

MATHEMATICAL MODEL OF SINGLE AND TWO COMPARTMENTAL
INDOOR AIR QUALITY

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To my beloved family.

Samsudin Bin Melan

Saadiah Binti Seis

Nurulhuda Bt Samsudin

Khairul Fadzli Bin Samsudin

Khairunnajwa Bt Samsudin

Muhd. Khairul Hakim Bin Radiman

Muhammad Luthfi Hakim Bin Muhd. Khairul Hakim

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ABSTRACT

Smoking does not only affect the smoker's health, but it also greatly influences the surrounding atmosphere, indoor and outdoor. Indoor air quality (concentration of carbon monoxide) in a compartmental room or building is a very important element to ensure the health and comfort level of the occupants. This study presents a mathematical model of two compartmental indoor air quality. The model is based on system of differential equation. Hence, the solutions is obtained by using the method of Laplace transform. The concentration of the pollutants are calculated and then compared with experimental data.

ABSTRAK

Merokok bukan sahaja memberi kesan buruk kepada kesihatan perokok, tetapi ia juga akan mempengaruhi persekitaran atmosfera, dalaman dan luaran. Kualiti udara dalaman (kepekatan karbon monoksida) di dalam ruangan bilik atau bangunan adalah elemen yang sangat penting untuk memastikan tahap kesihatan dan keselesaan penghuni bangunan. Kajian ini memperihalkan suatu model matematik bagi dua ruangan bahagian kualiti udara dalaman. Model ini adalah berdasarkan sistem persamaan pembezaan. Seterusnya, penyelesaian model tersebut diperoleh dengan menggunakan kaedah "Laplace transform". Kepekatan bahan pencemaran yang dikira kemudian dibandingkan dengan data eksperimen.

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LIST OF ABBREVIATION AND SYMBOLS

w_{OA}	-	The outdoors to room A airflow rate
w_{AO}	-	The airflow rate from room A to the outdoors
w_{BO}	-	The airflow rate from room B to the outdoors
w_{OB}	-	The outdoors to room B airflow rate
w_{AB}	-	Forward interzonal flow rate
w_{BA}	-	Reverse interzonal flow rate
v_A	-	Air volume of Room A
v_B	-	Air volume of Room B
ϕ_A	-	Global air change rate of Room A
ϕ_B	-	Global air change rate of Room B
α_{AB}	-	Proportion of Room B's intake air coming from Room A
α_{BA}	-	Proportion of Room A's intake air coming from Room B
α_{OA}	-	Proportion of Room A's intake air coming from outdoors

α_{OB}	-	Proportion of Room B's intake air coming from outdoors
NHAPS	-	National Human Activity Pattern Survey
IAQ	-	Indoor Air Quality
EPA	-	Environmental Protection Agency
ETS	-	Environmental Tobacco Smoke
CO	-	Carbon monoxide
VOCs	-	Volatile organic compounds

CHAPTER 1

INTRODUCTION

1.0 Background of the study

Majority of people spend large portion of time indoors or in the house. According to Klepeis *et al.* (2001), 68.7% spent their time in a residence, 7.6% outdoor, 5.5% in a vehicle, 5.4% office factory, 11% other indoor location and 1.8% in bar restaurant. Moreover, the total percentage time spent with a smoker in a residence is 42.7% compare to the outdoor that is 14.7%. Figure 1 show a pie chart showing the mean of time the NHAPS (National Human Activity Pattern Survey) respondents spent in six different locations on the diary day (weighted) with 9196 sample.

We work, study, eat, drink and sleep in enclosed environments where air circulation may be restricted. There are many sources of indoor air pollution. Tobacco smoke, cooking and heating appliances, and vapors from building materials, paints, wet or damp carpet, cabinetry or furniture made of certain pressed wood products, will cause pollution inside the house.

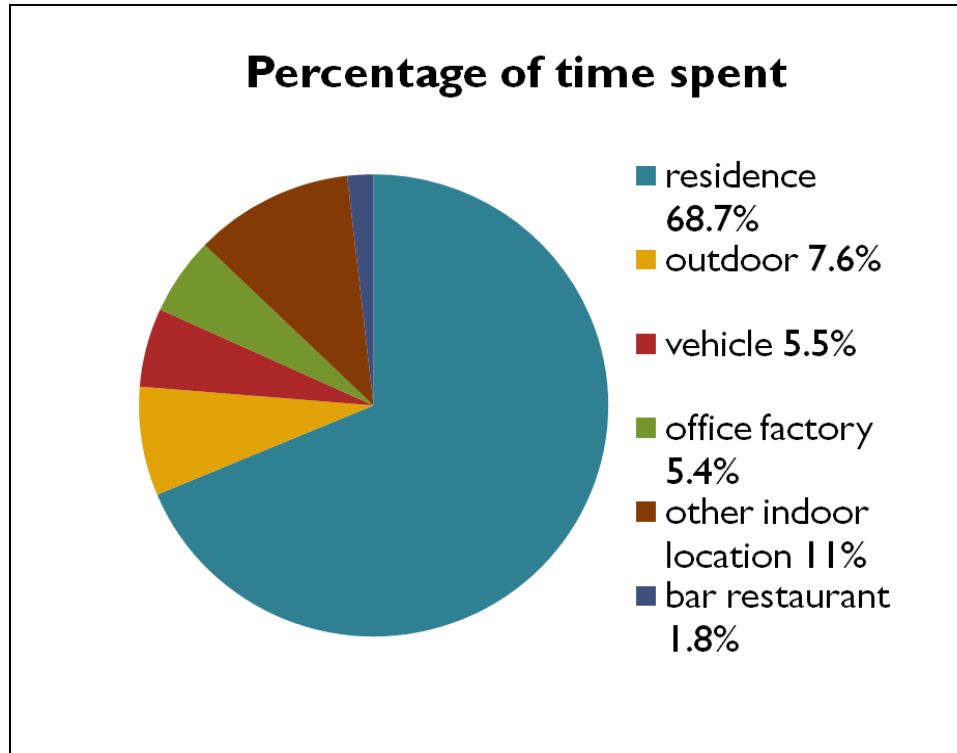


Figure 1.0: Pie chart showing the percentage of time spent

Indoor air quality (IAQ) is a term which refers to the air quality within and around buildings and structures, especially as it relates to the health and comfort of building occupants. Indoor environments can have pollutant levels higher than outdoor, as reported by Environmental Protection Agency EPA (1997).

Quality of outdoor air and emissions from the indoor environment and the buildings' occupants has made the indoor air quality affected by type and performance of heating, air conditioning, and ventilation technology, Jakola *et al.* (1994). According to Hodgson (2002), indoor environment in a restricted space is a complex and dynamic combination of physical, biological, and chemical factors that can affect the humans' health and physical reactions anytime whether we realize it or not.

Environmental tobacco smoke (ETS) is the smoke emitted from the burning of a cigarette, pipe, or cigar, and smoke inhaled by a smoker. It is a complex mix of more than 4000 chemical compounds, containing many known or suspected carcinogens and toxic agents, including particles, carbon monoxide, and formaldehyde.

The degradation of indoor air quality by harmful chemicals and other material, can be many times worse than outdoor air pollution. This is because contained areas enable potential pollutants to build up more than open spaces do. If we consider that people spend approximately 90 percent of their time indoors, and around 65 percent inside their homes in particular, we can see why indoor air pollution is an important issue.

1.1 Statement of problem

Smoking, cooking, consumer products, gas appliances, and building materials are some of indoor sources that can be found in home. Because of small volumes and low air change rates of most homes, indoor pollutant concentration from smoking activity in a home can be very high. So, it can give a bad exposure for human's breathing because the air was polluted.

Thus, this study is concerned with the prediction of indoor air pollutant concentrations from smoking activity and how much the exposure of the environmental tobacco smoke concentrating of carbon monoxide (CO).

1.2 Objectives of the research

The objectives of this research are;

- 1) To develop one and two compartmental model of indoor air pollutant concentration from smoking activity.
- 2) To solve one and two compartmental model of indoor air pollutant concentration.
- 3) To analyze and interpret the solution of one and two compartmental model obtained by Laplace transform method.

1.3 Scope of the research

In this study we will considered two compartmental indoor air quality from the research that have been done by Ott *et al.* titled ‘Analytical Solutions to Compartmental Indoor Air Quality Models with Application to Environmental Tobacco Smoke Concentrations Measured in a House’. The model will be based on system of differential equations.

1.4 Significance of the study

The smokers should make every effort to keep cigarette smoke away from the non-smokers especially in a house or building because inhaling the smoke can affected

the others too. Living with a smoker increases a non-smoker's chances of developing lung cancer by twenty percent to thirty percent, this fact is according to the United State Surgeon General.

The significance of the study is to provide better understanding and valuable information on how indoor air pollutant from smoking activity in a home can give bad effect to occupant's health due to its concentration from cigarette smoke that can persist for a long period of time.

Thus, smoking activities are better done outside the house to ensure the adverse effects of cigarette smoke inhaled by other occupants in the house can be avoided.

1.5 Dissertation Organizations

This study contains seven chapters started with introductory chapter. First chapter described briefly about the research background, problem statements, objectives, scope and significance of this study.

Literature review of this study will be considered in chapter two. This chapter explained briefly about all previous studies or research that has been done and related with the current project including the theories, models, method and figures that may support this project.

Then, chapter three discuss methodologies and procedure in completing this study. Next, the results and analysis of data for single compartment and two compartments are discussed in the fourth and fifth chapter. In chapter six, we discussed

the results and analysis of data for two compartments. Finally, some suggestions and conclusion of the study will be recommended in chapter seven.

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