream, the variation of the permeate pressure (P_L), θ and y_1 in radial directions can be taken as being negligible since the membrane length is much greater than the hollow-fiber diameter and the permeate flow along the membrane length is dominant. Therefore, equations (4) and (5) become:

$$\Delta\theta / \Delta l_t = \{1 - u_r / u_f\} \quad ..... (18)$$

$$\Delta(\theta y_1) / \Delta l_t = \{x_{1f} - x_1 u_r / u_f\} \quad ..... (19)$$

4.2 Solution Strategy of The Mathematical Model

Equations (15) to (17) are numerically integrated simultaneously after initialization steps are carried out. The integration of equations (15) to (17) is started at the feed inlet end of the hollow-fiber with initial values of y's calculated from the given feed composition using equation (10).

A numerical integration subroutine from NAG library under subroutine called D02BBF is invoked with the following initial conditions:

For area $Dl_t = 0$, i.e. for $l_t = 0$,
$u_o = u_f$, $x_1 = x_{1f}$, $P = P_H$ and $y' = y'_f$.

The algorithm for the iteration method is described below:

1) As first approximation, γ (=P_L/P_H) is assumed to be everywhere equal to γ_o . Then equations (16) to (17) are integrated from $l_t = 0$ to l_t = 1 give the profile of flow, composition and feed pressure, respectively (u, x_1 and P_H).

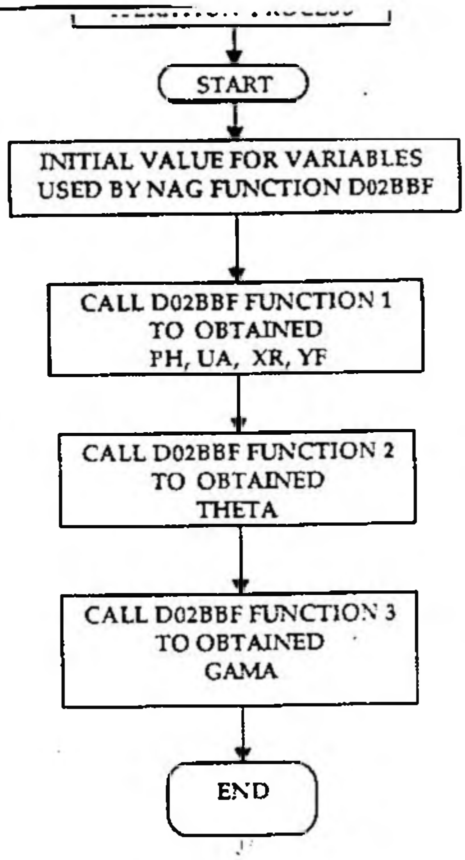


Figure 4.1 Sub-Module of Iteration Process

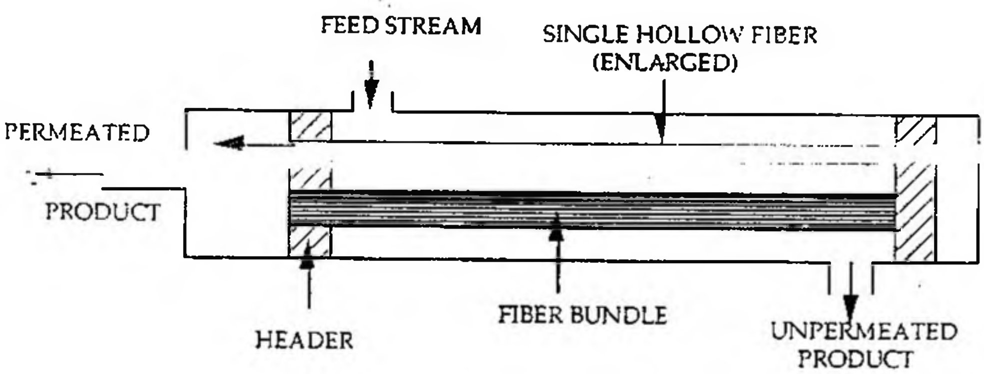


Figure 4.2 Hollow-Fiber Permeator

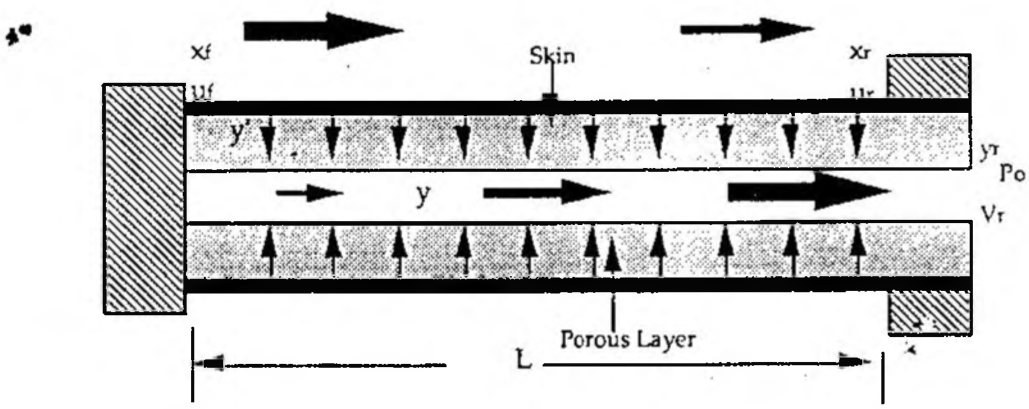


Figure 4.3 Gas Permeation through Asymmetric Hollow-Fiber Membrane in The Crossflow Mode with Feed Flow Outside of The Hollow-Fiber

2) The next step is to integrate equations (18) and (19) from $l_t = 0$ to $l_t = 1$ utilizing the results obtained above and the following initial conditions:

$\theta = 0$, and $y_{10} = y'_f$

after which a θ-y -l_t relation and the permeate outlet y_{10} is obtained.

3) The above sequence of steps are repeated until the calculated y_{1L} converge to the value at the permeate outlet y_{10}. The bulk permeate composition y at the fiber outlet can be calculated using material balance from the composition of flows of the feed and retentate streams.

Figure 4.1 shows a sub-module of the iteration process used in the computer program.

5.0 RESULTS AND DISCUSSIONS

5.1 Feed Pressure Profile

Figure 5.1 and 5.2 show a typical plot of the feed pressure profile along the membrane surface for feed gas composition 5% and 10% carbon dioxide respectively. As seen from figure 5.1 and 5.2, the predicted feed pressure profile decreased along the hollow-fiber length toward the outlet. This is due to frictional losses when fluid streams flowing through the narrow channels of permeators (Berman, 1953; Kovvali et al., 1992). Feed pressure drops were found to increase with an increased feed pressure.

Initial Value of Permeation Rates		
CO_2 Permeation rate	=	13.09×10^{-5} cm^3(STP)/cm^2.s.cmHg
CH_4 Permeation rate	=	0.294×10^{-5} cm^3(STP)/cm^2.s.cmHg

Commercial-size Membrane Permeator		
Membrane Area, *A*	=	232877.97 cm^2
Number of Active fiber, *N*	=	7200
Volumetric Feed Flow rate, *U* cm^3(STP)/s	=	1.83×10^6
Hollow Fiber Inside Diameter, D_i	=	0.016 cm
Hollow Fiber Outside Diameter, D_o	=	0.032 cm
Membrane Module Diameter, D_{mm}	=	5.08 cm
Length of Membrane Module, *L*	=	304.8 cm

Operating Condition Ranges		
Ranges of Feed Pressure	=	2068 cm Hg to 5170 cm Hg
Permeate Pressure	=	76 cm Hg
Operating Temperature	=	25 °C
Ranges of Feed Composition	=	5% to 50% CO_2 in the feed

Table 5.1 Input Variables Used in The Mathematical Model

In this study, it was found that the highest feed pressure drop was at about 107.31 cm Hg with feed pressure at the inlet and feed composition of 5170 cm Hg and 5% carbon dioxide in the feed, respectively.

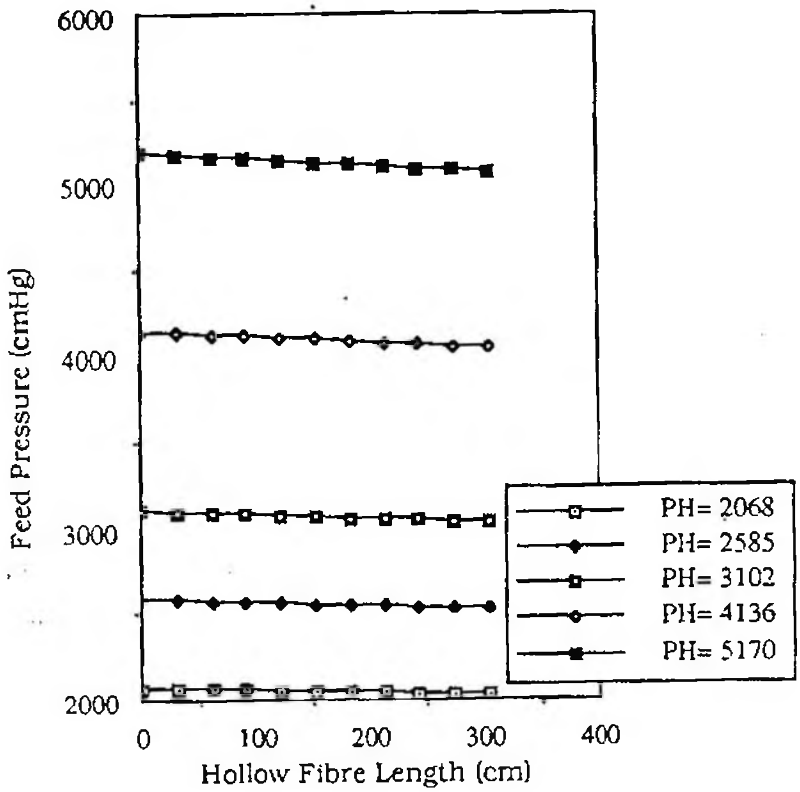


Figure: 5.1 Feed Pressure Profile
(T=25 deg. C, CO2=5%)

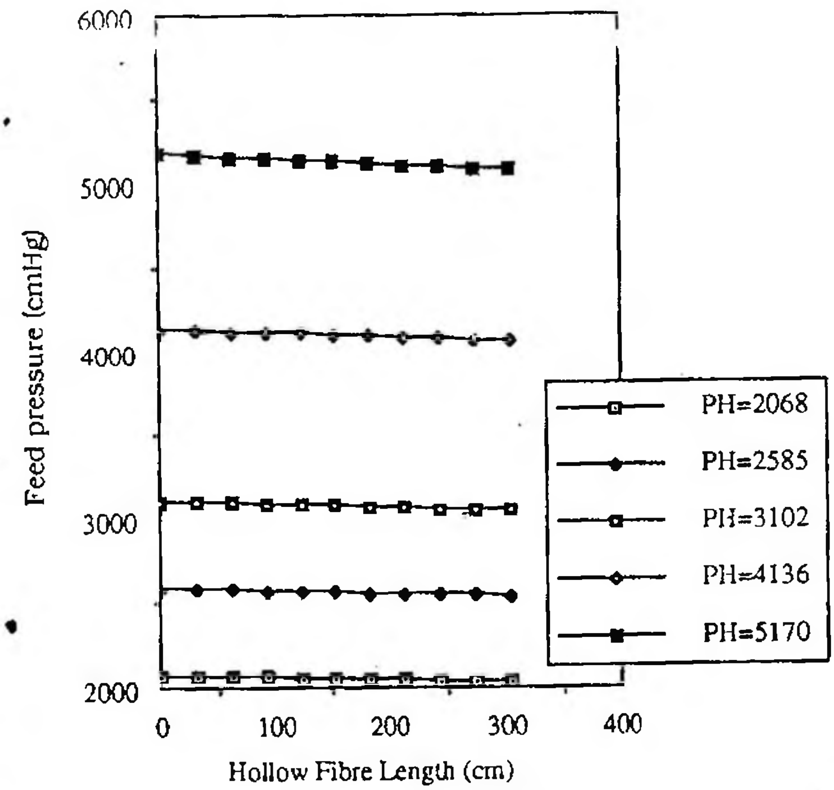


Figure 5.2: Feed Pressure Profile
(T=25 deg.C, CO2=10%)

5.2 Effect of Feed Pressure Drop on Permeate Composition

Typical plot of the effect of feed pressure drop on permeate composition is shown in Figure 5.3 for 5% carbon dioxide in the feed. The feed pressure drop seems to have a linear relation with the feed pressure at the inlet. These figures also show that the permeate composition increased in approximately a parabolic fashion with respect to the feed pressure at the inlet. This means that operating at higher feed pressure is desirable in order to increase the permeate purity.

Figure 5.4 shows the predicted outlet permeate and retentate composition as a function of feed pressure at the inlet. Higher feed pressure at the inlet resulted in higher carbon dioxide concentration in the permeate stream and the reverse is true for the retentate stream. As mentioned above the higher feed pressure at the inlet gives higher driving force for the more permeable component resulting in high carbon dioxide purity in the permeate stream

5.3 Effect of Feed Pressure on Permeation Rates

The effect of feed pressure on permeation rates for the individual components in the feed is shown in Tables 5.1 . The results given are based on 5% carbon dioxide in the feed at different values of feed pressures.

The results showed that the permeation rates of both carbon dioxide and methane increased with increasing feed pressure at the inlet. In this case, carbon dioxide is the faster permeating gas because of its higher solubility and diffusivity. Figures 5.5 and 5.6 are examples of plots of permeation rates for carbon dioxide and methane versus feed pressure at the inlet at different feed composition. Figures 5.5 and 5.6 show that feed pressure at the inlet seems to have a stronger influence on the permeation

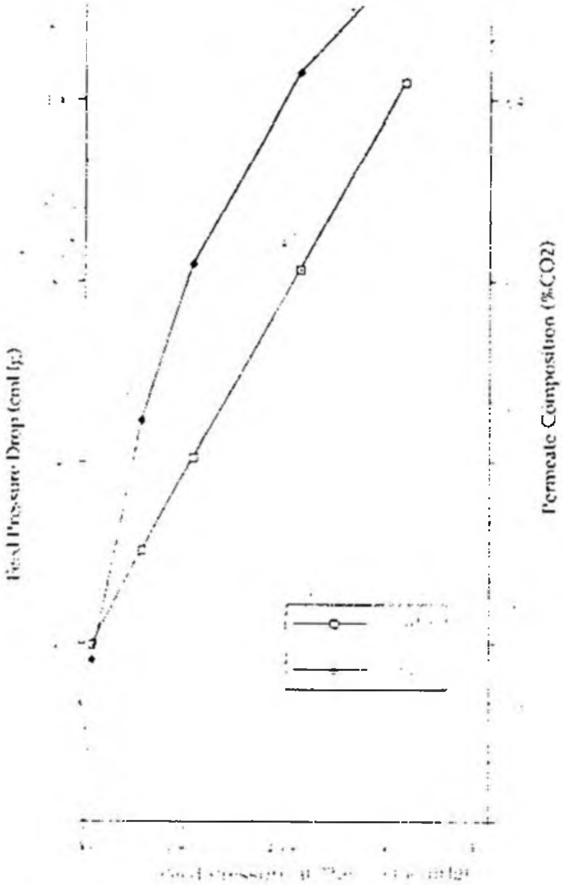


Figure 5.3 Effect of Feed Pressure Drop on Permeate Composition versus Feed Pressure
(T = 25 C, CO2 = 5%)

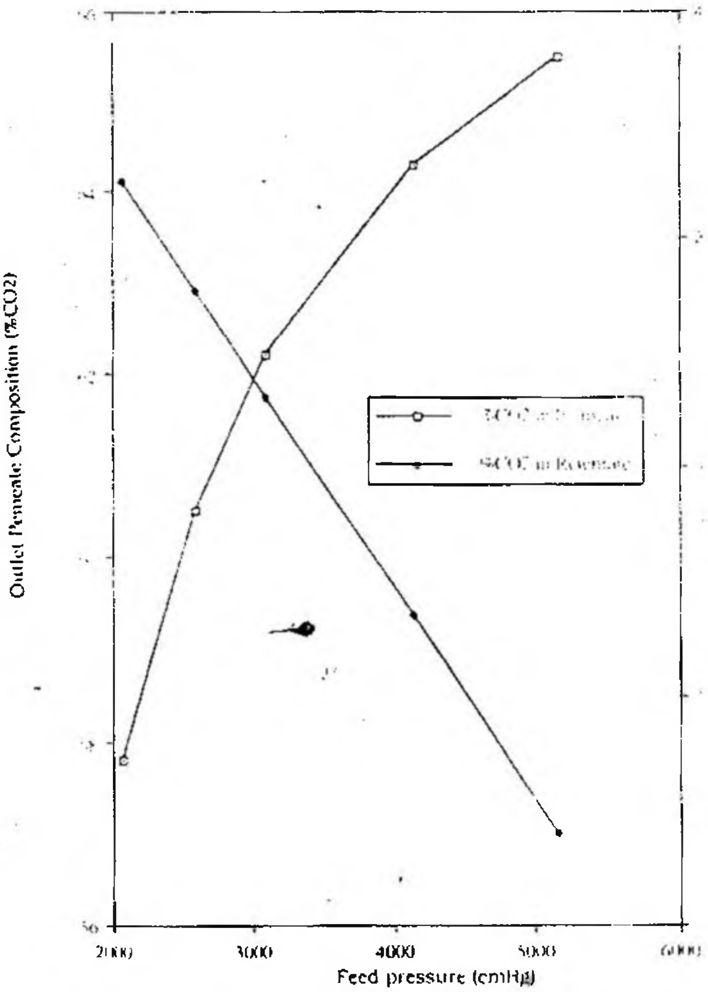


Figure 5.4 Predicted Outlet Composition in the Permeate and Retentate Streams at different Feed Pressures
(T = 25 C, CO2 = 5%)

rates of carbon dioxide, especially when the concentration of carbon dioxide is lower in the feed.

With reference to Figure 5.5, for 5% carbon dioxide in the feed, the permeation rates of carbon dioxide were found to increase at about 4.77% with increasing feed pressure from 2068 to 5170 cm Hg. For the same feed composition and feed pressure increment, the permeation rates of methane were found to increase by about 2.73% . When the carbon dioxide in the feed was increased to 50%, its permeation rates was found to increase by about 0.44%; whereas permeation rates of methane was increased to about 4.3%. Higher carbon dioxide concentration in the feed has lesser effects on its permeation rates. Therefore, feed pressure at the inlet has more influence on the permeation rate of carbon dioxide.

5.4 Effect of Feed Composition on Permeation Rates

Retentate and permeate compositions were found to vary along the hollow-fiber length and this directly affects the performance of the system. The retentate composition, as expected, decreased along the membrane length while the permeate composition shows increasing carbon dioxide concentration along the membrane length. The typical profile of both permeate and retentate streams are shown in Figure 5.7 and 5.8.

The permeate composition profiles show increasing carbon dioxide concentration along the fiber length toward the permeate outlet. This is due to the partial pressure difference of carbon dioxide at the permeate outlet is greater compared to that at the feed inlet end.

The concentration of carbon dioxide in the feed has less influence on its permeation rate at higher feed to permeate pressure ratio as compared at the lower feed to permeate pressure ratio. This is probably due to at

Feed pressure	Feed Pressure Drop	Permeate Pressure	Permeate Comp.	Retentate Comp.
		5% C)2 + 95%CH4 mixture.		
2068	42.02	76.	57.807	3.147
2585	54.90	76.	60.459	2.647
4136	85.55	76.	64.311	1.147
5170	107.31	76.	65.540	0.146
		10% :O2 + 90%CH4 mixture.		
2068	39.76	76.	78.068	6.147
2585	50.07	76.	79.254	5.146
4136	81.02	76.	80.874	2.146
5170	101.66	76.	81.373	0.146
		15% :O2 + 85%CH4 mixture.		
2068	37.49	76.	86.179	9.146
2585	47.24	76.	86.757	7.646
4136	76.49	76.	87.552	3.146
5170	95.99	76.	87.799	0.145
		25% :O2 + 75%CH4 mixture.		
2068	32.95	76.	92.786	15.146
2585	41.56	76.	92.985	12.645
4136	67.41	76.	93.264	5.145
5170	84.64	76.	93.353	0.144
		50% :O2 + 50%CH4 mixture.		
2068	21.54	76.	97.641	30.145
2585	27.30	76.	97.676	25.144
4136	44.60	76.	97.725	10.142
5170	56.12	76.	97.741	0.141

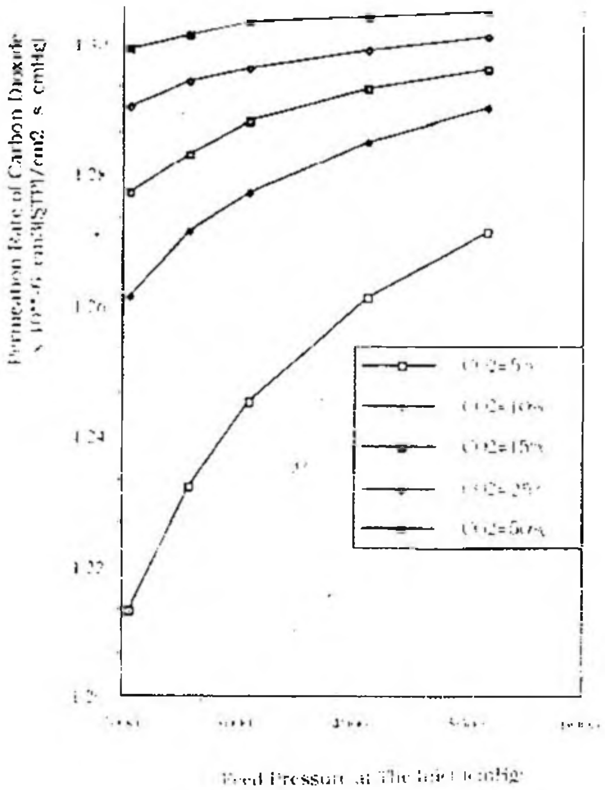


Figure 5.5 Effect of Feed Pressure on The Permeation Rates of Carbon Dioxide in Mixtures at Constant Temperature Composition versus Feed Pressure (T = 25 C)

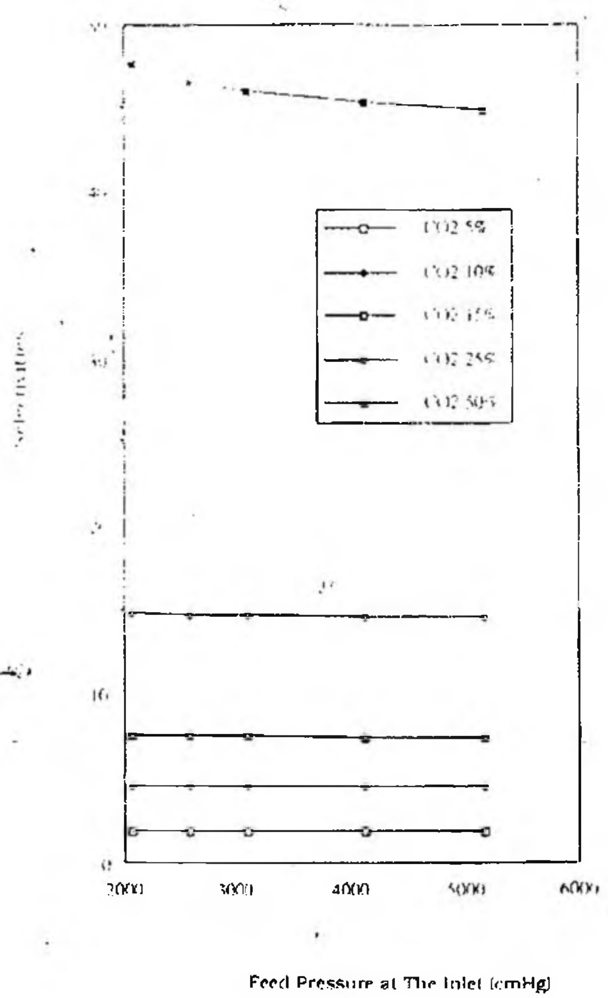


Figure 5.6 Effect of Feed Pressure on The Permeation Rates of Methane in Mixtures at Constant Temperature Composition versus Feed Pressure (T = 25 C)

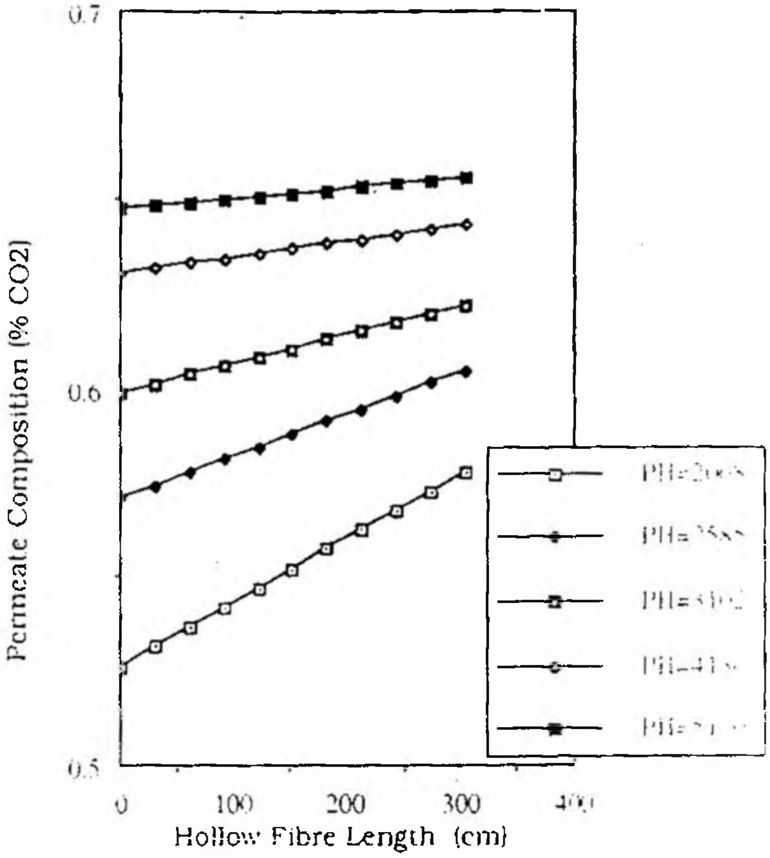


Figure 5.7 Permeate Composition Profile

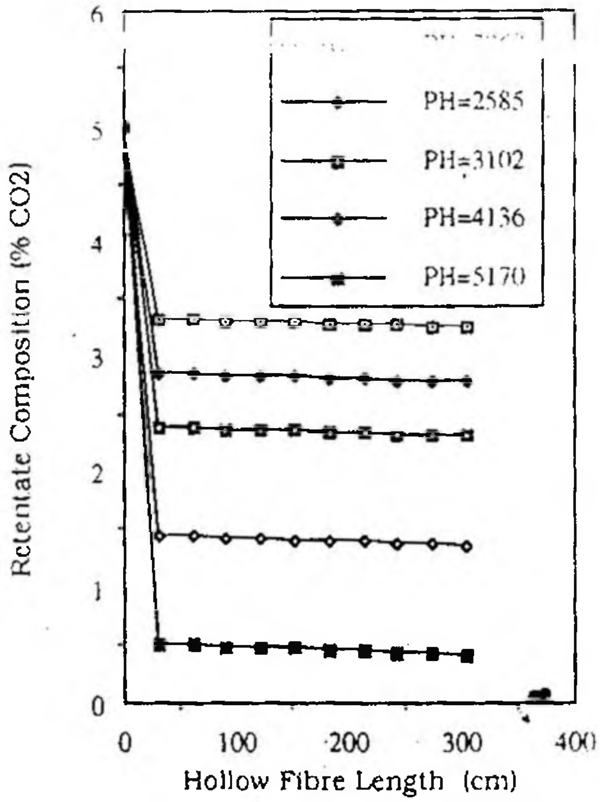


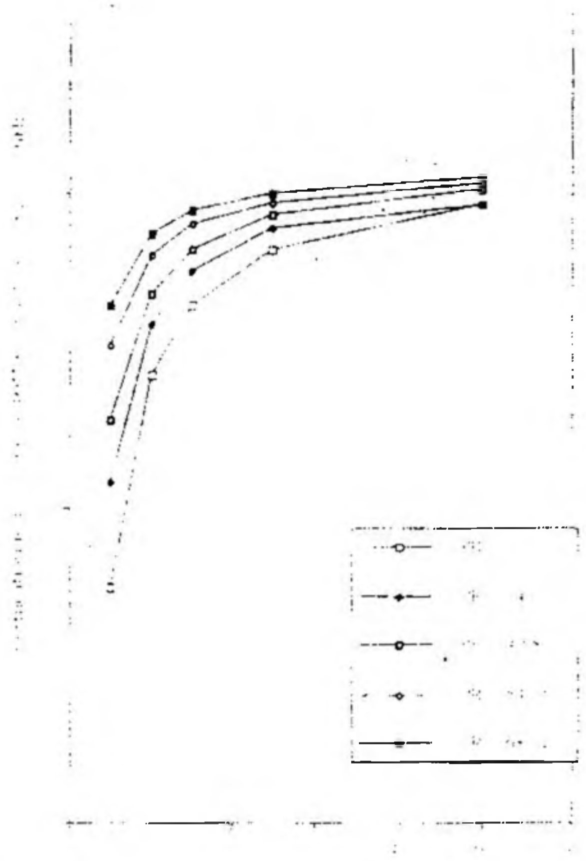
Figure 5.8 Retentate Composition Profile

higher feed to permeate pressure ratio, the permeation rates of carbon dioxide increased with increasing carbon dioxide content in the feed approaching its partial pressure saturation.

Therefore, the lower the methane concentration in the feed, the lesser is the effect of feed pressure at the inlet on its permeation rates. Better separation can be achieved at higher feed pressure at the inlet, and by using low concentration of methane in the feed.

With reference to Figure 5.9 increasing feed to permeate pressure ratio results in an increase in the permeation rate of carbon dioxide at constant carbon dioxide concentration, thus increasing the dependence of permeation rate on feed composition as indicated by the value of permeation rate at 27.2 and 68.02 pressure ratio, respectively. At 10% carbon dioxide in the feed mixture, the permeation rate at 27.2 pressure ratio is 1.2491×10^{-6} cm^3(STP)/cm^2.s.cmHg compared to 1.299×10^{-6} cm^3(STP)/cm^2.s.cmHg at 50% carbon dioxide composition in the feed. However, at a pressure ratio of 68.02 and at the same carbon dioxide composition in the feed, the permeation rate of carbon dioxide increased from 1.286×10^{-6} cm^3(STP)/cm^2.s.cm Hg to 1.304×10^{-6} cm^3(STP)/cm^2.s.cm Hg.

In contrast to the permeation rate of carbon dioxide, Figure 5.10 shows that the permeation rate of methane is significantly decreased with increasing carbon dioxide composition in the feed. The permeation rate of methane at 10% carbon dioxide in the feed and at 27.2 pressure ratio is about 2.79×10^{-7} cm^3(STP)/cm^2.s.cmHg compared to 0.291×10^{-7} cm^3(STP)/cm^2.s.cmHg at 50% carbon dioxide composition in the feed and at 68.02 pressure ratio.



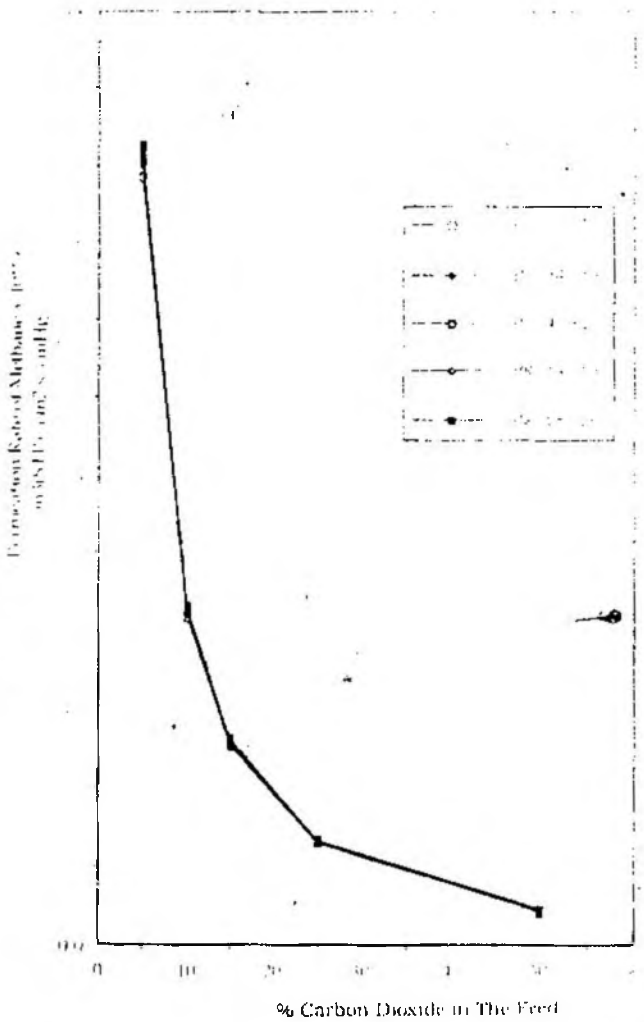


Figure 5.10 Effect of Feed Composition on Permeation Rates of CH_4 in Different Mixtures and at Constant Temperature (T = 25 C)

With reference to the above results, feed to permeate pressure ratios seems to have a little or no effects on the permeation rate of methane in the mixtures.

5.5 Effect of Feed Pressure on Selectivities

Selectivities is the ratio of the permeation rates of the fast and slower gas in the mixture. Results obtained shows that feed pressure have a less influence on selectivities. Figure 5.11 shows the effect of feed pressure on selectivities at 25 ^oC at various carbon dioxide content in the feed. As seen in the figure, selectivities slightly decreased for 50% concentration of carbon dioxide in the feed.

With reference to Figure 5.11, for 50% carbon dioxide composition in the feed and 25 ^oC, the carbon dioxide-methane selectivity decreases from 46.638 to 44.905, with pressure increment from 2068 cmHg to 5170 cmHg. This represents insignificant decreased in selectivity.

5.6 Effect of Feed Composition on Selectivities

Feed composition influenced both the modified permeation rates for carbon dioxide and methane , thus affecting the selectivities.

The results obtained show that selectivities are strongly dependent on feed composition. Selectivities tend to increase with increasing feed composition at the same feed pressure at the inlet. Figure 5.12 shows that an increase in feed to permeate pressure ratio results in a slight decrease in selectivities. Therefore, feed to permeate pressure ratio seems to have a little influence on selectivities. High concentration of carbon dioxide in the feed favoured higher selectivities.

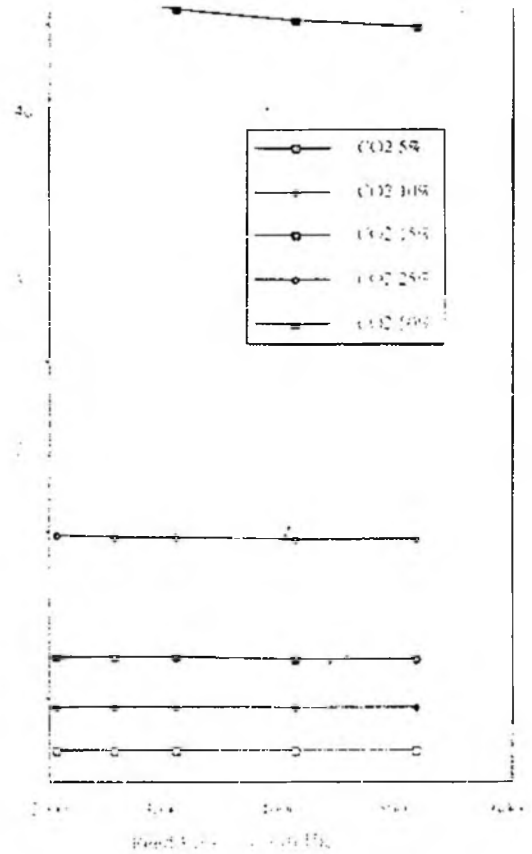
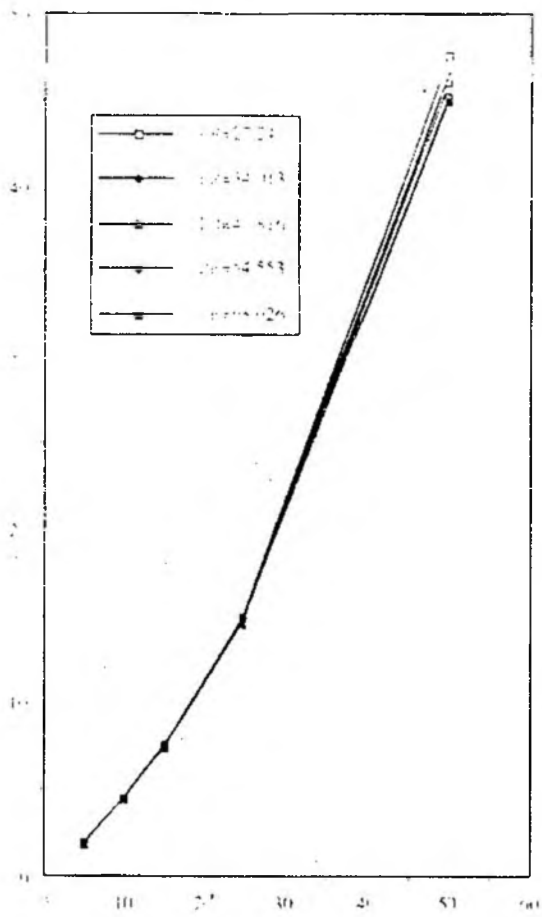


Figure 5.31 Effect of Feed Pressure on Selectivities of Separation of CO2-CH4 Mixture at Different Feed Pressure



Feed Composition (% CO2)

Figure 5.32 Effect of Feed Composition on Selectivities of CO2-CH4 Mixture Separation a Constant Temperature

5.7 Pressure Drop Prediction of A Membrane Gas Separation System.

The prediction of pressure drop of membrane gas separation system is an important aspect in order to avoid under-or over-design of a particular system of interest. Pressure is one of the most important parameters in a membrane gas separation. Since membrane gas permeation is a pressure-driven process, the pressure differential between the feed and permeate streams has significant effect on a particular membrane's performance.

This section discusses the predicted feed pressure drop in a commercial-sized membrane gas separation system using the mathematical model developed.

5.7.1 Case Study 1

In this study, the importance of pressure drops prediction in determining the separation performance of carbon dioxide from natural gas (methane) was shown. The data is taken from Shirley and Borzik (Shirley and Borzik, 1982) in which a single-stage membrane process was used for the separation of 7% carbon dioxide in the feed. The data is used to predict the applicability of the mathematical model in the prediction of the feed and permeate pressure drops. Results obtained are given in Table 5.2 below.

The results shows that the predicted feed pressure drop (DPH) is 89.74 cm Hg compared to 77.55 cm hg for the observed feed pressure drop. This different might be attributed to the number of active fiber and the hollow-fiber inner and outer diameter used in this study were not accurately determined.

5.7.2 Case study 2

Another case study of prediction of feed pressure profile utilizing data obtained from Prism separator for the separation of carbon dioxide/methane mixtures was made (Unpublished correspondence, 1988). There were two cases considered, 50% carbon dioxide and 7% carbon dioxide in the feed. Three ranges of operating pressure were used and the predicted results are given in table 5.3 below.

Based from this study, it is clear that feed pressure drop must be considered in membrane system design. If they are assumed constant, significant error might be introduced in designing the system.

6.0 CONCLUSIONS

The separation of carbon dioxide from natural gas (methane) using asymmetric hollow-fiber membrane permeator was found to be strongly affected by the operating conditions. The mathematical model developed in this study was solved numerically using a Runge-Kutta-Merson method from NAG subroutine library. The model which considered feed drop in calculating permeation rates and selectivities of carbon dioxide/methane mixtures, was successful in predicting the performance of the hollow-fiber membrane permeator used in this study. Based on the simulation or iteration process at different feed pressure, feed and composition profiles were established along the membrane surface. The predicted feed pressure profile decreased linearly along the hollow-fiber length toward the outlet whereas the predicted permeate composition increased with approximately a parabolic fashion with respect to the feed pressure at the inlet.

The permeation rates of both carbon dioxide and methane increased with increasing feed pressure at the inlet. However carbon dioxide permeation rate increased at faster rate as compared to methane. Therefore, permeation rate of carbon dioxide shows stronger dependence on feed pressure at the inlet.

Feed composition were found significantly to affect permeation rates and selectivities. High concentration of carbon dioxide in the feed gave higher selectivities.

The model was shown to accurately predict the feed pressure drop based on the case studies conducted.

Observed		Predicted
PH	DPH	DPH
(cm Hg)		
4394.5	77.55	89.74

Table 5.2 Prediction of Feed and Permeate Pressure Drops for Case Study 1 (7% CO_2 and 93% CH_4).

Observed		Predicted
PH	DPH	DPH
(cm Hg)		
50% CO2 + 27% CH4		
1551	155.1	15.20
4136	155.1	43.97
7% CO2 + 80% CH4		
1551	155.1	29.82
5170	155.1	104.16

Table 5.3 Prediction of Feed and Permeate Pressure Drops for Case Study 2

NOMENCLATURE

A	=	Membrane's area [cm^2]
C_k	=	Dimensionless constant
D_o	=	Hollow -fiber outside diameter, [cm]
D_{mm}	=	Hollow-fiber membrane module diameter, [cm]
F	=	Friction factor
g_c	=	Newton's law conversion factor [gm cm/dyne sec^2]
G	=	Mass flow rate per cross sectional area of the membrane [gm/s cm^2]
L	=	Hollow-fiber length, [cm]
l_m	=	Effective skin thickness of the asymmetric membrane, [cm]
l_h	=	Hollow-fiber length variable, [cm]
l_t	=	l_h/L, Hollow-fiber length variable, dimensionless
N	=	Total number of active fibers
P	=	Pressure, [cmHg]
P_H	=	Feed-side pressure, [cmHg]
P_L	=	Permeate-side pressure, [cmHg]
P_{Hi}	=	Feed partial pressure, [cmHg]
P_{Li}	=	Permeate partial pressure, [cmHg]
P_i/l_m	=	Permeation rate coefficient for component i, [cm^3(STP)/cm^2.s.cmHg]
Q_1	=	Volumetric flow rate of the permeate gas component, [cm^3 (STP)/s]
R	=	Universal gas constant, [cm^3 atm/gmole K]
R_H	=	Hydraulic radius , [cm]
T	=	Absolute temperature, [^oK]

u = Feed-side gas flow rate per unit length of the hollow-fiber in cross flowmode, [cm^3(STP)/s.cm]

u_f = Feed gas flow rate per unit length of the hollow-fiber in cross flow mode, [cm^3(STP)/s.cm]

u_r = Retentate gas flow rate per unit length of hollow-fiber in cross flow mode, [cm^3(STP)/s.cm]

U = Feed-side gas flow rate for the entire membrane, [cm^3(STP)/s]

U_f = Feed gas flow rate for hollow-fiber in cocurrent flow mode, [cm^3(STP)/s]

V = Volumetric permeate flow rate in equations (2.7) and (2.8), [cm^3(STP)/s]

x_1 = Mole fraction of the fast gas in the feed

y' = Local permeate concentration on the membrane surface, [mol fraction]

y = Permeate concentration in the bulk permeate stream, [mol fraction]

y_1 = Mole fraction of the fast gas in the permeate.

z = Nonideal gas compressibility factor

Greek symbols

α = Membrane selectivity (permeability of more permeable component/permeability of less permeable component)

γ = Ratio of permeate to feed pressure

γ_0 = Ratio of permeate to feed pressure at the permeate outlet

μ_1 = Viscosity of the less permeable component, [centipoise]

μ = Gas viscosity [centipoise]

μ_2 = Viscosity of the more permeable gas, [centipoise]

μ_m = Viscosity of the gas mixture [centipoise]

θ = Ratio of permeate to feed flow

Ω_1 = Molecular weight of the fast gas [gm]

Ω_2 = Molecular weight of the slow gas [gm]

Ω_m = Molecular weight of the gas mixture [gm]

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