DYNAMIC MODELLING OF A TWO-LINK FLEXIBLE MANIPULATOR

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Special dedication to
my wife, Hawani
who is really understanding and supporting me
in completing this work.
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father and mother,
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and inspired me
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ABSTRACT

Flexible manipulator systems exhibit many advantages over their traditional (rigid-arm) counterparts. However, they have not been favoured in production industries due to its obvious disadvantages in controlling the manipulator. Therefore, this thesis presents theoretical investigation into the dynamic modelling and characterisation of a flexible manipulator system. A constrained two-link flexible manipulator is considered that moves in horizontal plane only. A mathematical model of the system is developed based on finite element method. The final derived model of the system is simulated to investigate the behaviours of the system.

ABSTRAK

Sistem manipulasi mudah lentur mempunyai banyak kelebihan berbanding sistem tradisional yang tegar. Namun, ianya jarang digunakan dalam industi disebabkan kelemahannya yang ketara dalam pengawalan. Oleh kerana itu, tesis ini menumpukan kepada kajian pemodelan dinamik dan perwatakan sistem manipulasi mudah lentur secara teori. Kajian ini hanya mengambil kira sistem manipulasi mudah lentur dua lengan yang bergerak secara mendatar. Model matematik sistem ini dibangunkan berdasarkan kaedah unsur terhingga. Model tersebut disimulasi untuk mengkaji sifat-sifat sistem tersebut.

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LIST OF SYMBOLS

Subscript *ij*	-	*j*th element of link *i*
OXY	-	inertial system of coordinates.
$O_1X_1Y_1(O_2X_2Y_2)$	-	body-fixed system of coordinates attached to link 1(2). O_1X_1 is the direction of the inflexed link 1(2).
r_{02}	-	distance vector from *O* to the origin of the system of coordinates $O_2X_2Y_2$.
$x_1(x_2)$	-	a vector in body-fixed system $O_1X_1Y_1(O_2X_2Y_2)$ from origin $O_1(O_2)$ to a given point on the link.
n (n_1, n_2)	-	number of finite elements of an arbitrary link (link 1, link 2)
θ (α)	-	angular displacement at joint 1 (2)
l (l_1,l_2)	-	length of each element of an arbitrary link (link 1, link 2)
$L_1(L_2)$	-	total length of link 1(2)
$E_1(E_2)$	-	Young's modulus of link 1(link 2)
$I_1(I_2)$	-	second moment of area of link 1(link 2)
$\rho_1(\rho_2)$	-	mass density per unit volume of link 1(link 2)

$A_1(A_2)$	-	cross sectional area of link 1(link 2)
x, y	-	distances in body-fixed system $O_1X_1Y_1$ $(O_2X_2Y_2)$ of an arbitrary point on the finite element 'j' from the common junction between elements '(j-1)' and 'j' of link 1 (2)
M (M_{1j}, M_{2j})	-	generalised inertia matrix (also called simply inertia or mass matrix) of the arbitrary link (link 1, link 2)
K (K_{1j}, K_{2j})	-	stiffness matrix of the arbitrary link (link 1, link 2)
$\tau_1(\tau_2)$	-	applied torque at joint 1 (joint 2)
$w_1(x_1,t)\&w_2(x_2,t)$	-	Elastic deflection of link 1 & link 2
$N(x)(N_a(x),N_b(x))$	-	Shape function of the arbitrary link (link 1,link 2)
$Q(t)(Q_a(t),Q_b(t))$	-	Nodal displacement of the arbitrary link (link 1,link 2)
$y_1(x_1,t)$ & $y_2(x_2,t)$	-	Total displacement of link 1 & link 2
u_{n-1},u_n (u_{k-1},u_k)	-	flexural displacement and flexural slope, respectively, at the common junction between elements '(j-1)' and 'j' of the arbitrary link and link 1 (link 2) measured with respect to the axis $O_1X_1(O_2X_2)$
$\theta_{n-1},\theta_n(\theta_{k-1},\theta_k)$	-	flexural displacement and flexural slope, respectively, at the end-point of link 1 (link 2) measured with respect to the axis $O_1X_1(O_2X_2)$
θ_{n1}	-	flexural slope at the tip of the first link
u_{n1}	-	flexural displacement at the tip of the first link

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CHAPTER I

INTRODUCTION

1.1 Introduction

Many of today's robots are required to perform tasks, which demand high level of accuracy in end-effector positioning. The links of the robot connecting the joints are large, rigid and heavy. This prevents oscillations in the links, which is one of the main causes of errors in end-effector positioning.

Since the links are heavy, much of the joint motor's power is expended moving the link and holding them against gravity. Also payloads must be kept quite small compared to the mass of the robot itself, since large payloads will cause sagging and vibrations in the links, which will create uncertainty in the end-effector position. This results in a situation where these rigid robots are very inefficient and slow.

An attempt to solve these problems led to the development of flexible robots. A flexible robot is one, which has at least one of the links made up of lightweight material and is not rigid. In general, flexible robot manipulators exhibit many advantages over rigid robots: they require less material, are lighter in weight, consume less power, require smaller actuators, are more manoeuvrable and transportable, have less overall cost and higher payload to robot weight ratio [1]. Figure 1.1 and Figure 1.2 show examples of experimental two-link flexible manipulator system.

Weight reduction however, incurs a penalty in that the manipulator becomes more flexible and more difficult to control accurately. The control difficulty is caused by the fact that since the manipulator is a distributed system, a large number of flexible modes are required to accurately model its behaviour. Further complications arise due to highly nonlinear nature of the system. Due to lack of sensing, vibration due to system flexibility and incapability of precise positioning, an accurate model is very difficult to obtain.

Therefore, flexible manipulators have not been favoured in production industries, as the manipulator is required to have reasonable end-point accuracy in response to input commands. In this respect, a control mechanism that accounts for both rigid body and flexural motions of the system is required. If the advantages associated with lightness are not to be sacrificed, accurate models and efficient controllers have to be developed.

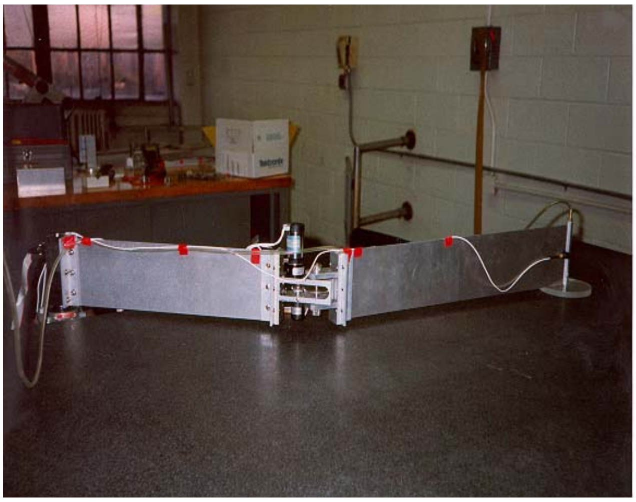

Figure 1.1: Two-link flexible manipulator at Polytechnic University, Brooklyn, New York.

Figure 1.2: Two-link flexible manipulator from University of Washington, USA

1.2 Objectives of the project

The main goal of this project is to derive a dynamic model of a two-link flexible manipulator by using Finite Element method. The emphasis is on obtaining a customized model that satisfies the aforementioned requirements about explicitness, completeness and accuracy of the Finite Element method that used to obtain the dynamic model. The project is also targeted to simulate and validate the dynamic model of the system. The simulation will be performed using MATLAB and SIMULINK. The last objective is to investigate the dynamic behaviour of the flexible manipulator according to the simulation results.

1.3 Scope of project

This project focuses in modelling a two-link flexible manipulator without payload and damping, which moves in the horizontal plane only, where the gravity effects are neglected. The dynamic model of the system is described by using Finite Element method. By using Lagrange's equation, the dynamic model is represented in state space form, so that it can be solved using control system approaches. The final derived model of the system is simulated using MATLAB and SIMULINK to investigate the behaviours of the system.

1.4 Outline of Thesis

This thesis is written to bring the readers step by steps going in the main core of the content. Chapter I introduces the objectives and scope of the project.

Chapter II familiarizes the readers with the literature review about this project. It discusses the brief review of all about previous researches in modelling flexible manipulator.

Chapter III discusses the research methodology in order to complete this project. Every stages of research methodology are briefly depicted in flow chart. It also explained the mathematical and simulation tools that used to realize this project.

Chapter IV explains the model development of a single-link flexible manipulator by using Finite Element method. It started by obtaining the total displacement of flexible link, the Kinetic Energy and the Potential Energy, the Element Mass Matrix and the Element Stiffness Matrix.

Chapter V explains the model development of a two-link flexible manipulator by implementing the same method in modelling single-link flexible manipulator. It also started by obtaining the total displacement of flexible link, the Kinetic Energy and the Potential Energy, the Element Mass Matrix and the Element Stiffness Matrix.

Chapter VI explains the simulation environment, the initial values of each parameter, the MATLAB simulation and SIMULINK used in simulation part of final derived model of two-link flexible manipulator. It also shows the input signal of each torque and the comparison of these two inputs.

Chapter VII shows and explains the simulation results of each link, the analysis of the simulation results, the validation of derived model and the discussion related to simulation results. This chapter is very important to show that this project is succeeded.

This paper work ends with Chapter VIII to conclude the whole project and this thesis. In this chapter, it will propose some recommendations and enhancements that can be made on this project in the future.

1.5 Summary of This Chapter

This chapter introduced this project with the advantages of flexible manipulator compared to rigid arm. This chapter also described the objectives and scope of this project. Then, the outline of thesis is discussed in final part of this chapter.

The flexible manipulator and the rigid manipulator can be combined in order to minimize the problem in controlling the flexible manipulator. In this project, both links are flexible. It can be suggested if one of the flexible manipulator is unbending, regarding to fact that the vibrations can been eliminated by increasing the rigidity of the arms.

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