NUMERICAL STUDY OF EKMAN BOUNDARY LAYER AND HEAT TRANSFER IN POROUS MEDIUM USING KELLER-BOX

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To Tears of my Mother and To Silence of my Father. For the patience of my wife

To my Soul (Ayham and Mohmmad).

To my brothers and sisters, and my friends who gave me all help.

To my beloved supervisor who gave me support and scientific assistance.

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ABSTRACT

In this research, the Ekman boundary layer flow and heat transfer in porous medium with large value of suction parameter is studied. The governing equations which are the momentum equation and energy equation are derived based on the principle of conservation law. The obtained dimensional governing equations are transformed into non-dimensional equations by using appropriate non-dimensional variables. The numerical solutions of the non-dimensional governing equations are obtained by using the implicit finite difference scheme known as the Keller-Box method. These numerical results of primary velocity, secondary velocity, and temperature profiles are displayed and analyzed through graphs. The solutions obtained satisfy all imposed boundary conditions. Results for wall shear stress and heat coefficient are displayed and analyzed through graphs and tables. The results show that, primary velocity increases with increasing Grashof number and Eckert number, while it decreases with increasing Prandtl number and permeability parameter. The secondary velocity increases with increasing of suction parameter, while the effect of Ekman number gives the opposite behavior. The fluid temperature is increasingly affected by Prandtl number. There is no effect of rotation on temperature. The primary shear stress at the wall increases in case of strong values of Grashof and Eckert numbers, while it decreases with the rise of Prandtl number, suction and permeability parameters. The secondary shear stress at the wall increases with rising suction parameter, while it decreases with increasing of Ekman number.

ABSTRAK

Dalam kajian ini, aliran lapisan sempadan Ekman dan pemindahan haba dalam bahantara berliang dengan nilai parameter sedutan yang besar dikaji. Persaman menakluk yang terdiri daripada persamaan momentum dan persamaan tenaga diterbitkan berdasarkan hukum prinsip keabadian. Persamaan berdimensi yang diperoleh diubah menjadi persamaan tidak berdimensi dengan menggunakan pembolehubah tidak berdimensi yang bersesuaian. Penyelesaian berangka bagi persamaan tidak berdimensi diperoleh dengan menggunakan skema beza terhingga tersirat yang dikenali sebagai kaedah kotak-Keller. Keputusan berangka bagi profil halaju utama, profil halaju sekunder dan profil suhu dipaparkan dan dianalisis melalui graf. Penyelesaian yang diperoleh memenuhi semua syarat sempadan yang dikenakan . Keputusan bagi tegasan ricih dinding dan pekali haba dipaparkan dan dianalisis melalui graf dan jadual. Keputusan menunjukkan bahawa, halaju utama meningkat dengan peningkatan nombor Grashof dan nombor Eckert, manakala berkurangan dengan peningkatan nombor Prandtl dan parameter kebolehtelapan. Halaju sekunder meningkat dengan peningkatan parameter sedutan, manakala kesan nombor Ekman menunjukkan fenomena yang bertentangan. Nombor Prandtl memberikan kesan yang mendalam terhadap suhu bendalir. Tiada kesan putaran pada suhu. Tegasan ricih utama di dinding meningkat bagi kes nilai nombor Grashof dan nombor Eckert yang besar, manakala berkurangan dengan peningkatan nombor Prandtl , parameter sedutan dan parameter kebolehtelapan. Tegasan ricih sekunder di dinding meningkat dengan peningkatan parameter sedutan, manakala berkurangan dengan peningkatan nombor Ekman.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

θ		Dimensionless temperature [–]
μ		Dynamic viscosity of the fluid [$kgm^{-1}s^{-1}$]
ρ		Density of the fluid [kgm^{-3}]
η		Dimensionless coordinate normal to the surface [–]
ψ		Stream function [m^2s^{-1}]
τ_x		Shearing stress at the wall [N/m^2]
C_f	-	Skin-friction coefficient
C_P		Specific heat at constant pressure [$Jkg^{-1}k^{-1}$]
E	-	Ekman number
E_c	-	Eckert number
F	-	Dimensionless stream function
f_w	-	Transpiration parameter
g	-	Acceleration due to gravity [ms^{-2}]
Gr	-	Grashof number
k	-	Thermal conductivity [$wm^{-1}k^{-1}$]
Nu	-	Nusselt number [–]
Pr	-	Prandtl number [–]
q_c	-	Conduction heat flux [w/m^2]
q_r	-	Radiative heat flux [w/m^2]
q_w	-	Heat flux at the surface [w/m^2]
Rd	-	Radiation parameter [–]
T	-	Temperature of the fluid in the boundary layer [K]
T_∞	-	Temperature of the ambient fluid [K]

T_w	-	Temperature at the surface [K]
U	-	Velocity component along the surface [ms^{-1}]
u	-	Dimensionless velocity along the surface [–]
V	-	Velocity component normal to the surface [ms^{-1}]
v	-	Dimensionless velocity normal to the surface [–]
X	-	Coordinate along the surface [m]
Y	-	Coordinate normal to the surface [m]
B	-	Coefficient of thermal expansion [K^{-1}]
N	-	Kinematic viscosity [m^2/s]
Ω	-	Angular velocity

LIST OF APPENDICES

CHAPTER 1

1.0 Introduction

The purpose of this chapter is to present the background of this research. Also, in section 1.1 the problem statement and objectives of this study will be presented. Additionally, the limitations and scope will be discussed in the following sections. In section 1.5 the significant of this research will be discussed. Finally, various expected result will be concluded in section 1.6.

1.1 Study Background

The transformation of body around fixed point is called rotation. Rotating fluid is similar to rigid body rotation, the rotation reference frame is non inertial reference that is rotating reference to inertial frame. As an earth example, surface is rotating reference frame.

In fluid mechanics, a boundary layer is the layer of fluid in the immediate nearness of a bounding surface where the effects of viscosity are considerable. Boundary layer theory is the platform of modern fluid dynamics, found by Ludwig Prandtl in 1904. It gives a physical explanation of the flow of air, sea, ocean and other fluids of small viscosity under circumstances of interest in many engineering applications.

There are many kind of boundary layer. In the Earth's atmosphere, the planetary boundary layer is the air layer near the ground affected by diurnal heat, moisture or momentum transfer to or from the surface. On an aircraft wing the boundary layer is the part of the flow close to the wing, where viscous forces distort the surrounding non-viscous flow. Laminar boundary layers can be loosely classified according to their structure and the circumstances under which they are created. The thin shear layer which develops on an oscillating body is an example of a Stokes boundary layer, while the Blasius boundary layer refers to the well-known similarity solution near an attached flat plate held in an oncoming unidirectional flow. In the theory of heat transfer, a thermal boundary layer occurs. A surface can have multiple types of boundary layer simultaneously.

The Ekman layer is the layer in a fluid where there is a force balance between pressure gradient force, Coriolis force and turbulent drag. It was first described by Vagn Walfrid Ekman in the 1902 in his doctoral thesis. The Ekman layer is a kind of viscous boundary layer in a rotating system, and refers to the area to which force applied to a horizontal boundary is transmitted. The thickness of this boundary layer is given by a depth to which the force is transmitted during one revolution of the earth, so as the viscosity coefficient grows it becomes thicker, and as the effect of rotation grows it becomes thinner.

When fluid move within a reference frame that is rapidly rotating, they are subject to new (gyroscopic) constraints that can significantly affect the behavior of the flow. In particular, this can introduce an anisotropic “stiffness” that can make the flow almost two-dimensional under some circumstances. This is of particular relevance in some engineering contexts (turbo machinery etc.) and also for atmospheres and oceans (for which the planetary rotation may be important).

Two dimensional flow models are interested to study, because fluid flows give researcher a lot of signs and behavior of flows in three dimensional which are more realistic.

In the Ekman layer caused by winds blowing across the sea's surface, the viscosity coefficient is not dependent on depth. In ocean theory the Ekman layer is horizontal boundary layer in a rotating fluid. Such layers exist at the top and bottom of the ocean and at the bottom of the atmosphere. In this study, we consider steady equations of motion for an unstratified geophysical flow of uniform depth in a rotating coordinate frame.

Mixed convection is the combination between forced and natural convection in boundary layer flow. Natural convection or also known as a free convection is caused naturally such as buoyancy effects due to density differences. This type of convection happens because of the temperature variations in the fluid. An example of natural convection is the cooling process in heat exchanger components. Meanwhile ,forced convection occurs when a fluid flow is induced by an external force such as pump, fan or mixer.

Convention flow due to thermal diffusion has received widespread attention due to the importance of heat transfer in engineering processes such as in petroleum and geothermal processes, drying, moisture migration in fibrous insulation, nuclear waste disposal and in the control of pollutant spread in ground water.

A porous medium is solid or matrix permeated by an interconnected network of pores filled with fluid. Porous media are broadly used in high temperature heat exchangers, turbine blades, jet nozzles, etc. In practice, cooling of porous structure is achieved by forcing the liquid or gas through capillaries of solid.

Actually, porous medium are used to insulate a heated body to maintain its temperature. Porous media are considered to be useful in diminishing the natural convection which would otherwise occur intensely on a vertical or horizontal heated surface. In order to make heat insulation of surface more effective, it is necessary to study the convection flow through a porous medium and to estimate its effect in heat transfer. Study of origin of flow through porous media is heavily based on Darcy's experimental law (Das *et al.*, 2006).

The mechanism of heat transfer due to the fluid motion is known as convection heat transfer. Convection heat transfer in porous media has been studied extensively for over 150 years now (Bhattacharya *et al.*, 2002). Mahdi *et al.* (2013) explained that, convection heat transfer in porous media have many theoretical and practical studies and all of these studies focused to show the effect of buoyancy phenomenon on the behavior of the flow and temperature fields through porous media.

The study of motion for viscous incompressible fluid has considerable interest in recent year due to its wide applications in cosmically, geophysical fluid dynamics and meteorology. The large scale and moderate motions of the atmosphere are greatly affected by vorticity of the earth's rotation. The motion in the earth's core is somehow responsible for the main geomagnetic field. It has been seen that, when the fluid is rotating near a flat plate, the pressure field of the flow far away from the plate also exists near the plate, but the Coriolis force near the plate is reduced owing to friction force. As a result, there exists a flow in the direction in which the pressure is falling until the Coriolis forces are compensated by viscous forces. Such a layer formed near the plate is known as Ekman layer (Manna *et al.*, 2007).

The deference of the effective factors on the heat transfer and fluid through porous media led to diversity of studies in this field, these factors are:

- Boundary conditions in porous media, which means either porous media penetrative as open cell aluminium foam or non-penetrative as closed cell aluminium foam.
- Thermal conditions in convection heat transfer, which means either convection heat transfer with constant temperature or with constant heat flux, or both together.
- Porous media shapes are either rectangular or triangular.
- Working fluid types are nanofluid (Al_2O_3+water), (SiO_2+water) or conventional fluid (air, water, oil, …).
- convection heat transfer types, free (natural), forced or mixed convection
- The method of data processing means numerical, analytical, or experimental.

1.2 Problem Statement

The aim of this study is to investigate the classical Ekman boundary layer and heat transfer through porous medium with large suction along semi-infinite flat plate. To solve this problem, they are questions need to be explored.

i. How to derive the conservation equations?
ii. How to solve the governing equations?
iii. How to analyse the relations of parameters inside the boundary layer?
iv. What is the behavior of velocity and temperature profiles when suction is very large?

1.3 Objectives of the Study

The objectives of this study:

1. To obtain the continuity, momentum and energy equations, with principle of conservation, boussinesqs approximation, and boundary layer approximation.
2. To obtain numerical results of the velocity and temperature profiles for non-dimensional equations by using finite difference method Keller-box scheme.
3. To analyze and discuss the velocity and temperature profiles as well as wall shear stress and heat transfer coefficient that have been affected by various parameters such as Prandtl number, Ekman number, Eckert number and permeability parameter.

1.4 Scope of the Study

The present study will focus on Ekman boundary layer mixed convective heat transfer through a porous medium with large suction. The plate is semi-infinite horizontal and non conducting. Newtonian fluid involved viscous and incompressible and steady state. All assumptions considered in a rotating reference frame. The equations will be solved numerically using Keller box method and the results obtained will be analyzed in form of graphs.

1.5 Significant of Study

Since we are living in the rotating system, the flood phenomenon, global warming and rising of the sea level have directly impact on the humans. Therefore this study can give assistant by explaining and predicting on how to reduce the existing phenomenon. Moreover, the researches in heat transfer of rotating system can give indications on nuclear contamination which directly damage the marine organisms.

In addition many applications related to this study, for examples but not limited, the oil engineer must understand the heat transfer in rotating reference frame to deduct the moment of oil through the tank. Ekman boundary layer exhibit in computer hard disc drives. Therefore, this study can help the computers manufactures on preventing dust deposition by using Ekman boundary layer (Tzeng *et al.* (1994).

1.6 Expected Results

According to the objectives of this study, the expected results are:

1. The governing equations, two dimensional Navier-stokes equations in rotating fluid which mean Ekman boundary layer will be derived.
2. The approximate dimensionless governing equations will be obtained.
3. The numerical solution of approximate non-dimensional governing equations will be determined.
4. The determined solution will be represented in the form of graphs for different value of non-dimensional parameters.

5. The influence of parameters to the velocity and temperature of fluid as well as wall shear stress and heat transfer coefficient will be observed and discussed.

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