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A Computational Intelligence Approach to Solve the Inverse Problem of Electrical DC Resistivity Sounding

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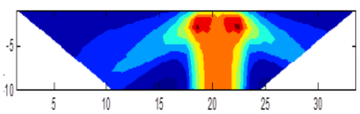
Graphical abstract



Abstract

Electrical methods have been widely used in geophysical surveying to obtain high-resolution information about subsurface conditions, since the last few decades. Resistivity is an important parameter in judging the ground properties, especially detecting buried objects of anomalous conductivity. Electrical DC (i.e. Direct Current) resistivity sounding is the commonly used technique to obtain the apparent 2-D resistivity of the region under investigation. Acquiring the true resistivity from collected data remains a complex task due to nonlinearity particularly due to contrasts distributed in the region. In this work, a radial basis function neural network (RBFNN) metamodelling approach is proposed to solve the 2-D resistivity inverse problem. The model was trained with synthetic data samples obtained for a homogeneous medium of 100Ω.m. The neural network was then tested on another set of synthetic data. The results show the ability of the proposed approach to estimate the true resistivity from the 2-D apparent resistivity sounding data with high correlation. The proposed technique, when executed, appears to be computationally-efficient, as it requires less processing time and produces less error than conventional method.

Keywords: DC resistivity sounding; 2-D resistivity; inversion problem; radial basis function; neural network; metamodelling

■1.0 INTRODUCTION

Geophysical surveys can be categorised into passive and active. Passive geophysical surveys involve the measurements of naturally occurring fields in the earth such as gravitational and magnetic fields, by means of measuring spatial variations in these fields to infer something about the subsurface geology. Active surveys are conducted by injecting a signal (e.g. an electrical current or an active radiometric source) into the earth and obtaining its responses to this signal. Electrical and electromagnetic techniques that employ active surveys represent the largest class of all geophysical methods.

DC electrical resistivity sounding method is the most popular technique to measure the subsurface resistivity. The conventional resistivity sounding is carried out on the earth's surface with a specified array of electrodes in order to obtain apparent resistivity data with respect to the variation of horizontal position and vertical depth. Typically, the apparent resistivity distribution is presented in a pseudosection using computer software, hence an inversion process is essential in order to determine the actual resistivity of the subsurface.

To represent the electrical resistivity, a number of approaches have been devoted to 1-D resistivity inversion (see, e.g. in [1] and [2]) while the 2-D inversion has also been used for many years (see, e.g in [3] and [4]) for over the last several decades. Of particular interest is the recent application of artificial neural networks for the inversion, as described in [5] and [6]. The former used a resilient back propagation algorithm to train the networks, but remained unable to resolve the inversion for the complicated geological structures. The latter focused on high resistivity contrast anomalies, however it is unclear of the ability to interpret resistivity in another range that may exist in the investigated region.

This paper addresses the 2-D DC resistivity inverse problem by using the radial basis neural network metamodelling approach. The inversion results are compared with the conventional inversion approaches to illustrate the effectiveness of the proposed technique for this purpose. The remainder of this paper is organised as follows. Section 2 presents the literature review on geophysical surveying and current metamodelling methodologies. The proposed technique, featuring a radial basis function neural network for 2-D resistivity inversion is described in Section 3. Section 4 shows the results, followed by some analysis and discussion. Finally, a conclusion is drawn in Section 5.

2.0 LITERATURE REVIEW

1.1 Geophysical Survey Tools

Electrical methods are popular in geophysical surveys, where they typically operate at frequencies ranging from direct current (DC) to >1GHz, to obtain information about the subsurface structure and composition [7]. Information of both structural and electrical properties provides important constraints for geophysical modelling. Structural information can be used to define for example the locations of faults and fractures, whereas electrical property information can be used to qualitatively characterise the rock, soil and other solid types as well as the fluid properties.

Electrical DC resistivity (ER) and induced polarisation (IP) are deployed in a similar manner, where the spatial distribution of low-frequency resistive and capacitive characteristics of soil can be determined [8]. Both ER and IP involve galvanic contact with the soil, hence may limit their application to appropriate sites and also require longer survey times. Modern resistivity and IP methods used today are very closely related to early developments in the 1970s, adopting the four-electrode measurement approach, called the Wenner four-pin method [9]. Extensive research in imaging prompted the development of multi-electrode measurement systems in the 1980s. From mid 1990s to date, most instrument manufacturers have been able to offer multiplexed instruments, and in some cases multi-measurement channel hardware, thus enabling a reduced survey time. A basic configuration of the electrical DC resistivity technique is illustrated in Figure 1.

The use of ground-penetrating radar (GPR) for geophysics applications was growing considerably in the 1970s. The foundations of GPR lie in the electromagnetic (EM) theory, using radar pulses in the microwave band (UHF/VHF frequencies) of the radio spectrum to detect the reflected signals from subsurface structures for subsurface imaging. GPR performs best in coarse-grained materials, such as sands and gravels, which are transparent to radio wave signals. Its sensitivity to water content could also provide a technique for mapping water table and perched water tables [10].

Electromagnetic induction (EM) is the most versatile of the airborne geophysical methods. It consists of both frequency domain EM (FDEM) and time domain EM (TDEM) approaches, measures the apparent electrical conductivity of the ground to depths ranging from a few to a few hundred meters, depending on the instrument chosen and the ground conductivity [11]. Both frequency and time domain EM methods employ a changing primary magnetic field created around a transmitter loop or coil to induce currents to flow in the ground, which in turn creates a secondary magnetic field sensed by the receiver coil. Unprocessed airborne EM data consist of the vertical (z) and two horizontal (x and y) components of secondary field strength (B) or decay rates (dB/dt) over time. Recent work has focused on inverting recorded EM airborne data into models that can accurately depict subsurface conductivity [12-14].

1.2 Metamodelling Techniques

Metamodeling, considered generally as an explicit description of how a domain-specific model is built for a complex system, has been successfully used in many fields where complicated computer models of an actual system exist but they may require a considerable amount of running time. Models involving finite element and fluid dynamics analysis or multi-objective optimisation algorithms with many parameters are some typical examples. Metamodelling evolves from the classical Design of Experiments (DOE) theory, in which polynomial functions are used as response surfaces, or metamodels. Nowadays there exist a number of metamodeling techniques, such as neural networks [15][16], Multivariate Adaptive Regression Splines (MARS)[17], Response Surface Modelling (RSM)[18], etc. Nevertheless, there is no conclusion about which model is definitely superior to the others. However, insights have been gained through a number of recent studies, whereby Kriging models, Gaussian and radial basis function (RBF) processes are intensively investigated [19].

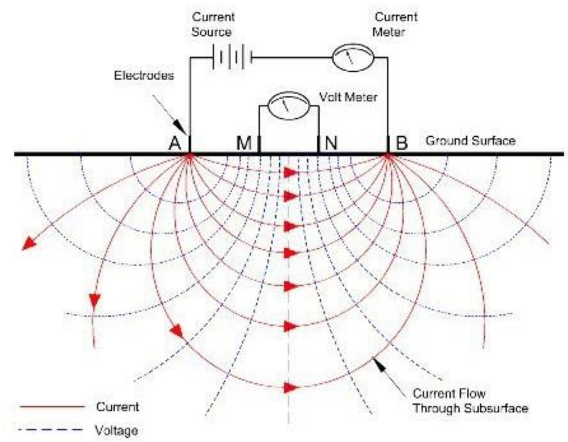


Figure 1 The basic configuration of electrical DC resistivity measurements [9]

In general the Kriging model is more accurate for nonlinear problems and also flexible in either interpolating sample points or filtering noisy data. However, it is difficult to obtain and use because a global optimization process need to be applied to identify maximum likelihood estimators. On the contrary, a polynomial model is easy to construct, clear on parameter sensitivity, and cheap to work with some slight reduction in accuracy [20].

The RBFs were first used in 1988 to design Artificial Neural Networks [21], with two layers: a hidden layer of radial basis function and a linear output layer. The input of the network is typically nonlinear, whereas the output is linear, representing the weight sum from the hidden neurons. The RBF model, especially the multi-quadric RBF, can interpolate sample points and at the same time is easy to construct. It thus seems to reach a trade-off between Kriging and polynomials. Recently, a new model called Support Vector Regression (SVR) was used and tested [22] with a higher accuracy than all other Metamodeling techniques including Kriging, polynomial, MARS, and RBF over a large number of test problems.

3.0 RBFNN METAMODEL FOR 2-D RESISTIVITY INVERSE PROBLEM

In this work, we will employ a Radial Basis Function Neural Network (RBFNN) as the proposed metamodel to approximate the resistivity inversion mapping and the apparent resistivity values obtained from electrical sounding measurements. Here, each hidden neuron of the RBF implements a radial activated function, and various types of function has been tested as the activation function such as Gausian, multi-quadratic, polyharmonic spline and thin-plate spline [23]. Generally, a thin-plate spline function is mostly used in time series modelling, whereas a Gaussian function is preferred with pattern classification problems. The Gaussian function has been used as a standard function for the radial basis function neural network in MATLAB. By denoting R the number of inputs while Q the number of outputs, the output of RBFNN, e.g. for $Q = 1$, is calculated as

$$\eta^1(x,w) = \sum_{k=1}^{S1} w_{1k}\phi\left(\|x-c_k\|_2\right), \tag{1}$$

where $x \in \Re^{R\times 1}$ is an input vector, $\phi(\cdot)$ is a basis function which $x - c_k = r$ is the scalar radius, $\|\cdot\|_2$ denotes the Euclidean norm, w_{1k} are the weights in the output layer, $S1$ is the number of neurons (and centers) in the hidden layer and $c_k \in \Re^{R\times 1}$ are the RBF centers in the input vector space. Equation (1) can also be written as,

$$\eta(x,w) = \phi^T(x)w \tag{2}$$

where

$$\phi^T(x) = \left[\phi_1\left(\|x-c_1\|\right) \quad \cdots \quad \cdots \quad \phi_{S1}\left(\|x-c_{S1}\|\right)\right] \tag{3}$$

and

$$w^T = \left[w_{11} \quad w_{12} \quad \cdots \quad w_{1S1}\right] \tag{4}$$

The output of the neuron in a hidden layer is a nonlinear function by means of Gaussian function that is given by:

$$\phi(x) = e^{-\frac{\|x-c_k\|^2}{\sigma^2}}, \tag{5}$$

where is the spread parameter of the RBF. For training, the least squares formula was used to find the second layer weights while the centres are set using the available data samples.

Advantages of RNFNN over the multilayer perceptrons include the ability to effectively generate multidimensional interpolative approximations, to yield robustness and reliability in a computationally-aided design. As compared to the conventional method, which typically incorporates with a fixed algorithm in order to solve a particular problem, the RBFNN metamodel will flexibly learn the nonlinear mapping between the input and output of the system, while solving that the inversion problem in the context of ER geophysical surveying.

In order to achieve a good performance of the designed network as well as fast convergence during the training stage, the selection of input and output data sets, a training method and test examples is crucial. In this work, we design the RBFNN metamodel by selecting apparent resistivity horizontal position (x-axis), apparent resistivity vertical position (z-axis) and apparent resistivity value (ohm.meter) as the input to the network. The true resistivity (ohm.meter) is used as the target output of the network. Synthetic data are generated by using a finite element forward modelling code by means of the RES2DMOD simulation software [24]. It is a 2-D forward modelling program which calculates the apparent resistivity pseudosection for a user defined 2-D subsurface model. Here, the finite-element algorithm [25] is chosen to calculate the apparent resistivity values as the synthetic data in this study. There are two generated outputs from the RES2DMOD in the forms of (*.dat) and (*.txt). The first form consists of a matrix with four columns, horizontal distance, electrode spacing, number of data levels and apparent resistivity value. By using a prepared MATLAB m-file, the four-column matrix is converted to a three-column matrix (comprising horizontal location and depth of a datum and respective apparent resistivity) to become the input matrix of the RBFNN. For the second form, it consists of a matrix with three columns, x-location, z-location and respective true resistivity value. Generally, the size (number of rows) is not equal for both forms. Hence, by using another prepared MATLAB m-file, this form will be converted to a one-column matrix (for target resistivity value) which is the target matrix of the RBFNN, having the same size as with the input matrix. Several test data sets are also generated from the forward modelling. The synthetic data sets (input data, target data and test data) are then normalised to the range of [0, 1] by using the mapminmax function in MATLAB, to allow the Gaussian activation function to squash all incoming data and to make the computer model execution more efficient.

Next, the RBFNN will be trained with several training iterations to achieve a prescribed mean square error (MSE) threshold. The designed network is tested on several test sets, which have not been used during the training stage. To review this section, the proposed algorithm for the 2-D resistivity mapping is summarised as follows:

1. Define the initial input data space, P, which consists of a set of three-column matrices from the (*.dat) file of the forward modelling.
2. Define the initial target data space, T, which consists of a set of one-column matrices from the (*.txt) file of the forward modelling.
3. Normalise the input and output of the training data set, and define them as P' and T'.
4. Fit the RBFNN using P' and T' until reaching the prescribed MSE threshold.
5. Define a testing data set, R.
6. Evaluate the designed network in (4) using the normalised testing data sets, R'.
7. Calculate system error, which is the difference between output in (6) and the target testing data. If the error is less than another prescribed goal, stop the process and plot the inversion results. Otherwise go to step 8.
8. Set the new iteration number. Add a new input and target data set (by changing the anomalous body location and distance between electrodes) to the previous training set, and go to step 3.

■4.0 RESULTS AND DISCUSSION

4.1 Generation of Synthetic Data

In this example, we evaluate the RBFNN metamodel in inverting the 2-D resistivity imaging data generated by using a hybrid Wenner-Schlumberger configuration [3]. In this study, in order to collect the synthetic data, we assume 36 electrodes of Wenner-Schlumberger arrays were used with several electrode separations: 1 m, 1.5 m, 2 m, 2.5 m, 3 m, 3.5 m and 4 m. The medium was homogeneous of 100 Ω.m with the embedded anomalous body of 1000 Ω.m, as typically depicted in Figure 2, where the location of the buried object was changeable within the range of the homogenous medium to generate several inputs and target outputs for the training data set. Seven (07) training data sets were generated by using the RES2DMOD software to provide apparent resistivity values correspondingly as in real-world measurements. Each data set consists of 240 datum points with a possible larger number could be iteratively added to be more sufficient to train the mapping. The data set arrangement for training and testing is shown in Table 1.

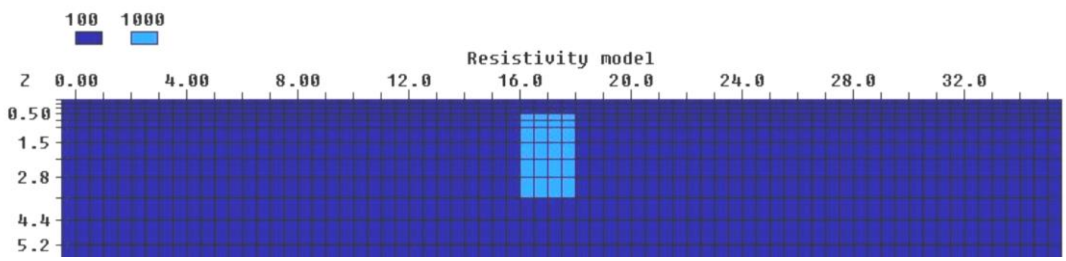


Figure 2 Model used to generate synthetic resistivity data set

Table 1 The arrangement of the training and testing data sets (un-normalised)

Training data sets					Test data sets				
i	Input data set, *P*			Target data set, *T*	i	Input data set, P_R			Target data set, T_R
	x-location (m)	*z*-location (m)	Apparent resistivity (Ω.m)	True resistivity (Ω.m)		x-location (m)	z-location (m)	Apparent resistivity (Ω.m)	True resistivity (Ω.m)
1	1.5	0.075	95.0654	100	1	1.5	0.075	85.0654	100
2	2.5	0.075	75.8188	100	2	2.5	0.075	45.8188	100
3	3.5	0.075	140.4341	1000	3	3.5	0.075	50.4341	100
…	…	…	…	…	…	…	…	…	…
240	…	…	…	…	240	…	…	…	…
Total number of data for 7 training sets = 1680					Total number of data for each test set = 240				

The RBFNN is designed with 3 radial basis centres, added one by one until reaching a prescribed error goal, set at 0.001. The spread parameter is finely tuned at 0.3 for best output results. Once the training stage has terminated upon reaching the mean square error (MSE) goal, the network is tested by using other data sets that were not used for training. The time required for completing the inversion process (Step 1 to 7 as described previously) is about 25 seconds on an INTEL® Core i3 PC. Notably, the technique can perform the inversion on the test data in only a few seconds without any more training.

4.2 Analysis and discussion

The network performance was verified using several synthetic 2-D data sets. An example of the results from the synthetic data is depicted in Figure 3. The top figure shows in pseudosection the apparent resistivity values collected by using finite element forward modelling in the RES2DMOD software.

The 2D resistivity inversion results are shown in Figure 3(b). Resistivity of the actual model, an anomalous body of 1000 Ω.m, is shown in Figure 3(c) for comparison. It is obvious that the subsurface object can be located by using the proposed method. Furthermore, the outer shape of the object is almost correlated to the actual model with the dot-line showing the exact location and dimension as referred to the actual model. The predicted anomalous body has a small blank area and the horizontal dimension is slightly wider than the actual dimension, however the location of the body is exactly the same.

To evaluate the performance of the proposed method, a comparison study is included here with results obtained from a conventional 2-D resistivity inversion technique based on a least square algorithm, available in the RES2DINV software [24] with the latest version, RES2DINV ver. 3.59, whereby it is claimed to give the inversion closely corresponding to reality.

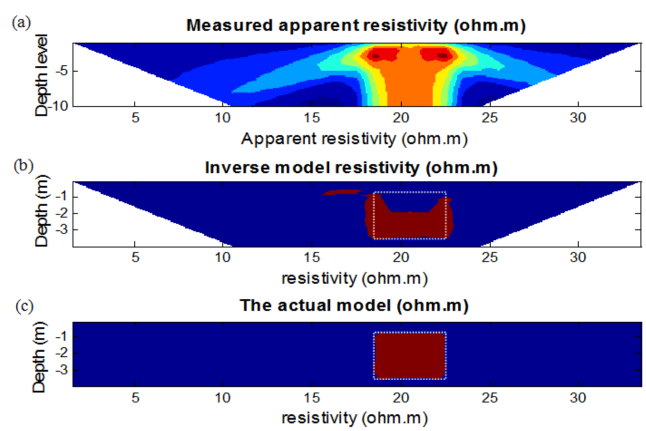


Figure 3 The inversion using synthetic data; (a) Apparent resistivity from the forward model, (b) Inversion result using RBFNN metamodel, (c) The actual test model

The comparison results are illustrated in Figure 4. There are three anomalies involved in this investigation. As can be seen, both inversion methods are able to locate correctly the position of the anomalies. We also determined the approximate vertical and horizontal dimensions of the anomalies, as presented in Table 2. The results show that both methods could provide mostly the right dimension as compared to the actual parameters, however, on average our proposed method contribute an average error of

20% less than the conventional method using the mentioned software.

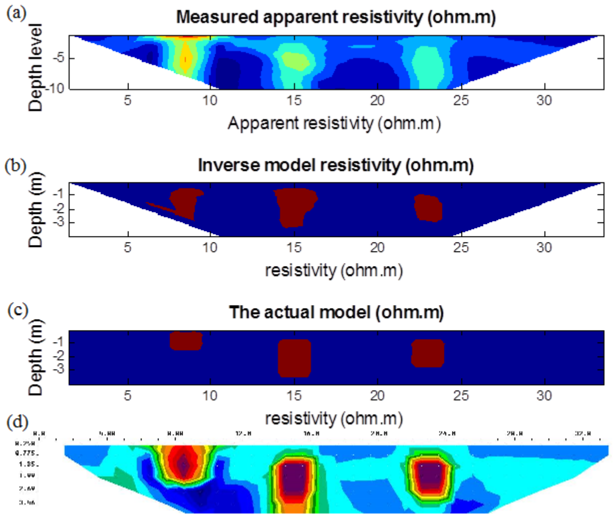


Figure 4 Comparison results (a) Apparent resistivity from the forward model, (b) Inversion result using RBFNN metamodel, (c) The actual test model (d) Inversion result using RES2DINV ver. 3.59 (least square with the smoothness-constrained)

Table 2 Comparison for the dimension from the inversion results (as in Figure 4)

	Methods	**Horizontal dimension (m)**	**% error**	**Vertical dimension (m)**	**% error**
Anomalous *A*	RBFNN	1.95	2.63	1.07	23.57
	Conventional	1.72	9.47	1.73	23.57
	Actual parameter	**1.90**		**1.40**	
Anomalous *B*	RBFNN	2.07	0.00	2.47	9.52
	Conventional	1.78	14.01	2.27	16.85
	Actual parameter	**2.07**		**2.73**	
Anomalous *C*	RBFNN	1.61	22.22	1.93	6.76
	Conventional	1.84	11.11	1.87	9.66
	Actual parameter	**2.07**		**2.07**	

■5.0 CONCLUSION

We have presented a radial basis function neural network metamodel for 2D resistivity mapping used in geophysical surveying. In our study, apparent resistivity values from known models are used for the inputs and the targets to train the proposed network. Testing with synthetic data indicate the ability of the proposed approach to converge between inputs and targets in a computationally-efficient manner and to exhibit good performance in the data inversion. The accuracy and efficiency of the proposed method is determined by comparing it with an existing conventional method using the least square with the smoothness-constrained and commercial software. The horizontal and vertical dimensions of the anomalous object, on average, are recovered rather accurately.

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